Long and pointy Poisson-Voronoi cells

Distribution tail of the circumradius of the typical cell

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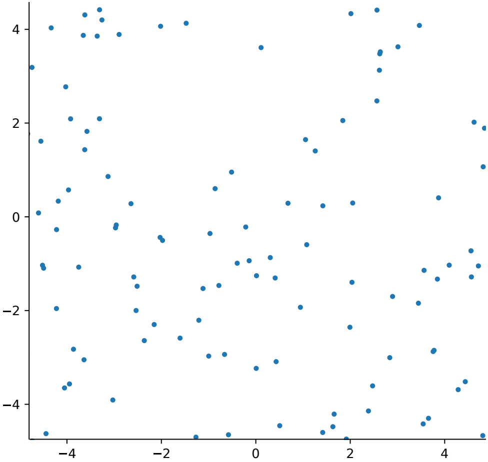
(0) Introduction and setup of the problem

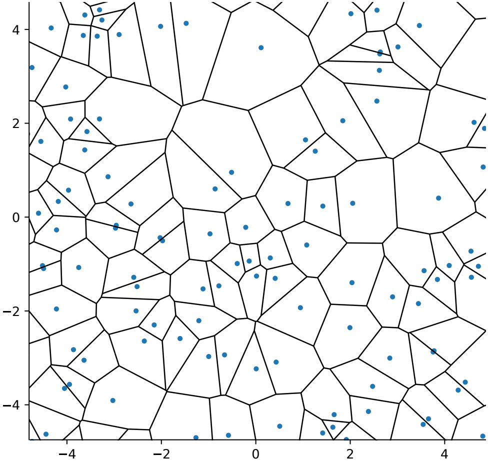
Construction of the homogeneous Poisson-Voronoi tessellation 1/2

Φ a Poisson point process on $\mathbb{R}^d$ with intensity measure the Lebesgue measure.



Construction of the homogeneous Poisson-Voronoi tessellation 2/2

To $X \in \Phi$ associate the cell $Cell(X, \Phi) = \{y \in \mathbb{R}^d : \forall X' \in \Phi, \|y - X\| \leq \|y - X'\|\}$.



The collection of all cells constitutes the homogeneous Poisson-Voronoi tessellation.

The typical cell and some observables

Typical cell

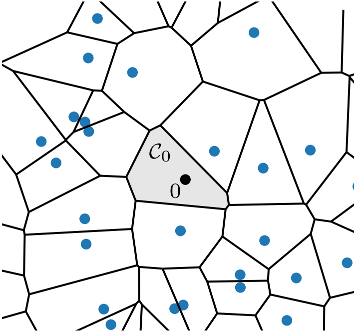
$$\mathcal{C}_0 = \mathcal{C}(0, \Phi \cup \{0\}),$$

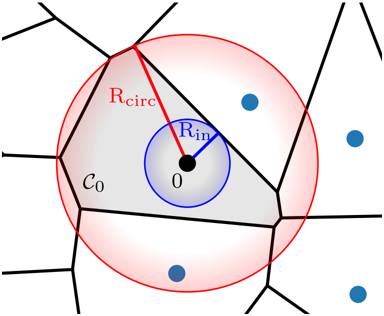
circumradius of the typical cell

$$R_{\text{circ}} = \min\{r \geq 0 : \mathcal{C}_0 \subseteq \mathcal{B}_r(0)\},$$

inradius of the typical cell

$$R_{\text{in}} = \max\{r \geq 0 : \mathcal{B}_r(0) \subseteq \mathcal{C}_0\}.$$

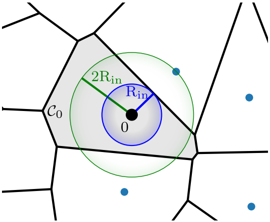




Previous works

Explicit law of the inradius:

$$\mathbb{P}\left(R_{\text{in}} \geq t\right) = \mathbb{P}(\Phi \cap \mathcal{B}(0, 2t) = \emptyset) = e^{-\text{Vol}_d(\mathcal{B}(0,2t))} = e^{-\kappa_d (2t)^d}.$$



In [1], double bound on the tail probability of the circumradius. For the 2-dimensional case, for t large enough,

