Abstract

Recent results on matrix weighted norm inqualities

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Matrix weighted inequalities for singular integrals and other operators have been of interest since the work of Nazarov, Treil and Volberg in the 1900s. They asked the question: can we generalize the classical Muckenhoupt A_p condition to matrix weights so that we have inequalities of the form

$$\int_{\mathbb{R}^n} |W(x)Tf(x)|^p\,dx \leq C\int_{\mathbb{R}^n} |W(x)f(x)|^p\,dx,$$

$1 < p < \infty$, where W is a $d \times d$ self-adjoint, positive semi-definite matrix, f is an $\mathbb{R}^d$-valued measurable function, and T is a singular integral or other linear operator from classical harmonic analysis? NTV proved bounds for the Hilbert transform on the real line, introducing a generalization of A_p in terms of norm functions on $\mathbb{R}^d$. Their results were extend to $\mathbb{R}^n$ and general Calderón-Zygmund singular integrals by Christ and Goldberg. Later, Roudenko introduced an equivalent definition of matrix $\mathcal{A}_p$ that more closely resembles the classical scalar condition:

$$[W]_{\mathcal{A}_p} = \sup_Q \left(\fint_Q \left(\fint_Q |W(x)W^{-1}(y)|_{\mathrm{op}}^{p'}\,dy\right)^{\frac{p}{p'}}\,dx\right)^{\frac{1}{p}} < \infty.$$

In the past 10 years there has been a great deal of research in the study of matrix weights. In this talk we will discuss recent work and open problems in this area, including: the sharp constant problem for singular integrals; Rubio de Francia extrapolation and the machinery of convex set-valued functions developed to prove this result; multiplier weak-type norm inequalities; extensions to Banach functions spaces, particularly the variable Lebesgue spaces; and convex body sparse domination for off-diagonal operators.

This talk will include work with my colleagues Marcin Bownik, Josh Isralowitz, and Kabe Moen, with postdocs Fatih Şirin and Brandon Sweeting, and with PhD students Michael Penrod and Aapo Laukkarinen.

Lecture notes for an earlier version of this talk can be found on arXiv:
`https://arxiv.org/abs/2508.13352`