Graduate Seminar on Differential Geometry SS 2019

Topic: Spectral geometry of the Steklov problem.

The seminar will be a block seminar which takes place at the MPI (exact place will be specified) at the following dates:

7th of May, 12.00 pm – 2.00 pm,
8th of May, 8.00 am – 12.00 pm,
10th of May, 12.00 pm – 2.00 pm.

In recent years the study of the Steklov problem has become a very active field. For a compact Riemannian manifold (M, g) with non-empty boundary ∂M, the Steklov problem on M is

$$\begin{aligned} \Delta u = 0 \qquad & \text{in}\, M, \\ \partial_\nu u = \sigma\, u \qquad & \text{on}\, \partial M. \end{aligned}$$

Here ∂_ν is the outward normal derivative along ∂M. Further, σ is referred to as Steklov eigenvalue.

In this seminar we would like to discuss some up to date results, e.g. spectral asymptotics, invariants of the Steklov spectrum, geometric inequalities, nodal geometry, rigidity results. We will roughly follow the survey article [5] and references therein as a guide for discussion.

References

[1] B. Colbois, A. El Soufi, A. Girouard, *Isoperimetric control of the Steklov spectrum.* J. Funct. Anal. **261** (2011), no. 5, 1384–1399.

[2] J. Edward. *An inverse spectral result for the Neumann operator on planar domains.* J. Funct. Anal. **111** (1993), no. 2, 312–322.

[3] A. Fraser, R. Schoen. *The first Steklov eigenvalue, conformal geometry, and minimal surfaces.* Adv. Math. **226** (2011), no. 5, 4011–4030.

[4] C. Gordon, P. Herbrich, D. Webb, *Robin and Steklov isospectral manifolds,* arXiv:1808.10741.

[5] A. Girouard, I. Polterovich. *Spectral geometry of the Steklov problem. Shape optimization and spectral theory,* 120–148, De Gruyter Open, Warsaw, 2017.

[6] A. Girouard, I. Polterovich. *Upper bounds for Steklov eigenvalues on surfaces.* Electron. Res. Announc. Math. Sci. **19** (2012), 77–85.

[7] A. Girouard, Parnovski, I. Polterovich, D. Sher. *The Steklov spectrum of surfaces: asymptotics and invariants.* Math. Proc. Cambridge Philos. Soc. **157** (2014), no. 3, 379–389.

[8] I. Polterovich, D. Sher. *Heat invariants of the Steklov problem.* J. Geom. Anal. **25** (2015), no. 2, 924–950.

[9] S. Zelditch. *Maximally degenerate Laplacians,* Annales de l'institut Fourier, tome **46**, no 2 (1996), p. 547–587.

If you have further questions, please contact Mihajlo Cekić (m.cekic AT mpim-bonn.mpg.de) or Anna Siffert (siffert AT mpim-bonn.mpg.de).

List of talks[1]

(1) Preliminaries, examples and Weyl's law ([5, Sections 1 and 3]). Definition of the DN map, Steklov spectrum, basic constructions (balls and cylinders), relations to other problems (sloshing, EIT), polygons (smooth vs non-smooth boundary).

(2) Spectral invariants and asymptotics for smooth surfaces ([5, Section 2]). [7], i.e. proof of Theorem 2.1.2 (asymptotics), sketch of Theorem 2.2.2. (invariants), counterexamples in higher dimensions (with details).

(3) Heat invariants ([5, Section 2.2]+[8, Sections 1-4]). In particular proof of Corollary 2.2.4 that the total mean curvature of the boundary is an invariant.

(4) Geometric inequalities ([5, Section 4]). Weinstock's inequality, isoperimetric control (sections 4.1-4.2.). Proof of Weinstock's inequality. Counterexamples for annuli. Either proof of Theorem 4.2.9. ([1], harder) or proof of Theorem 4.2.1. ([6], easier).

(5) Free boundary minimal surfaces 1 ([3, Sections 1-3]).

(6) Spectral rigidity ([5, Section 5]+[8, Chapter 5]). Rigidity of the disc in $\mathbb{R}^2$, examples of isospectral manifolds. Proof of Proposition 5.2.3. together with the Theorem of Zelditch on multiplicities using microlocal analysis ([9, Theorem A]). Heat invariants may be taken to be known. Extra: isospectral examples with connected boundary.

(7) Spectral rigidity II. Sunada method [4].

(8) Nodal geometry ([5, Section 6]). Courant's theorem, multiplicity bounds. Proof of Courant's theorem. Proof of Theorem 6.3.1. on the multiplicity bounds for eigenvalues. Extra: proof of Theorem 6.3.6. on the existence of a conformal class with prescribed spectrum.

[1]The statements with three digits are references to statements in [5].