Ax-Schanuel theorem and Nevanlinna theory —an application

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Abstract

Generalizing the conjecture of the algebraic independence of e and π, Schanuel's conjecture (1966) claims the the transcendence degree of $\alpha_1, \ldots, \alpha_n, e^{\alpha_1}, \ldots, e^{\alpha_n}$ is at least n for $\mathbf{Q}$-linearly independent (non-degeneracy condition) complex numbers $\alpha_j \in \mathbf{C}$ $(1 \leq j \leq n)$. J. Ax (1971/'72) proved a *formal function analogue* of the conjecture for $(\alpha_j) = (f_j(t)) \in \mathbf{C}[[t]]^n$, $\mathrm{tr.deg}_{\mathbf{C}}\, \widehat{\exp} f \geq n+1$, where $\widehat{\exp} f(t) := ((f_j(t)), (e^{f_j(t)}))$, and also dealt with the case of semi-abelian varieties.

In this talk we discuss the problem for an analytic $f : R \to \mathrm{Lie}(A)$ with exponential map $\exp_A : \mathrm{Lie}(A) \to A$ of a semi-abelian variety A, where R may be $\mathbf{C}$, an affine algebraic curve, a punctured disk, a parabolic Riemann surface, etc, to say, $R := \mathbf{C}$ here for simplicity. We study the value distribution of $\widehat{\exp}_A f := (f, \exp_A f) : \mathbf{C} \to \mathrm{Lie}(A) \times A$ from the Nevanlinna theoretic viewpoint, giving another proof of Ax-Schanuel theorem in the analytic case, such that $\mathrm{tr.deg}_{\mathbf{C}}\, \widehat{\exp}_A f \geq \dim A + 1$.