THE ANALYTIC TOPOLOGY SUFFICES FOR THE B_{dR}^+-GRASSMANNIAN

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ABSTRACT. The B_{dR}^+-affine Grassmannian was introduced by Scholze in the context of the geometric local Langlands program in mixed characteristic and is the Fargues–Fontaine curve analogue of the equal characteristic Beilinson–Drinfeld affine Grassmannian. For a reductive group G, it is defined as the étale (equivalently, v-) sheafification of the presheaf quotient LG/L^+G of the B_{dR}-loop group LG by the B_{dR}^+-loop subgroup L^+G. We combine algebraization and approximation techniques with the known cases of the Grothendieck–Serre conjecture to show that the analytic topology suffices for this sheafification, more precisely, that the B_{dR}^+-affine Grassmannian agrees with the analytic sheafification of the aforementioned presheaf quotient LG/L^+G.

1. The B_{dR}^+-affine Grassmannian

For a reductive group G over a ring R, the Beilinson–Drinfeld affine Grassmannian Gr_G plays an important role in the geometric Langlands program, as well as in other fields that feature reductive groups and their torsors. Letting LG (resp., L^+G) be the *loop functor* (resp., its *positive loop subfunctor*) that sends a variable R-algebra A to $G(A((t)))$ (resp., to $G(A[[t]])$), the affine Grassmannian Gr_G is defined as the étale (equivalently, fpqc) sheafification of the presheaf quotient LG/L^+G. By the recent results [Čes22, Theorems 1.7 and 1.8], based on the study of G-torsors over $\mathbb{A}^1_A$, Zariski sheafification gives the same result and, if G is totally isotropic (for instance, quasi-split), then no sheafification is needed at all: then Gr_G already agrees with the presheaf quotient LG/L^+G.

In his Berkeley lectures [SW20], Scholze adapted the definition of Beilinson–Drinfeld to the then-emergent geometric *local* Langlands program, and subsequently with Fargues applied it in their elaboration of this program in [FS21]. To review his definition, we let K be a nonarchimedean local

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field, let G be a smooth affine group scheme defined either over K or over its ring of integers $\mathcal{O}_K$, and recall that the B_{dR}^+-affine Grassmannian $\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}$ is a functor on the category of perfectoid $\mathcal{O}_K$-algebra pairs (A, A^+). For such a pair, we let $(A^\flat, A^{\flat+})$ denote its tilt, let $\varpi^\flat \in A^{\flat+}$ be a pseudouniformizer such that $\omega := (\omega^\flat)^\sharp$ satisfies $\omega^p \mid p$ in A^+ (see [SW20, Lemma 6.2.2] or [ČS21, Section 2.1.2]), let $W_{\mathcal{O}_K}(A^{\flat+})$ denote the $\mathcal{O}_K$-ramified Witt vectors of $A^{\flat+}$, and let

$$I := \mathrm{Ker}\left(W_{\mathcal{O}_K}(A^{\flat+}) \twoheadrightarrow A^+\right)$$

be the kernel of the Fontaine $\mathcal{O}_K$-algebra map sending any Teichmüller $[a]$ to $a^\sharp$ (compare with [ČS21, Equation (2.1.1.1)]). In the B_{dR}^+ context, the role of the formal power series ring $A[\![t]\!]$ is played by

$$B_{\mathrm{dR}}^+(A) := \varprojlim_n (W_{\mathcal{O}_K}(A^{\flat+})[\tfrac{1}{[\omega^\flat]}]/I^n)$$

compare with [SW20, page 138]. This notation is slightly abusive because $B_{\mathrm{dR}}^+(A)$ does depend on K, although not on A^+ nor on $\varpi^\flat$. The ideal I is principal, generated by a nonzerodivisor that is functorial in (A, A^+), and the role of the Laurent power series ring $A(\!(t)\!)$ is played by

$$B_{\mathrm{dR}}(A) := B_{\mathrm{dR}}^+(A)[\tfrac{1}{I}].$$

In the following special cases, we can be slightly more explicit.

(1) The field K is of characteristic 0 and A is a K-algebra: then we may choose $\varpi^\flat$ such that ϖ^p is a unit multiple of p (see [ČS21, Section 2.1.2]), so $B_{\mathrm{dR}}^+(A)$ and $B_{\mathrm{dR}}(A)$ are K-algebras.