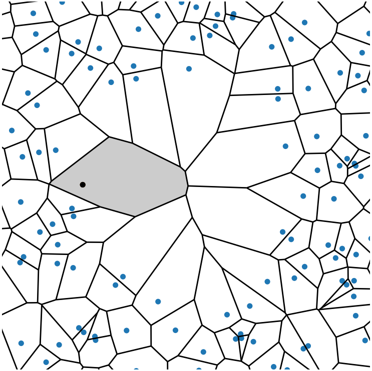
$$2\pi t^2 e^{-\pi t^2} \leq \mathbb{P}\left(R_{\text{circ}} \geq t\right) \leq 4\pi t^2 e^{-\pi t^2}.$$

Our question:

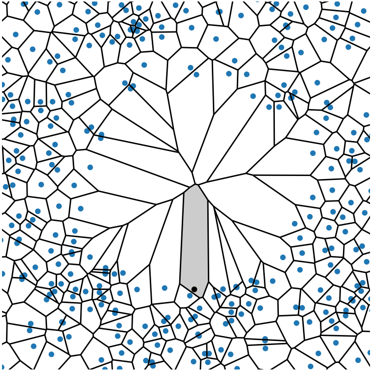
$$\mathbb{P}(R_{\text{circ}} \geq t) \overset{t\to\infty}{\sim} ?$$

[1] P. Calka. The distributions of the smallest disks containing the Poisson-Voronoi typical cell and the Crofton cell in the plane. *Adv. in Appl. Probab.* **34**, 702-717 (2002)

Cells with large circumradius



Cell with circumradius ≥ 3



Cell with circumradius ≥ 5

Our guess:

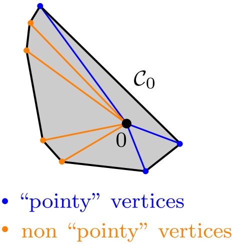
$$\mathbb{P}(R_{\text{circ}} \geq t) \underset{t\to\infty}{\sim} C_d \, t^{n_d} \, e^{-\kappa_d t^d}$$

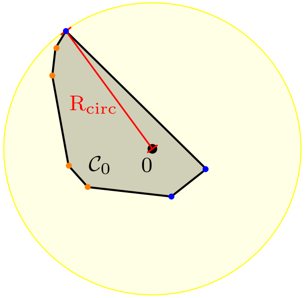
with C_d a constant, n_d a polynomial in d, $\kappa_d = \mathrm{Vol}_d(\mathcal{B}_{\mathbb{R}^d})$.

The circumradius as the distance to a "pointy" vertex

Notation. A vertex of $\mathcal{C}_0$ is said to be "pointy" if, locally around it, it is the point of $\mathcal{C}_0$ maximizing the distance to the nucleus 0.

$$R_{\text{circ}} = \text{dist}\left(0,\ \text{one of the "pointy" vertices of } \mathcal{C}_0\right)$$





Idea

Consider

$$\mathcal{V}_{\max}^{\geq t} = \{ \text{ all “pointy” vertices of } \mathcal{C}_0 \text{ at distance } \geq t \text{ from } 0 \ \}$$

then

$$\{R_{\text{circ}} \geq t\} = \{\#\mathcal{V}_{\max}^{\geq t} \geq 1\}.$$

We access the tail of the distribution of R_{circ} from the study of $\#\mathcal{V}_{\max}^{\geq t}$:

$$\mathbb{P}(R_{\text{circ}} \geq t) = \mathbb{P}(\#\mathcal{V}_{\max}^{\geq t} \geq 1) \underbrace{\overset{t\to\infty}{\sim}}_{(**)} \underbrace{\mathbb{E}\left[\#(\mathcal{V}_{\max}^{\geq t})\right]}_{(*)}.$$

(*) is computed in sections (1),(2).
(**) arises from the asymptotic negligibility of $\mathbb{E}[\#((\mathcal{V}_{\max}^{\geq t})^2_{\neq})]$ with respect to $\mathbb{E}[\#(\mathcal{V}_{\max}^{\geq t})]$ as $t \to \infty$: section (3).

