MAP 553
PC 9

Ada Boost

$$\hat{R}_{\varphi,m}\left(\hat{f}_{m-1}+\beta h\right) = \frac{1}{m}\sum_{i=1}^{m} e^{-y_i \hat{f}_{m-1}(x_i) - \beta y_i h(x_i)}$$

$$= \sum_{i=1}^{m} \omega_i^{(m)} e^{-\beta} \underbrace{\mathbb{1}_{Y_i = h(x_i)}}_{1-\mathbb{1}_{Y_i \neq h(x_i)}} + \sum_{i=1}^{m} \omega_i^{(m)} e^{\beta} \mathbb{1}_{Y_i \neq h(x_i)}$$

$$= e^{-\beta}\sum_{i=1}^{m} \omega_i^{(m)} + \left(e^{\beta} - e^{-\beta}\right) \sum_{i=1}^{m} \omega_i^{(m)} \mathbb{1}_{Y_i \neq h(x_i)}$$

Solution:

- $\hat{h}_{j_m} = \underset{h \in \{h_1,\dots,h_n\}}{\operatorname{argmin}} \sum_{i=1}^{m} \omega_i^{(m)} \mathbb{1}_{Y_i \neq h(x_i)}$

- ∂_β : $-e^{-\beta}\sum_i \omega_i^{(m)} + \left(e^{\beta}+e^{-\beta}\right)\sum_i \omega_i^{(m)} \mathbb{1}_{Y_i \neq h(x_i)}$

$$\Rightarrow \quad e^{2\hat{\beta}_m} = \frac{\sum_i \omega_i^{(m)} - \sum_i \omega_i^{(m)} \mathbb{1}_{Y_i \neq h_{j_m}(x_i)}}{\sum_i \omega_i^{(m)} \mathbb{1}_{Y_i \neq h_{j_m}(x_i)}} = \frac{1 - err_m(h_{j_m})}{err_m(h_{j_m})}$$

Enfin comme $-y_i h(x_i) = 2\,\mathbb{1}_{Y_i \neq h(x_i)} - 1$

$$\omega_i^{(m+1)} = \omega_i^{(m)} e^{-\beta_m y_i h_{j_m}(x_i)} = \omega_i^{(m)} e^{2\beta_m \mathbb{1}_{Y_i \neq h(x_i)}} \times e^{-\beta_m}$$

comme un facteur multiplicatif ~~[illegible]~~ dans $\omega_i^{(m+1)}$ ne change pas $err_{m+1}(h)$ on peut retirer $e^{-\beta_m}$