(2) The field K is of characteristic 0 and A is an algebra over its residue field k: then $A^{\flat+} \cong A^+$, so $I = (\pi)$ where $\pi \in \mathcal{O}_K$ is a uniformizer, and $B_{\mathrm{dR}}^+(A) = W_{\mathcal{O}_K}(A)$ with $B_{\mathrm{dR}}(A) = W_{\mathcal{O}_K}(A)[\frac{1}{\pi}]$.

(3) The field K is of characteristic $p > 0$. Then $\mathcal{O}_K \simeq k[\![\zeta]\!]$, the functor $W_{\mathcal{O}_K}(-)$ is the completed the tensor product over k with $k[\![\zeta]\!]$, and $B_{\mathrm{dR}}^+(A) \simeq A[\![\pi - \zeta]\!]$ with $B_{\mathrm{dR}}(A) \simeq A(\!(\pi - \zeta)\!)$, where $\pi \in \mathcal{O}_K$ is a uniformizer; in terms of this presentation, the ideal I is generated by $\pi - \zeta$. For the sake of uniformity of discussion, we do not exclude this case, but it will offer nothing new relative to the setting of the Beilinson–Drinfeld affine Grassmannian reviewed above.

We stress that the cases (1)–(3) are nonexhaustive, see, for instance, [SW20, Example 6.1.5 4.].

In the B_{dR}^+ context, the *loop functor* (resp., its *positive loop subfunctor*) is defined by

$$LG\colon (A, A^+) \mapsto G(B_{\mathrm{dR}}(A)) \quad \text{(resp., by} \quad L^+G\colon (A, A^+) \mapsto G(B_{\mathrm{dR}}^+(A))),$$

granted either that G is defined over $\mathcal{O}_K$ or that one restricts to (A, A^+) with A a K-algebra. We have reused the notation LG and L^+G because the power series context will henceforth play no role.

The B_{dR}^+-*affine Grassmannian* $\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}$ is defined as the étale sheafification of the presheaf quotient LG/L^+G, compare with [SW20, Definition 19.1.1]. Our goal in this article is to show that for reductive G, the sheafification for the much coarser analytic topology gives the same result, namely, that $\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}$ agrees with the analytic sheafification of the presheaf quotient LG/L^+G, see Theorem 3.1 below.

In the view of the analogy with the Beilinson–Drinfeld affine Grassmannian, it could be that, at least for totally isotropic G, no sheafification is needed at all, namely, that $\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}$ is the presheaf quotient LG/L^+G. However, we do not know how to approach this, nor how to find a counterexample.

2. The modular description

Our proof that the analytic sheafification suffices will hinge on the following modular description of the B_{dR}^+-affine Grassmannian. This modular description has already been given in [SW20, Proposition 19.1.2], although we prefer the slightly different argument for it given below.

Proposition 2.1. *For a nonarchimedean local field K and a smooth affine group scheme G defined either over K or over $\mathcal{O}_K$, letting (A, A^+) range over perfectoid $\mathcal{O}_K$-algebra pairs (resp., such that A is a K-algebra if G is only defined over K), we have the following functorial modular interpretation:*

$$\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}(A) \cong \left\{ (\mathcal{E}, \iota) \colon \begin{array}{l} \mathcal{E} \text{ is a } G\text{-torsor over } B_{\mathrm{dR}}^+(A), \\ \iota \in \mathcal{E}(B_{\mathrm{dR}}(A)) \text{ is its trivialization over } B_{\mathrm{dR}}(A) \end{array} \right\} / \sim .$$

Moreover, $\mathrm{Gr}_G^{B_{\mathrm{dR}}^+}$ is a sheaf for the v-topology and it contains the presheaf quotient LG/L^+G as the subfunctor that parametrizes those pairs $(\mathcal{E}, \iota)$ for which $\mathcal{E}$ is a trivial torsor.

