Feuille d'exercices numéro 4

Systèmes différentiels

Dans la question 1, lorsque les valeurs propres ne sont pas réelles, les solutions $X(t)$ dépendent de 4 constantes $c_1, \ldots, c_4$ qui doivent vérifier certaines relations. L'énoncé ne demande pas de les déterminer, mais on le fait dans ce corrigé.

1. (a) $\chi_A(z) = z^2 - 5z + 6$ $\boxed{\Delta > 0}$

$\lambda_1 = 3,\ \lambda_2 = 2\,;\ V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\boxed{\lambda_1 > \lambda_2 > 0}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} 2\,c_1\, e^{3t} - c_2\, e^{2t} \\ c_1\, e^{3t} + c_2\, e^{2t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(b) $\chi_A(z) = z^2 - 3z + 2$ $\boxed{\Delta > 0}$

$\lambda_1 = 2,\ \lambda_2 = 1\,;\ V_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ V_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ $\boxed{\lambda_1 > \lambda_2 > 0}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} -2\,c_1\, e^{2t} - 3\,c_2\, e^{t} \\ c_1\, e^{2t} + c_2\, e^{t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(c) $\chi_A(z) = z^2 - 3z - 4$ $\boxed{\Delta > 0}$

$\lambda_1 = 4,\ \lambda_2 = -1\,;\ V_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix},\ V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\boxed{\lambda_1 > 0 > \lambda_2}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} 2\,c_1\, e^{4t} - c_2\, e^{-t} \\ 3\,c_1\, e^{4t} + c_2\, e^{-t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(d) $\chi_A(z) = z^2 - 1$ $\boxed{\Delta > 0}$

$\lambda_1 = 1,\ \lambda_2 = -1\,;\ V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ V_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\boxed{\lambda_1 > 0 > \lambda_2}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} c_1\, e^{t} + c_2\, e^{-t} \\ 2\,c_1\, e^{t} + 3\,c_2\, e^{-t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(e) $\chi_A(z) = z^2 - 8z + 15$ $\boxed{\Delta > 0}$

$\lambda_1 = 5,\ \lambda_2 = 3\,;\ V_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix},\ V_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\boxed{\lambda_1 > \lambda_2 > 0}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} 3\,c_1\, e^{5t} + 2\,c_2\, e^{3t} \\ 2\,c_1\, e^{5t} + c_2\, e^{3t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(f) $\chi_A(z) = z^2 + z - 6$ $\boxed{\Delta > 0}$

$\lambda_1 = 2,\ \lambda_2 = -3\,;\ V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ V_2 = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$ $\boxed{\lambda_1 > 0 > \lambda_2}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} c_1\, e^{2t} + c_2\, e^{-3t} \\ c_1\, e^{2t} + 6\,c_2\, e^{-3t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(g) $\chi_A(z) = z^2 + 2z$ $\boxed{\Delta > 0}$

$\lambda_1 = 0,\ \lambda_2 = -2\,;\ V_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix},\ V_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\boxed{0 = \lambda_1 > \lambda_2}$

$X(t) = c_1\, V_1 + c_2\, e^{\lambda_2 t}\, V_2 = \begin{bmatrix} -3\,c_1 - 2\,c_2\, e^{-2t} \\ c_1 + c_2\, e^{-2t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(h) $\chi_A(z) = (z+2)^2$ $\boxed{\Delta = 0}$

$\lambda = -2\,;\ V_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\boxed{\dim\big(E_\lambda(A)\big) = 2}$

$X(t) = c_1\, e^{\lambda t}\, V_1 + c_2\, e^{\lambda t}\, V_2 = \begin{bmatrix} c_1\ e^{-2t} \\ c_2\ e^{-2t} \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(i) $\chi_A(z) = z^2 - 2z + 1$ $\boxed{\Delta = 0}$

$\lambda = 1\,;\ V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\boxed{\dim\big(E_\lambda(A)\big) = 1}$

$V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ et $(A - \lambda\, I_2) V_2 = 2\, V_1$

$X(t) = (c_1\, t + c_2)\, e^{\lambda t}\, V_1 + \frac{c_1}{2}\, e^{\lambda t}\, V_2 = \begin{bmatrix} (c_1\, t + c_1/2 + c_2)\ e^{t} \\ (c_1\, t - c_1/2 + c_2)\ e^{t} \end{bmatrix}$

