Let $X \subset \mathbb{P}^4_{\mathbb{C}}$ be a smooth hypersurface of degree $d \geq 5$, and let $S \subset X$ be a smooth hyperplane section. Assume that there exists a non trivial cycle $Z \in \mathrm{Pic(X)}$ of degree 0, whose image in $\mathrm{CH}_1(X)$ is in the kernel of the Abel-Jacobi map. The family of couples (X, S) containing such Z is a countable union of analytic varieties. We show that it has a unique component of maximal dimension, which is exaclty the locus of couples (X, S) satisfying the following condition: There is line $\Delta \subset S$ and a plane $P \subset \mathbb{P}^4_{\mathbb{C}}$ such that $P \cap X = \Delta$, and $Z = \Delta - dh$, where h is the class of the hyperplane section in $\mathrm{CH}_1(S)$.

The image of Z in $\mathrm{CH}_1(X)$ is 0. Thus, this construction provides evidence for a conjecture by Nori which predicts that the Abel-Jacobi map for 1-cycles on X is injective.