Proof. Let us temporarily write Gr'_G for the functor defined by the displayed modular description. Since LG parametrizes the possible ι for the trivial G-torsor and L^+G parametrizes the automorphisms of the trivial G-torsor over $B_{\mathrm{dR}}^+(A)$, the presheaf quotient LG/L^+G is identified with the subfunctor of Gr'_G that parametrizes those $(\mathcal{E}, \iota)$ for which $\mathcal{E}$ is a trivial G-torsor. Thus, all we need to show is that Gr'_G is a v-sheaf (in particular, an étale sheaf) and that each $(\mathcal{E}, \iota)$ lies in LG/L^+G étale locally on $\mathrm{Spa}(A, A^+)$, in other words, that $\mathcal{E}$ trivializes étale locally on $\mathrm{Spa}(A, A^+)$.

For the latter, of course, $\mathcal{E}$ inherits smoothness (and also affineness) from G, so it trivializes étale locally on A. However, this by itself does not suffice because a priori only *finite* étale covers of A induce étale covers of $\mathrm{Spa}(A, A^+)$. Instead, for each $x \in \mathrm{Spa}(A, A^+)$, we let $\mathrm{Spa}(A_i, A_i^+) \subset \mathrm{Spa}(A, A^+)$ range over all the rational opens containing x and recall from, for example, [Sch22, Example 5.2], that the residue pair (k_x, k_x^+) is perfectoid such that, letting $\varpi \in A^+$ be a pseudouniformizer,

$$k_x^+ \cong \left(\varinjlim_i A_i^+ \right)^\wedge \quad \text{with} \quad k_x \cong k_x^+[\tfrac{1}{\varpi}],$$

where the completion is ϖ-adic. Since k_x is a field and $\mathcal{E}$ is smooth, some finite separable field extension $\widetilde{k}/k_x$ satisfies $\mathcal{E}(\widetilde{k}) \neq \varnothing$. Each A_i^+ is ϖ-Henselian (even ϖ-adically complete) and $A_i \cong A_i^+[\frac{1}{\varpi}]$ (resp., $k_x \cong k_x^+[\frac{1}{\varpi}]$), so $\varinjlim_i A_i^+$ is ϖ-Henselian (see [SP, Lemma 0FWT]) with $\varinjlim_i A_i \cong (\varinjlim_i A_i^+)[\frac{1}{\varpi}]$. In particular, we may apply the algebraization result [BČ22, Corollary 2.1.20][1] (with $B := \varinjlim_i A_i$ and $B' := \varpi(\varinjlim_i A_i^+)$) and then use a limit argument as in [KL15, Remark 1.2.9] to find some i and some finite étale A_i-algebra $\widetilde{A}$ whose base change to k_x is $\widetilde{k}$. We consider the restriction of scalars

$$\widetilde{\mathcal{E}} := \mathrm{Res}_{\widetilde{A}/A_i}(\mathcal{E}_{\widetilde{A}}),$$

which is representable by a smooth affine A_i-scheme, see [BLR90, Section 7.6, Propositions 2 and 5, Theorem 4]. By construction, $\widetilde{\mathcal{E}}(k_x) \cong \mathcal{E}(\widetilde{k}) \neq \varnothing$. Therefore, the Elkik-style approximation result [BČ22, Theorem 2.2.2][2] shows that, at the cost of enlarging i, we also have

$$\widetilde{\mathcal{E}}(A_i) \cong \mathcal{E}(\widetilde{A}) \neq \varnothing.$$

This, however, implies the desired triviality of $\mathcal{E}$ over some étale neighborhood of x.[3]

For the remaining v-sheaf property of Gr'_G, we will use the trivialization ι. Firstly, thanks to ι and the ideal I from §1 being generated by a nonzerodivisor, the objects of the prestack in groupoids

$$(A, A^+) \mapsto \left\{ (\mathcal{E}, \iota) \colon \begin{array}{l} \mathcal{E} \text{ is a } G\text{-torsor over } B_{\mathrm{dR}}^+(A), \\ \iota \in \mathcal{E}(B_{\mathrm{dR}}(A)) \text{ is its trivialization over } B_{\mathrm{dR}}(A) \end{array} \right\}$$

[1] Or already its earlier version [GR03, Theorem 5.4.53].

[2] Or already its earlier version [GR03, Proposition 5.4.21].