Outline

To show:

$$\mathbb{P}\left(R_{\text{circ}} \geq t\right) \underset{t\to\infty}{\sim} \mathbb{E}\left[\#(\mathcal{V}^{\geq t}_{\max})\right] = C_d t^{n_d} e^{\kappa_d t^d}.$$

(0) Introduction and setup

(1) Computation of $\mathbb{E}[\#(\mathcal{V}^{\geq t}_{\max})]$ up to multiplication by a constant: find n_d

(2) Computation of the **expected volume of a random simplex**: find C_d

(3) Negligibility of $\mathbb{E}[\#((\mathcal{V}^{\geq t}_{\max})^2_{\neq})]$, **Theorem** and consequence regarding the **extremal index**

(1) Setup of $\mathbb{E}[\#(\mathcal{V}_{\max}^{\geq t})]$: finding n_d

Characterization of "pointy" vertices

Lemma 1. Let c be a vertex of $\mathcal{C}_0$ at equal distance from 0 and $X_1, ..., X_d \in \Phi$ and

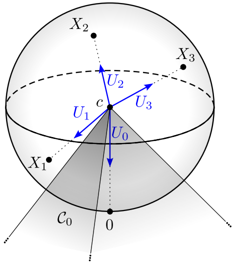
$$U_0 = \frac{c}{\|c\|} \text{ and } U_i = \frac{X_i - c}{\|X_i - c\|} \text{ for } i = 1..d$$

then, c is "pointy" iff

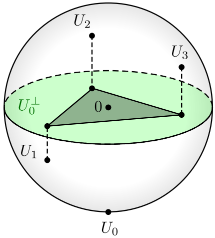
$$0 \in \mathrm{Conv}\left(P_{U_0^\perp}(U_1), .., P_{U_0^\perp}(U_d)\right)$$

where $P_{U_0^\perp}$ denotes the orthogonal projection onto $U_0^\perp$ and Conv the convex hull of the specified points.

"pointy" vertex c

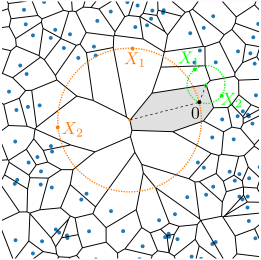


$0 \in \mathrm{Conv}(P_{U_0^\perp}(U_1), P_{U_0^\perp}(U_2), P_{U_0^\perp}(U_3))$



An integral expression for the expected number of locally maximal vertices 1/3

Wanted: $\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right]$.



First, we express $\#\mathcal{V}_{\max}^{\geq t}$ as a sum over the points of the process

$$\#\mathcal{V}_{\max}^{\geq t} = \frac{1}{d!} \sum_{(X_1,\ldots,X_d)\in\Phi^d_{\neq}} \mathbb{1}_{\mathrm{Center}(\mathrm{Ball}(0,X_1,\ldots,X_d))\in\mathcal{V}_{\max}^{\geq t}} .$$

This formulation allows us to use two powerful tools. The 1st tool is the Mecke formula

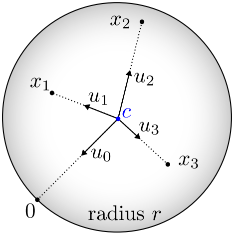
$$\mathbb{E}\left[\sum_{X\in\Phi} f(X,\Phi)\right] = \int_{\mathbb{R}^d} \mathbb{E}\left[f(x,\Phi\cup\{x\})\right] dx$$

and gives an explicit integral expression

$$\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right] = \frac{1}{d!} \int_{(\mathbb{R}^d)^d} \mathbb{E}\left[\mathbb{1}_{\mathrm{Center}(\mathrm{Ball}(0,x_1,\ldots,x_d))\in\mathcal{V}_{\max}^{\geq t}(\Phi\cup\{x_1,\ldots,x_d\})}\right] dx_1\ldots dx_d$$

An integral expression for the expected number of locally maximal vertices 2/3

$$\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right] = \frac{1}{d!}\int_{(\mathbb{R}^d)^d} \mathbb{E}\left[1_{\mathrm{Center}(\mathrm{Ball}(0,x_1,\ldots,x_d))\in\mathcal{V}_{\max}^{\geq t}(\Phi\cup\{x_1,\ldots,x_d\})}\right] dx_1\ldots dx_d$$

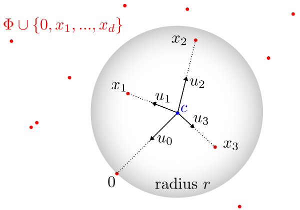


... and the 2nd tool is a spherical Blaschke-Petkantschin change of variables, that gives

$$\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right] = \int_0^\infty \int_{(\mathcal{S}_{\mathbb{R}^d})^d} \mathbb{E}\left[1_{c\in\mathcal{V}_{\max}^{\geq t}(\Phi\cup\{x_1,\ldots,x_d\})}\right] r^{d^2-1}\Delta_d(u_0,..,u_d)du_0\ldots du_d dr$$

where $\Delta_d(u_0,..,u_d) = \mathrm{Vol}_d(\mathrm{Conv}(u_0,..,u_d))$.

