When we focus on the case where the support of $\mu$ is upper-bounded by some constant $M$, in this case, $C(\nu) \leq M$.

When $M \leq 0$, it is easy to show that $C(\nu) = 0$.

Rescaling property

Assume that $X$ has distribution $\nu$ and consider $\lambda > 0$. Let $\nu_\lambda$ denote the distribution of the random variable $\lambda X$. It holds that

$$C(\nu_\lambda) = \lambda \cdot C(\nu).$$

The particular case of Barak-Erdős graphs

In terms of heaviest paths, $X_{ij} = -\infty$ is equivalent to removing the edge $(i, j)$ from the graph. Therefore, if $\nu$ is supported by $1$ and $\infty$, the problem consists in studying the length of a longest path in a directed acyclic version of an Erdős-Rényi random graph; a Barak-Erdős graph.

In [23], Maléin and Ramassamy proved that the time constant for Barak-Erdős graphs is an analytic function in the probability of an edge to be in the graph. The two results on the analyticity of the time constant presented here extend that result:

- The strict monotonicity tells us something about the geometry of the heaviest paths: because $\mu(\cdot|\lambda) > \mu(\cdot|\lambda^*)$, all the edges $W_{i,j}$ that may contribute to a heaviest path are also necessarily in the path.
- The map $\nu \mapsto C(\nu)$ is continuous for the metric $\delta_\infty$ defined by

$$\delta_\infty(\nu, \nu') = \max\{|\nu - \nu'|(\{i, j\}) | \nu(\{i, j\}) = 1, \nu'(\{i, j\}) = 0\}.$$

Then, $\nu \mapsto C(\nu)$ is continuous for the metric $\delta_d$ on the set of probability measures $\nu$ such that $M_\nu < \infty$ in the Lévy-Prokhorov metric on the set of probability measures on $\mathbb{R}^n$.

Interpretation of those results

- The strict monotonicity tells us something about the geometry of the heaviest paths: because $\mu(\cdot|\lambda) > \mu(\cdot|\lambda^*)$, all the edges $W_{i,j}$ that may contribute to a heaviest path are also necessarily in the path.
- The map $\nu \mapsto C(\nu)$ is continuous for the metric $\delta_d$ on the set of probability measures $\nu$ such that $M_\nu < \infty$.

References

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