[3] A slightly different approach to the étale local triviality of $\mathcal{E}$ is to apply the same approximation result over the strict Henselizations of $\mathrm{Spa}(A, A^+)$. However, arguing that the residue field of the maximal ideal of this strict Henselization is separably closed involves the aforementioned algebraization result for finite étale algebras, so the argument would not be any simpler overall, compare with [Hub96, Section 2.5] or with [AGV22, Lemma 1.4.26].

have no nontrivial automorphisms. Thus, it suffices to show that this prestack is a v-stack, in fact, that v-descent is effective for its objects. To this end, we consider maps

$$(A, A^+) \to (A', A'^+) \rightrightarrows (A'', A''^+)$$

of perfectoid $\mathcal{O}_K$-algebra pairs such that the first one induces a v-cover on adic spectra whose self-(fiber product) is $\mathrm{Spa}(A'', A''^+)$, and we consider a pair $(\mathcal{E}', \iota')$ over $B^+_{\mathrm{dR}}(A')$ equipped with a descent datum with respect to this cover. Since the ideal I is functorial in (A, A^+) (see §1), the reduction modulo I of this descent datum equips the G-torsor $\mathcal{E}'|_{B^+_{\mathrm{dR}}(A')/I}$ over A' with a descent datum with respect to the v-cover $(A, A^+) \to (A', A'^+)$. By the Tannakian formalism for G-torsors over perfectoid spaces [SW20, Theorem 19.5.2] combined with the v-descent for vector bundles on perfectoid spaces [SW20, Corollary 17.1.9], this last descent datum is effective, so $\mathcal{E}'|_{B^+_{\mathrm{dR}}(A')/I}$ descends to a G-torsor E over A.

The invariance under Henselian pairs for G-torsors [BČ22, Theorem 2.1.6][4] ensures that E lifts uniquely to a G-torsor $\mathcal{E}$ over $B^+_{\mathrm{dR}}(A)$ and, by uniqueness, $\mathcal{E}|_{B^+_{\mathrm{dR}}(A')} \simeq \mathcal{E}'$. In fact, by transferring the rigidification ι' along any choice of this G-torsor isomorphism over $B^+_{\mathrm{dR}}(A')$, we see that $\mathcal{E}$ is actually a descent of $\mathcal{E}'$ relative to the descent datum that we started from: indeed, the relevant diagram over $B^+_{\mathrm{dR}}(A'')$ will commute because it only involves G-torsor isomorphisms that respect the rigidifications, which determines such isomorphisms uniquely. It remains to argue that ι' descends as well, for which it now suffices to show that

$$\mathcal{E}(B_{\mathrm{dR}}(A)) \to \mathcal{E}(B_{\mathrm{dR}}(A')) \rightrightarrows \mathcal{E}(B_{\mathrm{dR}}(A''))$$

is an equalizer diagram. For this, since $\mathcal{E}$ inherits affineness from G, it suffices to show that

$$B_{\mathrm{dR}}(A) \to B_{\mathrm{dR}}(A') \rightrightarrows B_{\mathrm{dR}}(A'')$$

is an equalizer diagram. Since the ideal I is functorial and principal, it suffices to show the same for

$$B^+_{\mathrm{dR}}(A) \to B^+_{\mathrm{dR}}(A') \rightrightarrows B^+_{\mathrm{dR}}(A'').$$

This, however, may be checked modulo powers of I, where, since I is generated by a nonzerodivisor, it follows from the structure presheaf being a v-sheaf on perfectoid spaces [Sch22, Theorem 8.7]. ☐

3. The analytic topology suffices

We are ready for our promised main result that the analytic sheafification suffices in the forming of the B^+_{dR}-affine Grassmannian. This hinges on algebraization and approximation techniques from [BČ22, Section 2] and on the discrete valuation ring case of the Grothendieck–Serre conjecture.

Theorem 3.1. *For a nonarchimedean local field K and a reductive group scheme G defined either over K or over $\mathcal{O}_K$ as in §1, the B^+_{dR}-affine Grassmannian $\mathrm{Gr}_G^{B^+_{\mathrm{dR}}}$ is the sheafification of the presheaf quotient LG/L^+G with respect to the analytic topology on perfectoid $\mathcal{O}_K$-algebra pairs (A, A^+).*

We recall that the *analytic topology* is the one whose covers $\{(A, A^+) \to (A_j, A_j^+)\}_{j \in J}$ are characterized by the maps $\mathrm{Spa}(A_j, A_j^+) \to \mathrm{Spa}(A, A^+)$ being jointly surjective open immersions.

