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Rappel de la définition de quelques lois

Loi Gamma : $Y \sim \mathcal{G}(\alpha, \beta)$

$$f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y) \mathbb{1}_{\mathbb{R}^{+*}}(y)$$

$\mathbb{E}(Y) = \alpha/\beta$, $\text{var}(Y) = \alpha/\beta^2$, Mode$=(\alpha-1)/\beta$

Loi Inverse Gamma : $V \sim \mathcal{IG}(\alpha, \beta)$

$$f(v) = \frac{\beta^\alpha}{\Gamma(\alpha)} v^{-\alpha-1} \exp(-\beta/v) \mathbb{1}_{\mathbb{R}^{+*}}(v)$$

$\mathbb{E}(V) = \beta/(\alpha-1)$, $\text{var}(V) = \beta^2/[(\alpha-1)^2(\alpha-2)]$, Mode$=\beta/(\alpha+1)$

Loi Student : $T \sim \mathcal{T}(k, \theta, \tau^2)$

$$f(t) = \frac{1}{\sqrt{k\pi\tau^2}} \frac{\Gamma((k+1)/2)}{\Gamma(k/2)} \left(1 + \frac{(t-\theta)^2}{k\tau^2}\right)^{-\frac{k+1}{2}}$$

$\mathbb{E}(T) = \theta$, $\text{var}(T) = k/(k-2)$ si $k > 2$, Mode$=\theta$

Loi Student multivariée : $X \sim \mathcal{T}_p(k, \theta, \Sigma)$

$$f(x) = \frac{\Gamma((k+p)/2)/\Gamma(k/2)}{(det\Sigma)^{1/2}(k\pi)^{p/2}} \left(1 + \frac{(x-\theta)'\Sigma^{-1}(x-\theta)}{k}\right)^{-\frac{k+p}{2}}$$

Si $X \sim \mathcal{T}_p(k, \theta, \Sigma)$, $(X-\theta)'\frac{\Sigma^{-1}}{p}(X-\theta) \sim \mathcal{F}(p, k)$

Loi Beta : $Y \sim \mathcal{B}e(\alpha, \beta)$

$$f(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)} \mathbb{1}_{[0,1]}(y) \quad \text{avec } B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$\mathbb{E}(Y) = \alpha/(\alpha+\beta)$, $\text{var}(Y) = \alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$, Mode$=(\alpha-1)/(\alpha+\beta-2)$

Loi de Cauchy : $Y \sim \mathcal{C}(\mu, a)$

$$f(y) = \frac{1}{\pi a \left[1 + \left(\frac{y-\mu}{a}\right)^2\right]}$$

$\mathbb{E}(Y)$ non définie, $\text{var}(Y)$ non définie, Mode$=\mu$