

Emarks Spring session, 2017
Université de Lyon, May 22-24

Scheduled talks:

- **Hugues Auvray**, *Bergman kernels on punctured Riemann surfaces.*
- **Yoshinori Hashimoto**, *Quantisation of extremal Kähler metrics.*
- **Hendrik Süß**, *On Kaehler-Ricci solitons on Gorenstein del Pezzo surfaces* (j.w.w. Jacob Cable).
- **Zakarias Sjöström-Dyrefelt**, *K-stability of constant scalar curvature Kähler manifolds.*
- **Carl Tipler**, *Asymptotic stability of extremal metrics.*

Programme:

Talks are held at Institut Camille Jordan (Université Lyon I), in Room 112.

	<i>Monday 22</i>	<i>Tuesday 23</i>	<i>Wednesday 24</i>
9:30–10:00	Welcome!		
10:00–11:00	Süß 1	Süß 2	Sjöström-Dyrefelt 2
11:00–11:30	Coffee	Coffee	Coffee
11:30–12:30	Tipler	Hashimoto 2	Auvray
12:30	Lunch	Lunch	Lunch
14:30–15:30	Hashimoto 1	Sjöström-Dyrefelt 1	Free time slot
15:30	Discussion	Discussion	Discussion/Good bye!
19:30		<i>À la carte</i> dinner	

Talks are 55mns long + 5 mns question. There are long breaks to ask more questions to the speakers.

Abstracts

H. Auvray, *Bergman kernels on punctured Riemann surfaces*. — In a joint work with X. Ma (Paris 7) and G. Marinescu (Cologne), we obtain refined asymptotics for Bergman kernels computed from singular data on Riemann surfaces. More precisely, we work on the complement of a finite set of points, seen as singularities, on a compact Riemann surface, that we endow with a metric extending Poincaré's cusp metric around the singularities. As for the polarization line bundle, it comes equipped with a positively curved Hermitian metric, whose curvature is the base metric near the singularities. I shall thus explain how an advanced description of the model geometry (given by Poincaré's metric on the punctured unit disc), and localization techniques in the spirit of Bismut-Lebeau in a weighted analysis context, allow us to describe the Bergman kernels attached to these punctured Riemann surfaces, up to their singularities. If time allows, I shall also mention an arithmetic interpretation of these results, in terms of modular forms.

Y. Hashimoto, *Quantisation of extremal Kähler metrics*. — A foundational result in the study of constant scalar curvature Kähler (cscK) metrics is what is known as Donaldson's quantisation, in which cscK metrics are approximated by "finite dimensional" Fubini-Study metrics when the automorphism group is discrete. We propose in this talk a new "quantising" equation, which generalises various key results in Donaldson's quantisation when the automorphism group is no longer discrete, and can be applied more generally to extremal Kähler metrics; instead of considering Fubini-Study metrics with constant Bergman functions, we shall consider the ones whose Bergman function generates a holomorphic vector field. The first part of the talk will be an introduction to this result, with a brief discussion on the similarities and differences to the related works that appeared in the literature. In the second part we shall review various versions of relative Chow stability and its connection to the centre of mass defined by the Kodaira embedding.

Z. Sjöström-Dyrefelt, *K-stability of constant scalar curvature Kähler manifolds*. — In this talk we introduce a variational/pluripotential approach to the study of K-stability of Kähler manifolds with transcendental cohomology class, extending a classical picture for polarised manifolds. Our approach is based on establishing a formula for the asymptotic slope of the K-energy along certain geodesic rays, from which we deduce that cscK manifolds are K-semistable. Combined with a recent properness result of R. Berman, T. Darvas and C. Lu we further deduce uniform K-stability of cscK manifolds with discrete automorphism group, thus confirming one direction of the YTD conjecture in this setting.

If time permits we also discuss possible extensions of these results to the case of compact Kähler manifolds admitting holomorphic vector fields.

H. Süß, *On Kaehler-Ricci solitons on Gorenstein del Pezzo surfaces*. — This is a joint work with Jacob Cable. For smooth del Pezzo surface it is known that they either admit a Kaehler-Einstein metric or they are toric and therefore admit a Kaehler-Ricci soliton. Moreover, by results of Odaka-Spotti-Sun we know which Gorenstein del Pezzo surfaces admit a Kaehler-Einstein metric. Hence, it is natural to ask which of the remaining ones admit a (non-trivial) Kahler-Ricci soliton. We are approaching this question by determining K-stability of pairs (X, V) consisting of a del Pezzo surfaces X and holomorphic vector field V on X . Moreover, we classify all Gorenstein del Pezzo surfaces, which can be complemented to a K-stable pair. The main tool for achieving this classification is a combinatorial description of varieties with torus actions of complexity one and their degenerations. In the first lecture I will introduce this combinatorial description and the second lecture I will apply it to determine K-stability of pairs (X, V) , where X admits a complexity-1 torus action.

C. Tipler, *Asymptotic stability of extremal metrics*. — A result of Donaldson states that on a projective manifold with discrete automorphism group, a constant scalar curvature Kähler metric is the limit of some projective metrics, called balanced metrics. In this talk, we will prove a conjecture of Apostolov-Huang: an extremal metric on a polarized Kahler manifold is the limit of some specific relatively balanced metrics. This result extends Donaldson's work to projective manifolds with holomorphic vector fields. Along the proof, we will present a moment map interpretation of weighted balanced metrics. We also characterize a canonical weight for these metrics, providing a quantization of the extremal vector field action. As a corollary, we obtain a simpler proof of uniqueness of extremal metrics, as well as a splitting result. Joint work with Yuji Sano.