(avec $c_1, c_2 \in \mathbb{R}$)

Un autre choix possible est $V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, qui donne $X(t) = \begin{bmatrix} (c_1\, t + c_1 + c_2)\ e^{t} \\ (c_1\, t + c_2)\ e^{t} \end{bmatrix}$

(j) $\chi_A(z) = z^2 - z$ $\boxed{\Delta > 0}$

$\lambda_1 = 1,\ \lambda_2 = 0\,;\ V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ V_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\boxed{\lambda_1 > \lambda_2 = 0}$

$X(t) = c_1\, e^{\lambda_1 t}\, V_1 + c_2\, V_2 = \begin{bmatrix} c_1\, e^{t} + 2\,c_2 \\ 2\,c_1\, e^{t} + 3\,c_2 \end{bmatrix}$ (avec $c_1, c_2 \in \mathbb{R}$)

(k) $\chi_A(z) = z^2 - 2z + 4$ $\boxed{\Delta < 0}$

$\lambda_1 = 1 + i\,\sqrt{3},\ \lambda_2 = 1 - i\,\sqrt{3}\,;\ \alpha = 1$ et $\beta = \sqrt{3}$ $\boxed{\alpha > 0}$

$X(t) = \begin{bmatrix} e^{\alpha t}\,\big(c_1\, \cos(\beta\, t) + c_3\, \sin(\beta\, t)\big) \\ e^{\alpha t}\,\big(c_2\, \cos(\beta\, t) + c_4\, \sin(\beta\, t)\big) \end{bmatrix} = \begin{bmatrix} e^{t}\,\big(c_1 \cos(\sqrt{3}\, t) + c_2 \sin(\sqrt{3}\, t)\big) \\ e^{t}\,\big(c_2 \cos(\sqrt{3}\, t) - c_1 \sin(\sqrt{3}\, t)\big) \end{bmatrix}$

(avec $c_3 = c_2$, $c_4 = -c_1$ et $c_1, c_2 \in \mathbb{R}$)

(ℓ) $\chi_A(z) = z^2 + 1$ $\boxed{\Delta < 0}$

$\lambda_1 = \mathrm{i}$, $\lambda_2 = -\mathrm{i}$; $\alpha = 0$ et $\beta = 1$ $\boxed{\alpha = 0}$

$$X(t) = \begin{bmatrix} c_1\, \cos(\beta\, t) + c_3\, \sin(\beta\, t) \\ c_2\, \cos(\beta\, t) + c_4\, \sin(\beta\, t) \end{bmatrix} = \begin{bmatrix} c_1 \cos(t) + (-2\, c_1 + 5\, c_2) \sin(t) \\ c_2 \cos(t) + (-c_1 + 2\, c_2) \sin(t) \end{bmatrix}$$

(avec $c_3 = -2\, c_1 + 5\, c_2$, $c_4 = -c_1 + 2\, c_2$ et $c_1, c_2 \in \mathbb{R}$)

(m) $\chi_A(z) = z^2 - 2\, z + 3$ $\boxed{\Delta < 0}$

$\lambda_1 = 1 + \mathrm{i}\, \sqrt{2}$, $\lambda_2 = 1 - \mathrm{i}\, \sqrt{2}$; $\alpha = 1$ et $\beta = \sqrt{2}$ $\boxed{\alpha > 0}$

$$X(t) = \begin{bmatrix} \mathrm{e}^{\alpha\, t} \left(c_1\, \cos(\beta\, t) + c_3\, \sin(\beta\, t)\right) \\ \mathrm{e}^{\alpha\, t} \left(c_2\, \cos(\beta\, t) + c_4\, \sin(\beta\, t)\right) \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{t} \left(c_1 \cos(\sqrt{2}\, t) + \sqrt{2}\, c_2 \sin(\sqrt{2}\, t)\right) \\ \mathrm{e}^{t} \left(c_2 \cos(\sqrt{2}\, t) - c_1 \sin(\sqrt{2}\, t)/\sqrt{2}\right) \end{bmatrix}$$

(avec $c_3 = \sqrt{2}\, c_2$, $c_4 = -c_1/\sqrt{2}$ et $c_1, c_2 \in \mathbb{R}$)