An integral expression for the expected number of locally maximal vertices 3/3



$$\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right] = \int_0^\infty \int_{(\mathcal{S}_{\mathbb{R}^d})^d} \mathbb{E}\left[1_{c \in \mathcal{V}_{\max}^{\geq t}(\Phi \cup \{x_1, ..., x_d\})}\right] r^{d^2-1} \Delta_d(u_0, .., u_d) du_0 ... du_d dr$$

The condition $\{c \in \mathcal{V}_{\max}^{\geq t}(\Phi \cup \{x_1, ..., x_d\})\}$ is satisfied precisely when

1. c is a vertex of the tessellation of $\Phi \cup \{x_1, ..., x_d\}$ i.e. no points of Φ falls in $\mathrm{Ball}(0, x_1, ..., x_d)$: an event of probability $\mathbb{P} = e^{-\kappa_d r^d}$
2. c is sufficiently far, i.e. $r \geq t$: deterministic
3. c is "pointy", i.e. $0 \in \mathrm{Conv}(P_{u_0^\perp}(u_1), ..., P_{u_0^\perp}(u_d))$: deterministic

The integral splits and we obtain

$$\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right] = \int_t^\infty r^{d^2-1} e^{-\kappa_d r^d} dr \cdot \int_{(\mathcal{S}_{\mathbb{R}^d})^d} \Delta_d(u_0, .., u_d) 1_{0 \in \mathrm{Conv}(P_{u_0^\perp}(u_1), ..., P_{u_0^\perp}(u_d))} du_0 ... du_d$$

Summary and value of n_d

Denoting $\kappa_d = \mathrm{Vol}_d(\mathcal{B}_{\mathbb{R}^d})$, $\sigma_d = d\kappa_d = \mathrm{Vol}_{d-1}(\mathcal{S}_{\mathbb{R}^d})$ and $U_0, .., U_d$ i.i.d. $\mathrm{Unif}(\mathcal{S}_{\mathbb{R}^d})$,

$$\mathbb{E}\left[\#(\mathcal{V}_{\max}^{\geq t})\right] = \left(\frac{1}{d\kappa_d}\sum_{i=0}^{d-1} t^{di}\kappa_d^{i-d+1}\frac{(d-1)!}{i!}e^{-\kappa_d t^d}\right)\sigma_d^{d+1}\mathbb{E}\left[\Delta_d(U_0, .., U_d)1_{0\in\mathrm{Conv}(P_{U_0^\perp}(U_1,..,U_d))}\right]$$

$$\underset{t\to\infty}{\sim} C_d t^{d(d-1)} e^{-\kappa_d t^d}$$

so

$$n_d = d(d-1),$$

$$C_d = \sigma_d^d \mathbb{E}\left[\Delta_d(U_0, .., U_d)1_{0\in\mathrm{Conv}(P_{U_0^\perp}(U_1,..,U_d))}\right].$$

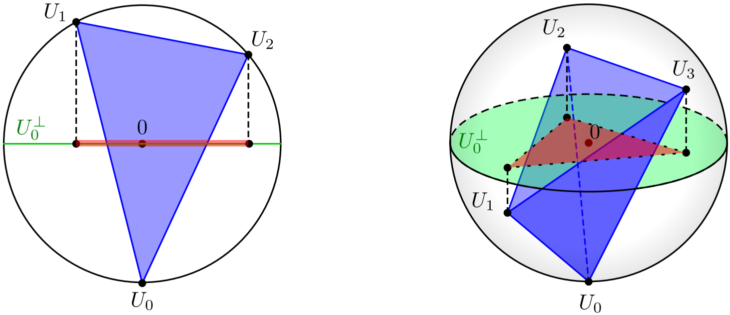
(2) Expectation of a random simplex: finding C_d

Previous work (Miles)

