Reduction by stage for finite W-algebras

(Work in progress)

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Big picture of finite W-algebras

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$$\begin{CD}
\mathscr{U}(\mathfrak{g}) @>{\text{classical limit}}>> \mathscr{S}(\mathfrak{g}) \\
@V{\text{Hamiltonian}}V{\text{reduction}}V @V{\text{Hamiltonian}}V{\text{reduction}}V \\
\mathscr{W}_{\mathrm{qu}}(\mathfrak{g}, f) @>>{\text{classical limit}}> \mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f)
\end{CD}$$

where

- $\mathscr{W}_{\mathrm{qu}}(\mathfrak{g}, f)$ is the quantum finite W-algebra associated to $(\mathfrak{g}, f)$ (associative algebra),
- $\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f)$ is the classical finite W-algebra associated to $(\mathfrak{g}, f)$ (Poisson algebra).

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Definition (Slodowy slice)
$S_f := \Phi(f + \operatorname{Ker}\operatorname{ad}(e)) = \chi + \operatorname{Ker}\operatorname{ad}^*(e)$.

Classical Hamiltonian reduction

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One can find a unipotent subgroup M of G such that one has a Hamiltonian action

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- [Gan-Ginzburg] construction of M for the Dynkin grading.
- [Brundan-Goodwin] construction for any good grading.

Reduction by stage

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With some "stage conditions", one gets a Poisson diffeomorphism:

$$X/\!/_\mu M \cong (X/\!/_{\mu_N} N)/\!/_{\mu_K} K.$$

Stage reduction for Slodowy slice

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Question
$f_1, f_2 \in \mathfrak{g}$ are two nilpotent elements.

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M_i denotes the unipotent group and μ_i the moment map associated to f_i, $i = 1, 2$.

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2. $M_0 \times S_{f_2} \cong {\mu_0}^{-1}(0)$,
3. *the following reduction by stage holds:*

$$\underbrace{\mathfrak{g}^* /\!/_{\mu_2} M_2}_{S_{f_2}} \cong (\underbrace{\mathfrak{g}^* /\!/_{\mu_1} M_1}_{S_{f_1}}) /\!/_{\mu_0} M_0.$$

Translation in Poisson algebra setting

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Definition (Classical W-algebra)
$\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_i) := (\mathscr{S}(\mathfrak{g})/I_{\mathrm{cl},i})^{\mathrm{ad}(\mathfrak{m}_i)}$, where $I_{\mathrm{cl},i}$ is the ideal spanned by $y - \chi_i(y)$, $y \in \mathfrak{m}_i$.

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Remark
$\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_i) \cong \mathbf{C}[S_{f_i}]$.

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Remark
$\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_i) \cong \mathbf{C}[S_{f_i}]$.

Theorem (Genra-J.)
Under the same assumptions as in the previous theorem, we have a Poisson algebra isomorphism

$$(\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_1)/I_{\mathrm{cl},0})^{\mathrm{ad}(\mathfrak{m}_0)} \xrightarrow{\sim} \mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_2)$$

$$(F \bmod I_{\mathrm{cl},1}) \bmod I_{\mathrm{cl},0} \longmapsto F \bmod I_{\mathrm{cl},2}$$

where $I_{\mathrm{cl},0}$ is the ideal of $\mathscr{W}_{\mathrm{cl}}(\mathfrak{g}, f_1)$ spanned by

$$y - \chi_2(y) \bmod I_{\mathrm{cl},1},$$

for $y \in \mathfrak{m}_0$.

Stage reduction for quantum W-algebras

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Definition (Quantum W-algebra)
$\mathscr{W}_{\mathrm{qu}}(\mathfrak{g}, f_i) := (\mathscr{U}(\mathfrak{g})/I_{\mathrm{qu},i})^{\mathrm{ad}(\mathfrak{m}_i)}$, where $I_{\mathrm{qu},i}$ is the left ideal spanned by $y - \chi_i(y)$, $y \in \mathfrak{m}_i$.

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Under the same assumptions as in the previous theorem, we have an algebra isomorphism

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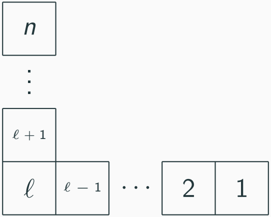
For $0 \leqslant \ell \leqslant n$, let us consider the nilpotent orbit $\mathbf{O}_\ell$ corresponding to the following Young diagram.



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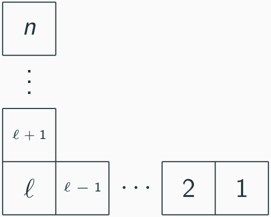

Theorem (Genra-J.)
Pick two hook type nilpotent elements. Then both reduction by stage theorems hold. For $\ell_1 < \ell_2$:

$$\begin{array}{ccc} & \mathscr{S}(\mathfrak{g}) \text{ or } \mathscr{U}(\mathfrak{g}) & \\ \swarrow & & \searrow \\ \mathscr{W}(\mathfrak{g}, \mathbf{O}_{\ell_1}) & \longrightarrow & \mathscr{W}(\mathfrak{g}, \mathbf{O}_{\ell_2}) \end{array}$$

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Question. Can we find links between categories of representations of different W-algebras thanks to this stage reduction? (we have a Skryabin equivalence by stage statement)

Thank you for your attention!