(n) $\chi_A(z) = z^2 - 2\, z + 4$ $\boxed{\Delta < 0}$

$\lambda_1 = 1 + \mathrm{i}\, \sqrt{3}$, $\lambda_2 = 1 - \mathrm{i}\, \sqrt{3}$; $\alpha = 1$ et $\beta = \sqrt{3}$ $\boxed{\alpha > 0}$

$$X(t) = \begin{bmatrix} \mathrm{e}^{\alpha\, t} \left(c_1\, \cos(\beta\, t) + c_3\, \sin(\beta\, t)\right) \\ \mathrm{e}^{\alpha\, t} \left(c_2\, \cos(\beta\, t) + c_4\, \sin(\beta\, t)\right) \end{bmatrix} = \begin{bmatrix} \mathrm{e}^{t} \left(c_1 \cos(\sqrt{3}\, t) + (c_1 + 2\, c_2) \sin(\sqrt{3}\, t)/\sqrt{3}\right) \\ \mathrm{e}^{t} \left(c_2 \cos(\sqrt{3}\, t) - (2\, c_1 + c_2) \sin(\sqrt{3}\, t)/\sqrt{3}\right) \end{bmatrix}$$

(avec $c_3 = (c_1 + 2\, c_2)/\sqrt{3}$, $c_4 = -(2\, c_1 + c_2)/\sqrt{3}$ et $c_1, c_2 \in \mathbb{R}$)

(o) $\chi_A(z) = z^2 + 1$ $\boxed{\Delta < 0}$

$\lambda_1 = \mathrm{i}$, $\lambda_2 = -\mathrm{i}$; $\alpha = 0$ et $\beta = 1$ $\boxed{\alpha = 0}$

$$X(t) = \begin{bmatrix} c_1\, \cos(\beta\, t) + c_3\, \sin(\beta\, t) \\ c_2\, \cos(\beta\, t) + c_4\, \sin(\beta\, t) \end{bmatrix} = \begin{bmatrix} c_1 \cos(t) - c_2 \sin(t) \\ c_2 \cos(t) + c_1 \sin(t) \end{bmatrix}$$

(avec $c_3 = -c_2$, $c_4 = c_1$ et $c_1, c_2 \in \mathbb{R}$)

4. (a) $c_1 = 0$, $c_2 = -1$
   (b) $c_1 = -2$, $c_2 = 1$
   (c) $c_1 = 0$, $c_2 = -1$
   (d) $c_1 = 4$, $c_2 = -3$
   (e) $c_1 = -3$, $c_2 = 5$
   (f) $c_1 = 7/5$, $c_2 = -2/5$
   (g) $c_1 = 1$, $c_2 = -2$
   (h) $c_1 = 1$, $c_2 = -1$
   (i) $c_1 = 2$, $c_2 = 0$
   (j) $c_1 = -5$, $c_2 = 3$
   (k) $c_1 = 1$, $c_2 = -1$
   (ℓ) $c_1 = 1$, $c_2 = -1$
   (m) $c_1 = 1$, $c_2 = -1$
   (n) $c_1 = 1$, $c_2 = -1$
   (o) $c_1 = 1$, $c_2 = -1$

   Pour (i), le choix alternatif $V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ donne $c_1 = 2$, $c_2 = -1$.

5. (a) $X(t) = \begin{bmatrix} \mathrm{e}^{2t} \\ -\mathrm{e}^{2t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} 2\, \mathrm{e}^{2t} \\ -2\, \mathrm{e}^{2t} \end{bmatrix}$

   $X(-1) \approx \begin{bmatrix} 0.135335 \\ -0.135335 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 0.270671 \\ -0.270671 \end{bmatrix}$

   $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

   $X(1) \approx \begin{bmatrix} 7.389056 \\ -7.389056 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} 14.778112 \\ -14.778112 \end{bmatrix}$

   (b) $X(t) = \begin{bmatrix} 4\, \mathrm{e}^{2t} - 3\, \mathrm{e}^{t} \\ -2\, \mathrm{e}^{2t} + \mathrm{e}^{t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} 8\, \mathrm{e}^{2t} - 3\, \mathrm{e}^{t} \\ -4\, \mathrm{e}^{2t} + \mathrm{e}^{t} \end{bmatrix}$

   $X(-1) \approx \begin{bmatrix} -0.562297 \\ 0.0972089 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -0.0209561 \\ -0.173462 \end{bmatrix}$