Our question:

$$\mathbb{E}\left[\Delta_d(U_0,..,U_d)1_{0\in \mathrm{Conv}(P_{U_0^\perp}(U_1,..,U_d))}\right] = ?$$

$\Delta_2(U_0,U_1,U_2)1_{0\in \mathrm{Conv}(P_{U_0^\perp}(U_1,U_2))}$ $\Delta_3(U_0,U_1,U_2,U_3)1_{0\in \mathrm{Conv}(P_{U_0^\perp}(U_1,U_2,U_3))}$



By [3], for $U_0,..,U_d$ i.i.d. uniformly distributed random variables on the $(d-1)$-dimensional sphere $\mathcal{S}_{\mathbb{R}^d}$:

$$\mathbb{E}\left[\Delta_d(U_0,..,U_d)\right] = \frac{1}{\sqrt{\pi}d!}\frac{\Gamma(\frac{d}{2})\Gamma(\frac{d^2+1}{2})}{\Gamma(\frac{d^2}{2})}\left(\frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d+1}{2})}\right)^d$$

[3] R.E. Miles. Isotropic random simplices. *Adv. in App. Prob.* **Vol. 3, No. 2**, 353-382 (1971)

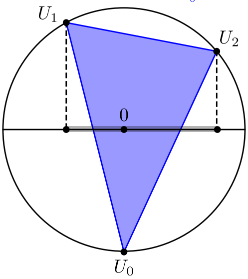
Reduction to the expectation of a subsimplex

Lemma 2. For $\mathrm{U}_0, \ldots, \mathrm{U}_d$ i.i.d. Unif($\mathcal{S}_{\mathbb{R}^d}$):

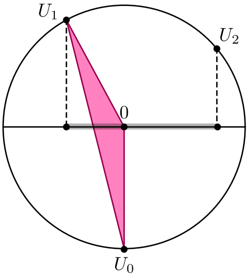
$$\mathbb{E}\left[\Delta_d(\mathrm{U}_0, \ldots, \mathrm{U}_d) 1_{0 \in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), \ldots, P_{\mathrm{U}_0^\perp}(\mathrm{U}_d))}\right]$$

$$= d\, \mathbb{E}\left[\Delta_d(0, \mathrm{U}_0, \ldots, \mathrm{U}_{d-1}) 1_{0 \in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), \ldots, P_{\mathrm{U}_0^\perp}(\mathrm{U}_d))}\right]$$

$\Delta_2(U_0, U_1, U_2) 1_{0 \in \mathrm{Conv}(P_{U_0^\perp}(U_1, U_2))}$



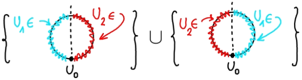
$\Delta_2(0, U_0, U_1) 1_{0 \in \mathrm{Conv}(P_{U_0^\perp}(U_1, U_2))}$

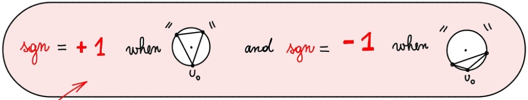


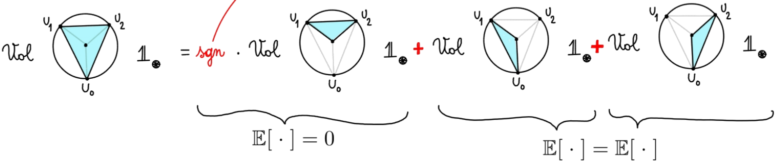
Reduction to the expectation of a subsimplex - proof for $d = 2$

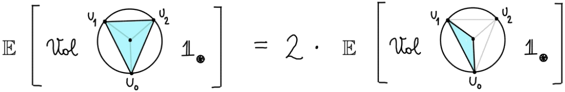
$$\mathbb{E}\left[\Delta_d(\mathrm{U}_0,\mathrm{U}_1,\mathrm{U}_2)1_{0\in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1),P_{\mathrm{U}_0^\perp}(\mathrm{U}_2))}\right] = 2\,\mathbb{E}\left[\Delta_d(0,\mathrm{U}_0,\mathrm{U}_1)1_{0\in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1),P_{\mathrm{U}_0^\perp}(\mathrm{U}_2))}\right]$$

Proof.

