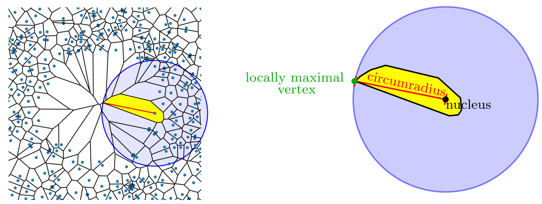
Some features of the typical Poisson-Voronoi / Poisson-Delaunay cell

Part 1 - Distribution tail of the circumradius of the typical Poisson-Voronoi cell.
We consider the homogeneous Poisson-Voronoi tessellation on $\mathbb{R}^d$, constructed from a Poisson point process on $\mathbb{R}^d$ with the Lebesgue measure as the intensity measure. The typical cell of the Poisson-Voronoi tessellation corresponds to a cell chosen uniformly at random from the set of cells intersecting a large window. We study the *circumradius* $\mathrm{R}_{\mathrm{circ}}$ of the typical Poisson-Voronoi cell: it is the smallest radius that a ball centered on the nucleus must have to fully cover it, see figure below.

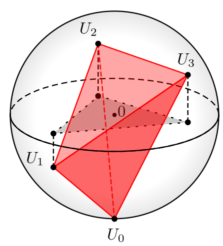


We obtain the tail probability of the circumradius of the typical Poisson-Voronoi cell

$$\mathbb{P}(\mathrm{R}_{\mathrm{circ}} \geq t) = C_d (d\kappa_d)^d t^{d(d-1)} e^{-\kappa_d t^d} + \mathcal{O}(t^{d(d-2)} e^{-\kappa_d t^d}) \text{ as } t \to \infty \tag{1}$$

where κ_d is the volume of the d-dimensional unit ball. The constant C_d arises from the study of the local shape of the cell around the vertex realizing the circumradius. Its value is given by the expectation of the volume of the random simplex $\mathrm{Conv}(U_0, ..., U_d)$ determined by i.i.d. uniform random points on the $(d-1)$-dimensional unit sphere $U_0, ..., U_d$ that satisfy an additional geometric condition, see figure below. More precisely,

$$C_d = \mathbb{E}\left[\mathrm{Vol}_d(\mathrm{Conv}(U_0, ..., U_d)) \mathbb{1}_{\{0 \in \mathrm{Conv}(\mathrm{Proj}_{U_0^\perp}(U_1), ..., \mathrm{Proj}_{U_0^\perp}(U_1))\}}\right] = \frac{1}{2^{d-1}\sqrt{\pi}(d-1)!} \frac{\Gamma\left(\frac{d}{2}\right)^d}{\Gamma\left(\frac{d+1}{2}\right)^{d-1}}. \tag{2}$$



Incidentally, the knowledge of the exact value of C_d proves a long-standing conjecture: the extremal index of the circumradius for the Poisson-Voronoi tessellation in $\mathbb{R}^d$ is equal to $\frac{1}{2d}$.

Part 2 - Law of the length of some edges of the typical Poisson-Delaunay cell
The homogeneous Poisson-Delaunay tessellation is the dual of the homogeneous Poisson-Voronoi tessellation. It is constructed from a Poisson point process on $\mathbb{R}^d$ of intensity measure the Lebesgue measure: whenever $(d+1)$ points lie on a sphere that contains no points of the process in its interior, their convex hull is a cell of the Poisson-Delaunay tessellation. The typical Poisson-Delaunay cell is a cell chosen uniformly at random within the set of cells intersecting a large window. Let $X_0, ..., X_d$ be the vertices of the typical Poisson-Delaunay cell (up to translation). We find a representation of the joint law of the lengths $(\|X_1 - X_0\|, ..., \|X_d - X_0\|)$. Explicitly,

$$(\|X_1 - X_0\|^2, ..., \|X_d - X_0\|^2) \overset{\text{law}}{=} R^2 (V_{\sigma(1)}, ..., V_{\sigma(d)}) \tag{3}$$

where R is a random variable following a generalized Gamma distribution, σ is a uniform random permutation and $(V_1, ..., V_d) \overset{\text{law}}{=} (CB_1D_1, ..., CB_{d-1}D_{d-1}, CB_d)$ where the random variables $B_1, ..., B_d$, $D_1, ..., D_{d-1}$, C are all independent and Beta distributed with explicit parameters depending only on the dimension d.

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