Mould calculus.

Alien calculus presents us at every step with elementary resurgence *mono-mials* $\mathcal{U}^{\omega_1,\dots,\omega_r}(z)$ or scalar *monics* $U^{\omega_1,\dots,\omega_r}$ that have their own symmetries and undergo numerous operations that either preserve or exchange these symmetries. The systematization of these operations gradually led to *mould calculus* (*moulds* are simply objects $M^{\omega_1,\dots,\omega_r}$ indexed by scalar sequences of any length), which later found many applications outside its native context, in such fields as combinatorics, differential geometry, Lie or pre-Lie algebra, etc.

Apart from its twelve basic *operations* and many *symmetries* (four main types, scores of secondary ones), mould calculus relies on

- some thirty-odd fundamental *special moulds* that tend to crop up everywhere and combine, as universal building blocks, to form practically all moulds of practical importance

- the self-dual *arborification-coarborification transform*

$$\sum A^{\boldsymbol{\omega}} B_{\boldsymbol{\omega}} \rightarrow \sum A^{\boldsymbol{\omega}^<} B_{\boldsymbol{\omega}^<} \qquad \left(A^{\bullet} = mould\,, B_{\bullet} = comould \right)$$

  that replaces totally ordered index sequences $\boldsymbol{\omega}$ by arborescent ones, and tends to restore convergence in the ubiquitous mould-comould expansions.

While at one level it would be tempting to dismiss moulds with all their wherewithal as just a glorified system of notations, the fact is that they often allow us to make fully explicit what would otherwise remain hopelessly implicit, and to go beyond mere 'existence theorems' (that all too often are sterile dead-ends) by illuminating the innards of the object whose bare 'existence' has been proved. This had led to notable advances, e.g. by dispelling the chimera of *super-multiple small denominators* in KAM theory. More generally, once the Taylor coefficients of a power series have been analysed down to their last *mould constituents*, the issue of convergence/divergence usually resolves itself and, in case of divergence, the proper resummation strategy naturally emerges.