Writing $(*) = \{0 \in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), P_{\mathrm{U}_0^\perp}(\mathrm{U}_2))\} =$ 



we have: 

hence: 

□

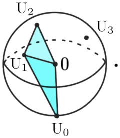
Recursive relation on the expectation of the subsimplex

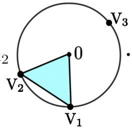
Lemma 3. Let $\mathrm{U}_0, \dots, \mathrm{U}_d$ be i.i.d. $\mathrm{Unif}(\mathcal{S}_{\mathbb{R}^d})$ and $\mathrm{V}_1, \dots, \mathrm{V}_d$ be i.i.d. $\mathrm{Unif}(\mathcal{S}_{\mathbb{R}^{d-1}})$. Then

$$\mathbb{E}\left[\Delta_d(0, \mathrm{U}_0, \dots, \mathrm{U}_{d-1}) 1_{0 \in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), \dots, P_{\mathrm{U}_0^\perp}(\mathrm{U}_d))}\right]$$

$$= \frac{1}{2d}\left(\frac{B(\frac{d}{2}, \frac{1}{2})}{B\left(\frac{d-1}{2}, \frac{1}{2}\right)}\right)^{d-1} \mathbb{E}\left[\Delta_{d-1}(0, \mathrm{V}_1, \dots, \mathrm{V}_{d-1}) 1_{0 \in \mathrm{Conv}(P_{\mathrm{V}_1^\perp}(\mathrm{V}_2), \dots, P_{\mathrm{V}_1^\perp}(\mathrm{V}_d))}\right]$$

where $B(\cdot, \cdot)$ denotes the Beta function.

$\mathbb{E}\Big[\mathrm{Vol}_3$ 

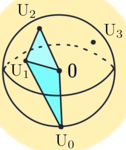
$\cdot \mathbb{1}_{0 \in \mathrm{Conv}\left(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), P_{\mathrm{U}_0^\perp}(\mathrm{U}_2), P_{\mathrm{U}_0^\perp}(\mathrm{U}_3)\right)}\Big] = \frac{1}{3 \cdot 2}\left(\frac{B\left(\frac{3}{2}, \frac{1}{2}\right)}{B\left(\frac{2}{2}, \frac{1}{2}\right)}\right)^2 \mathbb{E}\Big[\mathrm{Vol}_2$ 

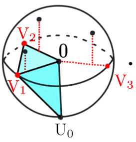
$\cdot \mathbb{1}_{0 \in \mathrm{Conv}\left(P_{\mathrm{V}_1^\perp}(\mathrm{V}_2), P_{\mathrm{V}_1^\perp}(\mathrm{V}_3)\right)}\Big]$

Sketch of proof from $d = 3$ to $d = 2$

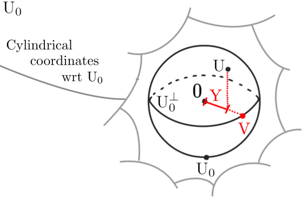
$$\mathbb{E}\left[\Delta_3(0,\mathrm{U}_0,\mathrm{U}_1,\mathrm{U}_2)1_{0\in\mathrm{Conv}\left(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1),P_{\mathrm{U}_0^\perp}(\mathrm{U}_2),P_{\mathrm{U}_0^\perp}(\mathrm{U}_3)\right)}\right] = \frac{1}{3\cdot 2}\left(\frac{B\left(\frac{3}{2},\frac{1}{2}\right)}{B\left(\frac{2}{2},\frac{1}{2}\right)}\right)^2 \mathbb{E}\left[\Delta_2(0,\mathrm{V}_1,\mathrm{V}_2)1_{0\in\mathrm{Conv}\left(P_{\mathrm{V}_1^\perp}(\mathrm{V}_2),P_{\mathrm{V}_1^\perp}(\mathrm{V}_3)\right)}\right]$$

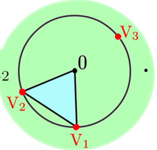
Proof.