   $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

   $X(1) \approx \begin{bmatrix} 21.4014 \\ -12.0598 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} 50.9576 \\ -26.8379 \end{bmatrix}$

   (c) $X(t) = \begin{bmatrix} \mathrm{e}^{-t} \\ -\mathrm{e}^{-t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} -\mathrm{e}^{-t} \\ \mathrm{e}^{-t} \end{bmatrix}$

   $X(-1) \approx \begin{bmatrix} 2.71828 \\ -2.71828 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -2.71828 \\ 2.71828 \end{bmatrix}$

   $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

   $X(1) \approx \begin{bmatrix} 0.367879 \\ -0.367879 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -0.367879 \\ 0.367879 \end{bmatrix}$

   (d) $X(t) = \begin{bmatrix} 4\, \mathrm{e}^{t} - 3\, \mathrm{e}^{-t} \\ 8\, \mathrm{e}^{t} - 9\, \mathrm{e}^{-t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} 4\, \mathrm{e}^{t} + 3\, \mathrm{e}^{-t} \\ 8\, \mathrm{e}^{t} + 9\, \mathrm{e}^{-t} \end{bmatrix}$

   $X(-1) \approx \begin{bmatrix} -6.68333 \\ -21.5215 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 9.62636 \\ 27.4076 \end{bmatrix}$

   $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 7 \\ 17 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} 9.76949 \\ 18.4353 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} 11.9768 \\ 25.0572 \end{bmatrix}$

(*e*) $X(t) = \begin{bmatrix} -9\ \mathrm{e}^{5t} + 10\ \mathrm{e}^{3t} \\ -6\ \mathrm{e}^{5t} + 5\ \mathrm{e}^{3t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} -45\ \mathrm{e}^{5t} + 30\ \mathrm{e}^{3t} \\ -30\ \mathrm{e}^{5t} + 15\ \mathrm{e}^{3t} \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 0.437229 \\ 0.208508 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 1.1904 \\ 0.544668 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -15 \\ -15 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -1134.86 \\ -790.051 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -6076.03 \\ -4151.11 \end{bmatrix}$

(*f*) $X(t) = \begin{bmatrix} 7\ \mathrm{e}^{2t}/5 - 2\ \mathrm{e}^{-3t}/5 \\ 7\ \mathrm{e}^{2t}/5 - 12\ \mathrm{e}^{-3t}/5 \end{bmatrix}$; $X'(t) = \begin{bmatrix} 14\ \mathrm{e}^{2t}/5 + 6\ \mathrm{e}^{-3t}/5 \\ 14\ \mathrm{e}^{2t}/5 + 36\ \mathrm{e}^{-3t}/5 \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} -7.84475 \\ -48.0158 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 24.4816 \\ 144.995 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} 10.3248 \\ 10.2252 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} 20.7491 \\ 21.0478 \end{bmatrix}$

(*g*) $X(t) = \begin{bmatrix} -3 + 4\ \mathrm{e}^{-2t} \\ 1 - 2\ \mathrm{e}^{-2t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} -8\ \mathrm{e}^{-2t} \\ 4\ \mathrm{e}^{-2t} \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 26.5562 \\ -13.7781 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -59.1124 \\ 29.5562 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -2.45866 \\ 0.541341 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -1.08268 \\ 0.729329 \end{bmatrix}$

(*h*) $X(t) = \begin{bmatrix} \mathrm{e}^{-2t} \\ -\mathrm{e}^{-2t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} -2\ \mathrm{e}^{-2t} \\ 2\ \mathrm{e}^{-2t} \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 7.38906 \\ -7.38906 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -14.7781 \\ 14.7781 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} 0.135335 \\ -0.135335 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -0.270671 \\ 0.270671 \end{bmatrix}$

(*i*) $X(t) = \begin{bmatrix} (2t+1)\ \mathrm{e}^{t} \\ (2t-1)\ \mathrm{e}^{t} \end{bmatrix}$; $X'(t) = \begin{bmatrix} (2t+3)\ \mathrm{e}^{t} \\ (2t+1)\ \mathrm{e}^{t} \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} -0.367879 \\ -1.10364 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 0.367879 \\ -0.367879 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} 8.15485 \\ 2.71828 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} 13.5914 \\ 8.15485 \end{bmatrix}$

