Exercice 1. We recall the formulas for the Christoffel symbols: One has then:

$$\Gamma^c_{ab} = \frac{1}{2} g^{cd}(\partial_a g_{bd} + \partial_b g_{ad} - \partial_d g_{ab}),$$

and

$$\begin{aligned} \nabla_a T^b &= \partial_a T^b + \Gamma^b_{ac} T^c, \\ \nabla_a T_b &= \partial_a T_b - \Gamma^c_{ab} T_c. \end{aligned}$$

Check that if X^a is a vector field then

$$\nabla_a X^a = |g|^{-\frac{1}{2}} \partial_a (|g|^{\frac{1}{2}} X^a).$$

Exercice 2. Prove that if $u \in C_0^\infty(M)$ (and P is the Klein-Gordon operator), solves $Pu = 0$ then $u = 0$.