Proof. By Proposition 2.1, the B^+_{dR}-affine Grassmannian $\mathrm{Gr}_G^{B^+_{\mathrm{dR}}}$ is a sheaf for the analytic topology (even for the v-topology) and contains the presheaf quotient LG/L^+G as a subfunctor that parametrizes those pairs $(\mathcal{E}, \iota)$ in which $\mathcal{E}$ is a trivial torsor. Thus, we only need to show that for every perfectoid $\mathcal{O}_K$-algebra pair (A, A^+) and every G-torsor $\mathcal{E}$ over $B^+_{\mathrm{dR}}(A)$ that becomes trivial

[4] Or already its earlier version [GR03, Theorem 5.8.14].

over $B_{\mathrm{dR}}(A)$, each $x \in \mathrm{Spa}(A, A^+)$ lies in some rational open subset $\mathrm{Spa}(A', A'^+) \subset \mathrm{Spa}(A, A^+)$ such that $\mathcal{E}|_{B^+_{\mathrm{dR}}(A')}$ is trivial. For this, since $B^+_{\mathrm{dR}}(A')$ is complete with respect to the kernel of its surjection onto A', by the smoothness of $\mathcal{E}$ inherited from G (or by [BČ22, Theorem 2.1.6]), it suffices to show that for each $x \in \mathrm{Spa}(A, A^+)$, the G-torsor $E := \mathcal{E}|_A$ trivializes over some A' as above.

To find the desired A', we first let $\mathrm{Spa}(A_i, A_i^+) \subset \mathrm{Spa}(A, A^+)$ range over all the rational open subsets containing x and, as in the proof of Proposition 2.1, consider the perfectoid residue pair (k_x, k_x^+) with

$$k_x^+ \cong \left(\varinjlim_i A_i^+\right)^\wedge \quad \text{and} \quad k_x \cong k_x^+[\tfrac{1}{\varpi}],$$

where $\varpi \in A^+$ is a pseudouniformizer and the completion is ϖ-adic. Similarly to there, the algebraization and approximation result [BČ22, Corollary 2.1.22 (b)][5] shows that

$$\varinjlim_i H^1(A_i, G) \cong H^1(\varinjlim_i A_i, G) \xrightarrow{\sim} H^1(k_x, G)$$

(for the first isomorphism in this display, see, for instance, [Čes15, Lemma 2.1]).[6] Thanks to this, all that remains is to show that the G-torsor $E|_{k_x}$ is trivial granted that we know that it lifts to a G-torsor over $B^+_{\mathrm{dR}}(k_x)$, namely, to $\mathcal{E}|_{B^+_{\mathrm{dR}}(k_x)}$, that becomes trivial over $B_{\mathrm{dR}}(k_x)$.

However, $B^+_{\mathrm{dR}}(k_x)$ is a discrete valuation ring and G is reductive, so the Nisnevich case of the Grothendieck–Serre conjecture [Nis82, Chapter II, Theorem 4.2], [Nis84, théorème 2.1] (see also [Guo22, Theorem 1]) implies that no nontrivial G-torsor over $B^+_{\mathrm{dR}}(k_x)$ trivializes over $B_{\mathrm{dR}}(k_x)$. ☐

Remark 3.2. The only way in which we used the assumption that our group G is reductive, as opposed to, say, merely smooth and affine, is to apply the Grothendieck–Serre type result that no nontrivial G-torsor over the discrete valuation ring $B^+_{\mathrm{dR}}(k_x)$ trivializes over $B_{\mathrm{dR}}(k_x)$. Granted that this input is obtained, the same argument would give Theorem 3.1 for other classes of smooth affine groups, for instance, it would be interesting to know whether the same holds for parahoric G.

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[5] Or already its earlier version [GR03, Theorem 5.8.14].

[6] In fact, we will only use the injectivity of the displayed map, which also follows from the Elkik-style approximation result [BČ22, Theorem 2.2.2], or already from its earlier version [GR03, Proposition 5.4.21].

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