Erratum for "Asymptotic completeness in quantum field theory. Massive Pauli-Fierz Hamiltonians"

We use the notation of the above mentioned paper. The proof of Lemmas 3.3 and 3.4 is incorrect as it is. In fact if we assume *(I1)* or even the stronger condition *(I1')*, it is not true that $(1-j_0^R)v$ and $j_\infty^R v$ tend to 0 in *norm* in $\mathcal{B}(\mathcal{K},\mathcal{K}\otimes\mathfrak{h})$ when $R\to\infty$. Therefore the estimates (3.8) and (3.11) are incorrect as stated.

There are two ways to correct this: first one can impose the condition

$$\|(1\!\!1_{[R,\infty[})(x)v\| \in o(R^0), \text{ when } R\to\infty.$$

Then the proofs are correct. The second way is to impose the condition *(H0)*, namely that $(K+1)^{-1}$ is compact in $\mathcal{BK}$. Note that this condition has to be imposed to obtain the asymptotic completeness, which is the main result of the paper.

Let us explain the modifications needed to prove Lemmas 3.3 and 3.4, if we add the asumption *(H0)*.

Proof of Lemma 3.3:

Since $(1-j_0^R)$ and j_∞^R tend to 0 *strongly* in $\mathcal{B}(\mathfrak{h})$, and $(K+1)^{-1}$ is compact, we obtain that $(1-j_0^R)v(K+1)^{-\frac{1}{2}}$ and $j_\infty^R v(K+1)^{-\frac{1}{2}}$ tend to 0 *in norm* in $\mathcal{B}(\mathcal{K},\mathcal{K}\otimes\mathfrak{h})$ when $R\to\infty$. Therefore instead of (3.8), we obtain (with obvious notations)

$$[V, Q_k(f^R)] \in o(R^0)(N+1)^{\frac{1}{2}}(K+1)^{\frac{1}{2}}.$$

Now under hypotheses *(I1)* and *(H1)*, H is selfadjoint on $\mathcal{D}(H_0)$ hence $(z-H)^{-1}(K+1)^{\frac{1}{2}} = (z-H)^{-1}(H_0+1)^{-1}(K+1)^{-\frac{1}{2}} \in O(|\mathrm{Im}z|^{-1})$ for $z\in U \Subset \mathbb{C}$. This yields

$$N^m(z-H)^{-1}[H, Q_k(f^t)](z-H)^{-1}\chi(H) \in o(t^0)|Imz|^{-p}.$$

One can then complete the proof of Lemma 3.3 as in the paper.

Proof of Lemma 3.4

by the same argument, instead of (3.11) one obtains

$$V\otimes 1\!\!1\check{\Gamma}(j^R) - \check{\Gamma}(j^R)V \in \check{o}_N(R^0)(N+1)^{\frac{1}{2}}(K+1)^{\frac{1}{2}},$$

and (3.12) has to be replaced by

$$H^{\mathrm{ext}}\check{\Gamma}(j^R) - \check{\Gamma}(j^R)H \in \check{o}_N(R^0)(N+1)^{\frac{1}{2}}(K+1)^{\frac{1}{2}}.$$

This still implies statement (i) in Lemma 3.4, by the same argument as above (using the resolvent of H to absorb the extra factor of $(K+1)^{\frac{1}{2}}$). The rest of the proof of Lemma 3.4 can be modified similarly.