Vol_3 

$\cdot\, \mathbb{1}_{0\in\mathrm{Conv}\left(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1),P_{\mathrm{U}_0^\perp}(\mathrm{U}_2),P_{\mathrm{U}_0^\perp}(\mathrm{U}_3)\right)} = \mathrm{Y}_1\mathrm{Y}_2\,\mathrm{Vol}_3$ 

$\cdot\, \mathbb{1}_{0\in\mathrm{Conv}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})} =$

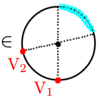


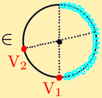
$= \frac{1}{3}\mathrm{Y}_1\mathrm{Y}_2\,\mathrm{Vol}_2$ 

$\cdot\, \mathbb{1}_{0\in\mathrm{Conv}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}$

hence: $\mathbb{E}\left[\Delta_3(0,\mathrm{U}_0,\mathrm{U}_1)1_{0\in\mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1),P_{\mathrm{U}_0^\perp}(\mathrm{U}_2))}\right] = \frac{1}{3}\mathbb{E}\left[Y_1\right]^2\mathbb{E}\left[\Delta_2(0,\mathrm{V}_1,\mathrm{V}_2)1_{0\in\mathrm{Conv}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}\right]$

In expectation and with an argument of symmetry, we can transform

$\{0\in\mathrm{Conv}\,(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})\} = \Big\{\mathbf{V_3}\in$ 

$\Big\} \xRightarrow{\text{into}} \{0\in\mathrm{Conv}\left(P_{\mathrm{V}_1^\perp}(\mathrm{V}_2),P_{\mathrm{V}_1^\perp}(\mathrm{V}_3)\right)\} = \Big\{\mathbf{V_3}\in$ 

$\Big\}$ $\square$

The expectation of the number of "pointy" vertices at large distance

Proposition 1. Let $\mathrm{U}_0, ..., \mathrm{U}_d$ be i.i.d. uniform random variables on the $(d-1)$-dimensional sphere $\mathcal{S}_{\mathbb{R}^d}$, then

$$\mathbb{E}\left[\Delta_d(\mathrm{U}_0, ..., \mathrm{U}_d) 1_{0 \in \mathrm{Conv}(P_{\mathrm{U}_0^\perp}(\mathrm{U}_1), ..., P_{\mathrm{U}_0^\perp}(\mathrm{U}_d))}\right] = \frac{1}{\sqrt{\pi} 2^{d-1}(d-1)!} \frac{\Gamma\left(\frac{d}{2}\right)^d}{\Gamma\left(\frac{d+1}{2}\right)^{d-1}}$$

and therefore,

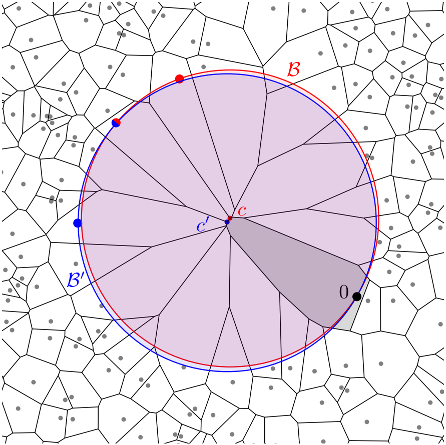
$$\mathbb{E}\left[\#(\mathcal{V}_{\max}^{\geq t})\right] \underset{t \to \infty}{\sim} C_d t^{d(d-1)} e^{-\kappa_d t^d}$$

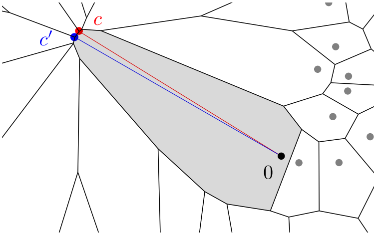
where

$$C_d = \frac{2\pi^{\frac{d^2-1}{2}}}{(d-1)!\Gamma\left(\frac{d+1}{2}\right)^{d-1}}$$

(3) Negligibility of $\mathbb{E}[\#((\mathcal{V}_{\max}^{\geq t})^2_{\neq})]$ and Theorem

Most likely realization of a distinct pair of "pointy" vertices





With similar methods, one can show that $\mathbb{E}\left[\#\left(\mathcal{V}_{\max}^{\geq t}\right)^2_{\neq}\right]$ is negligible wrt $\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right]$.

