

Feuille d'exercices numéro 1

Systèmes linéaires et matrices

Exercice 1

$$(a) (x, y) = \left(-\frac{3}{5}, -\frac{2}{5}\right)$$

(b) Pas de solution

$$(c) (x, y) = \left(\frac{5b+2}{ab-2}, -\frac{a+5}{ab-2}\right) \text{ si } ab \neq 2$$

$$(x, y) = (-by-1, y), y \in \mathbb{C}, \text{ si } ab = 2 \text{ et } a = -5$$

Pas de solution si $ab = 2$ et $a \neq -5$

$$(d) (x, y) = \left(-\frac{b-1}{a-2}, \frac{ab-2}{a-2}\right) \text{ si } a \neq 2$$

$$(x, y) = (x, -2x+b), x \in \mathbb{C}, \text{ si } a = 2 \text{ et } b = 1$$

Pas de solution si $a = 2$ et $b \neq 1$

$$(e) (x, y, z) = (-7, -9, 1)$$

$$(f) (x, y, z) = (4, 2z+3, z), z \in \mathbb{C}$$

$$(g) (x, y, z) = (0, 0, 0) \text{ si } a \neq -3; \text{ base : } \emptyset$$

$$(x, y, z) = (9z, -5z, z), z \in \mathbb{C}, \text{ si } a = -3; \text{ base : } V_1 = \begin{bmatrix} 9 \\ -5 \\ 1 \end{bmatrix}$$

$$(h) (x, y, z) = \left(\frac{2a-b-5c}{4}, \frac{b+3c}{2}, -c\right)$$

$$(i) (x, y, z) = (0, 0, 0) \text{ si } abc \neq -1; \text{ base : } \emptyset$$

$$(x, y, z) = (abz, -bz, z), z \in \mathbb{C}, \text{ si } abc = -1; \text{ base : } V_1 = \begin{bmatrix} ab \\ -b \\ 1 \end{bmatrix}$$

$$(j) (x, y, z) = (z-2, -3z+2, z), z \in \mathbb{C}$$

$$(k) (x, y, z) = \left(\frac{5a-3}{a^2-5a+1}, -\frac{12a+1}{a^2-5a+1}, -\frac{2a+7}{a^2-5a+1}\right) \text{ si } a^2-5a+1 \neq 0$$

Pas de solution si $a^2-5a+1 = 0$

$$(\ell) (x, y, z) = (0, 0, 0) \text{ si } a^2-3a-13 \neq 0; \text{ base : } \emptyset$$

$$(x, y, z) = ((a+3)z, -(2a+2)z, z), z \in \mathbb{C}, \text{ si } a^2-3a-13 = 0;$$

$$\text{base : } V_1 = \begin{bmatrix} a+3 \\ -(2a+2) \\ 1 \end{bmatrix}$$

$$(m) (x, y, z) = (0, 0, 0) \text{ si } a^2 \neq 1; \text{ base : } \emptyset$$

$$(x, y, z) = (-y, y, 0), y \in \mathbb{C}, \text{ si } a = 1; \text{ base : } V_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$(x, y, z) = (z, 0, z), z \in \mathbb{C}, \text{ si } a = -1; \text{ base : } V_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$(n) (x, y, z, t) = \left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

$$(o) (x_1, x_2, x_3, x_4) = (-x_4+3, 4x_4-9, x_4-2, x_4), x_4 \in \mathbb{C}$$

$$(p) (x_1, x_2, x_3, x_4, x_5) = (-2x_2-7x_5, x_2, 3x_5, -x_5, x_5), (x_2, x_5) \in \mathbb{C}^2;$$

$$\text{base : } V_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} -7 \\ 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}$$

$$(q) (x_1, x_2, x_3, x_4, x_5) = \left(-x_4 - \frac{x_5}{3}, -10x_4 - \frac{17x_5}{3}, -4x_4 - 2x_5, x_4, x_5\right),$$

$$(x_4, x_5) \in \mathbb{C}^2; \text{ base : } V_1 = \begin{bmatrix} -1 \\ -10 \\ -4 \\ 1 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} -1/3 \\ -17/3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

Exercice 2

$$2A - 3B = \begin{bmatrix} 3 & 4 & -12 & 2 \\ -1/2 & -2 & -23/2 & -1 \\ 2 & -6 & 9 & 1 \\ -9 & 8 & -3/2 & -5/3 \end{bmatrix}$$

Exercice 3

$$AX = \begin{bmatrix} 10 \\ -9/4 \\ 3 \\ -7 \end{bmatrix} \quad BX = \begin{bmatrix} -3 \\ -7/2 \\ 7 \\ -3/2 \end{bmatrix}$$

Exercice 4

$$(a) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(b) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(c) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(d) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(e) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(f) \ S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}; \text{ base : } \emptyset$$

$$(g) \ S = \left\{ \begin{bmatrix} x_1 \\ 2x_3 - x_5 \\ x_3 \\ -3x_5 \\ x_5 \\ 0 \\ 0 \end{bmatrix}, (x_1, x_3, x_5) \in \mathbb{C}^3 \right\}; \text{ base : } V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$(h) \ S = \left\{ \begin{bmatrix} x_1 \\ 3x_3 \\ x_3 \\ x_7 \\ 0 \\ -x_7 \\ x_7 \end{bmatrix}, (x_1, x_3, x_7) \in \mathbb{C}^3 \right\}; \text{ base : } V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$