Éléments de théorie spectrale et Analyse, Cours intensif, M2AAG - Résumé/Summary

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I do the summary and I'll prepare notes in english, and if I understand the class will be held in english unless essentially no one is disturbed by French.

The purpose of this class is to give you bases on spectral theory. Some of you may already know about this; sorry then. I should discuss the following items:
- Compact operators, basic stability results, some examples, and a description of the spectrum.
- A little more about the spectrum for bounded operators; two words about the resolvent.
- The spectral decomposition (one could say, diagonalization) of compact self-adjoint operators
- Then a little more on bounded self-adjoint operators, leading to the functional calculus (how to define functions $f(u)$ when, say, f is a continuous function and u is a self-adjoint operator).
- Finally at least definitions and statements about the spectral resolution of self-adjoint operators (i.e., consider $f(u)$ when f is a characteristic function of interval, then a bounded measurable function). We'll talk about bounded operators only, to avoid technical issues (like, dealing with domains), but the application we have in mind concerns the unbounded operator $-\Delta$ in a domain.

In a second part, I will try to explain what happens with the theorems above in the case of the Laplacian on a domain. This will give me an excuse to present some results in analysis (and then say things on the spectrum of $-\Delta$). Items that I could treat, if time allows, are:
- Theorems of Lusin and Egorov (useful general tools)
- Maximal functions and the Lebesgue differentiation theorem (very good to know)
- Sobolev spaces (just one derivative), Poincaré, and Sobolev inequalities (this one I have to treat)
- The compactness theorem of Rellich-Kondrachov
- How to solve of $-\Delta u = f$ in a bounded domain (simple cases)
- and the application to the diagonalization of $-\Delta$ in such a domain.

I my notes of last year on my web page, last item of list of publications of
https://www.imo.universite-paris-saclay.fr/∼guy.david (mind the homemade ∼)
but they are very rough; I'll try to improve them a little before I start.