Proposition 2. As $t \to \infty$,

$$\mathbb{E}\left[\#\left(\mathcal{V}_{\max}^{\geq t}\right)^2_{\neq}\right] = \mathcal{O}\left(t^{d(d-2)}e^{-\kappa_d t^d}\right) = o\left(\mathbb{E}\left[\#\mathcal{V}_{\max}^{\geq t}\right]\right).$$

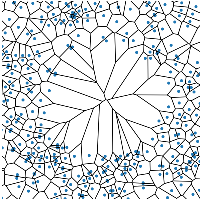
Theorem and implications 1/2

Theorem. As $t \to \infty$,

$$\mathbb{P}(R_{\text{circ}} \geq t) = C_d t^{d(d-1)} e^{-\kappa_d t^d} + \mathcal{O}(t^{d(d-2)} e^{-\kappa_d t^d})$$

where κ_d is the volume of the d-dimensional unit ball and $C_d = \frac{2\pi^{\frac{d^2-1}{2}}}{(d-1)!\Gamma(\frac{d+1}{2})^{d-1}}$.

$$\text{extremal index} = \frac{1}{\mathbb{E}[\#\text{ exceedances in a cluster of exceedances }]}$$



distribution tail of the circumradius of the typical cell

limit law of the max circumradius in an increasing window (properly rescaled)

[2] P. Calka & N. Chenavier. Extreme values for characteristic radii of a Poisson-Voronoi tessellation. *Extremes* **17**, 359-385 (2014)

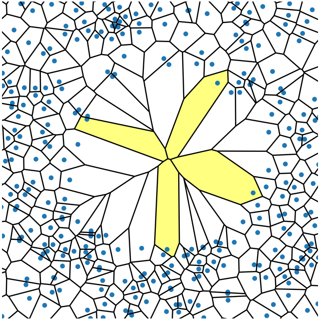
Theorem and implications 2/2

By merging our result with [2] we conclude that

Theorem. For the circumradius of the homogeneous Poisson-Voronoi tessellation:

$$\text{extremal index} = \frac{1}{2d}$$

proving the conjecture proposed in [1].



[1] P. Calka. The distributions of the smallest disks containing the Poisson-Voronoi typical cell and the Crofton cell in the plane. *Adv. in Appl. Probab.* **34**, 702-717 (2002)

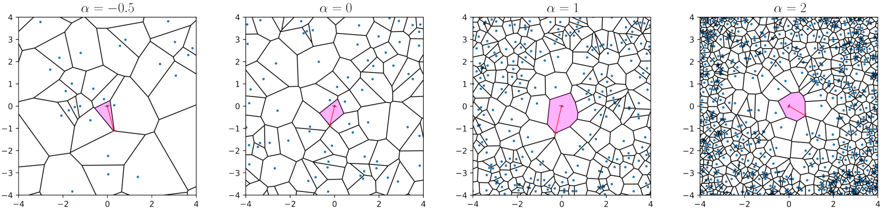
[2] P. Calka & N. Chenavier. Extreme values for characteristic radii of a Poisson-Voronoi tessellation. *Extremes* **17**, 359-385 (2014)

Generalization to a parametric model

For $\alpha > -d$ let Φ_α be a Poisson point process on $\mathbb{R}^d$ with isotropic measure

$$m_\alpha(dx) = \|\mathrm{x}\|^\alpha dx,$$

where dx denotes the Lebesgue measure. Let R^α_{circ} be the maximal distance from 0 to a vertex of the cell associated to 0 in the Voronoi tessellation generated by $\Phi_\alpha \cup \{0\}$.



Theorem. As $t \to \infty$,

$$\mathbb{P}\left(R^\alpha_{\text{circ}} \geq t\right) = C_{d,\alpha} t^{(d+\alpha)(d-1)} e^{-\kappa_{d,\alpha} t^{d+\alpha}} + \mathcal{O}\left(t^{(d+\alpha)(d-2)} e^{-\kappa_{d,\alpha} t^{d+\alpha}}\right)$$

where

$$C_{d,\alpha} = \frac{2^{d^2+(d-1)(\alpha-2)} \pi^{\frac{d(d-1)}{2}}}{(d-1)!} \left(\frac{\Gamma\left(\frac{d+\alpha}{2}\right)}{\Gamma\left(d+\frac{\alpha}{2}\right)} \right)^{d-1} \text{ and } \kappa_{d,\alpha} = \frac{2^{d+\alpha} \pi^{\frac{d-1}{2}}}{(d+\alpha)} \frac{\Gamma\left(\frac{d+\alpha+1}{2}\right)}{\Gamma\left(d+\frac{\alpha}{2}\right)}.$$

Thank you for your attention!