(*j*) $X(t) = \begin{bmatrix} -5\ \mathrm{e}^{t} + 6 \\ -10\ \mathrm{e}^{t} + 9 \end{bmatrix}$; $X'(t) = \begin{bmatrix} -5\ \mathrm{e}^{t} \\ -10\ \mathrm{e}^{t} \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 4.1606 \\ 5.32121 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -1.8394 \\ -3.67879 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -5 \\ -10 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -7.59141 \\ -18.1828 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -13.5914 \\ -27.1828 \end{bmatrix}$

(*k*) $X(t) = \begin{bmatrix} \mathrm{e}^{t}\left(\cos(\sqrt{3}\,t) - \sin(\sqrt{3}\,t)\right) \\ -\mathrm{e}^{t}\left(\cos(\sqrt{3}\,t) + \sin(\sqrt{3}\,t)\right) \end{bmatrix}$;

$X'(t) = \begin{bmatrix} \mathrm{e}^{t}\left((1-\sqrt{3})\cos(\sqrt{3}\,t) - (1+\sqrt{3})\sin(\sqrt{3}\,t)\right) \\ \mathrm{e}^{t}\left(-(1+\sqrt{3})\cos(\sqrt{3}\,t) - (1-\sqrt{3})\sin(\sqrt{3}\,t)\right) \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 0.304041 \\ 0.422172 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 1.03527 \\ -0.104443 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) \approx \begin{bmatrix} -0.732051 \\ -2.73205 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -3.11945 \\ -2.24658 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -7.01064 \\ 3.15648 \end{bmatrix}$

(ℓ) $X(t) = \begin{bmatrix} \cos(t) - 7\sin(t) \\ -\cos(t) - 3\sin(t) \end{bmatrix}$; $X'(t) = \begin{bmatrix} -7\cos(t) - \sin(t) \\ -3\cos(t) + \sin(t) \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 6.4306 \\ 1.98411 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} -2.94065 \\ -2.46238 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -7 \\ -3 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -5.34999 \\ -3.06472 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -4.62359 \\ -0.779436 \end{bmatrix}$

(m) $X(t) = \begin{bmatrix} e^t\left(\cos(\sqrt{2}\,t) - \sqrt{2}\sin(\sqrt{2}\,t)\right) \\ e^t\left(-\cos(\sqrt{2}\,t) - \sin(\sqrt{2}\,t)/\sqrt{2}\right) \end{bmatrix}$;

$X'(t) = \begin{bmatrix} e^t\left(-\cos(\sqrt{2}\,t) - 2\sqrt{2}\sin(\sqrt{2}\,t)\right) \\ e^t\left(-2\cos(\sqrt{2}\,t) + \sin(\sqrt{2}\,t)/\sqrt{2}\right) \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 0.571264 \\ 0.199579 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 0.970422 \\ -0.371685 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -3.3733 \\ 0.199579 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -8.0183 \\ 1.0508 \end{bmatrix}$

(n) $X(t) = \begin{bmatrix} e^t\left(\cos(\sqrt{3}\,t) - \sin(\sqrt{3}\,t)/\sqrt{3}\right) \\ e^t\left(-\cos(\sqrt{3}\,t) - \sin(\sqrt{3}\,t)/\sqrt{3}\right) \end{bmatrix}$;

$X'(t) = \begin{bmatrix} -4\,e^t\sin(\sqrt{3}\,t)/\sqrt{3} \\ e^t\left(-2\cos(\sqrt{3}\,t) + 2\sin(\sqrt{3}\,t)/\sqrt{3}\right) \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} 0.150574 \\ 0.268705 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 0.838559 \\ -0.301149 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} -1.98548 \\ -1.1126 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -6.19616 \\ 3.97096 \end{bmatrix}$

(o) $X(t) = \begin{bmatrix} \cos(t) + \sin(t) \\ -\cos(t) + \sin(t) \end{bmatrix}$; $X'(t) = \begin{bmatrix} \cos(t) - \sin(t) \\ \cos(t) + \sin(t) \end{bmatrix}$

$X(-1) \approx \begin{bmatrix} -0.301169 \\ -1.38177 \end{bmatrix}$, $X'(-1) \approx \begin{bmatrix} 1.38177 \\ -0.301169 \end{bmatrix}$

$X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$X(1) \approx \begin{bmatrix} 1.38177 \\ 0.301169 \end{bmatrix}$, $X'(1) \approx \begin{bmatrix} -0.301169 \\ 1.38177 \end{bmatrix}$