# A bounded planar set for which the Steiner problem has no solution

Given a set $A \subset \mathbb{R}^2$, consider the problem of minimizing the one-dimensional Hausdorff measure of a set $S$ such that $A \cup S$ is connected. For compact $A$, existence of a minimizer is known.

We construct a bounded open planar set $A$ with countably many connected components for which the infimum is not attained. More precisely, $A$ can be connected by adding sets of arbitrarily small positive length, so the infimum is zero, but no set of zero length connects $A$.

I will describe and explain the construction. I will also discuss why infinitely many connected components are necessary.

Based on joint work with Eugene Stepanov. See the preprint "A planar set that can be connected by arbitrarily short sets but by no zero-length set".

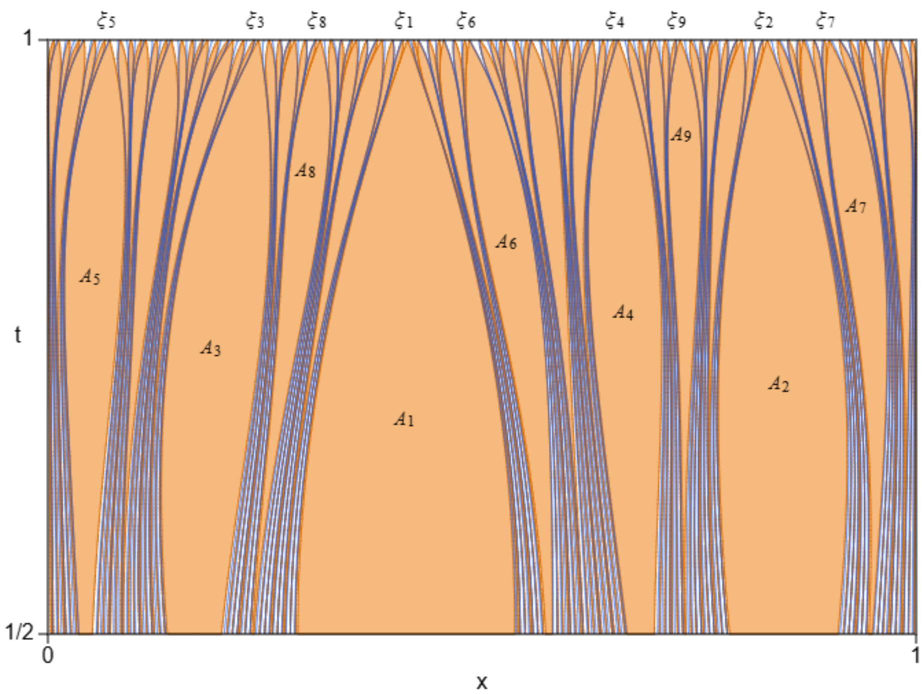


Figure 1: The first hundred components $A_1, \ldots, A_{100}$ (orange sets).