STRATIFICATION BY ISOTOPY CLASSES.

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ABSTRACT. Given a monic polynomial $P = z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_0$ with complex coefficients, the set $I(P) = P^{-1}(\mathbb{R} \cup i\mathbb{R}) \subset \mathbb{C}$ is a planar graph with many combinatorial properties. Maximal families of monic polynomials having ambient isotopic images $I(P)$ are contractible, and allow a common root-finding strategy, which can be used in steering robots.

Let S be a smooth surface of genus $g > 1$, twice punctured in $p, q \in S$. Let J be a conformal structure on S and let $f = f_{J,p,q}$ be the real harmonic function on S such that the holomorphic 1-form $df - idf \circ J$ has only poles at p, q with residues $+1, -1$ respectively. Let $I(J, p, q)$ be the union of the singular levels of the function $f_{J,p,q}$. Define a decomposition D_J of the space $S \setminus \Delta$ of pairs by declaring two pairs (p, q), (p', q') as J-equivalent if $I(J, p, q)$ and $I(J, p', q')$ are ambient isotopic. For generic p, q, a minimal system of simple closed curves that separate the connected components of $I(J, p, q)$ gives a pants decomposition $P_{J,p,q}$ with $2g$ pants of the punctured surface (S, p, q).

Declare two conformal structures J, J' as equivalent if the decompositions D_J, $D_{J'}$ are ambient isotopic. In fact one uses a finer decomposition that also keeps track of some singular gradient lines. As result one obtains a mapping class invariant decomposition of Teichmüller space $T(S)$. Especially interesting is the union $B(S) \subset T(S)$ of all bounded equivalence classes.

A similar construction works in case of $g = 1$. One uses an auxilary choice of a level two structure. The resulting subset $B(S)$ is the Farey graph in hyperbolic plane. In work with co-authors Athanase Papadopoulos and Sumio Yamada and hopefully in progress we study the higher genus cases. Especially interesting is already the genus 2 case with its Bolza surface.

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