Analysis of Local Objects: the Bridge equation.

Stricto sensu, 'local objects' are germs of analytic equations or equation systems (*analytic* in both variables and unknowns) of any conceivable type: differential, difference, q-difference, general functional. *Lato sensu*, 'local objects' include things like local vector fields on $\mathbb{C}^\nu$ that vanish at 0 or local self-mappings of $\mathbb{C}^\nu$ with 0 as fixed point, together with the equations of motion attached to such fields or mappings.

Singular local objects (they characteristically include singular ODE's and resonant vector fields or mappings) typically give rise to a special resurgence regimen, which is described by the so-called Bridge equation:

$$\mathbf{BE_1} \qquad \boldsymbol{\Delta}_\omega\, Y(z,\boldsymbol{u}) = \mathbf{A}_\omega\, Y(z,\boldsymbol{u}) \qquad (\omega \in \Omega_1)$$

- $Y(z,\boldsymbol{u})$ is the formal, parameter-saturated, general solution attached to the 'local object', with z the resurgence-carrying variable, and $\boldsymbol{u} := (u_1, u_2, ...)$ 'inert' parameters.
- $\boldsymbol{\Delta}_\omega$ is an alien derivation in 'invariant' form ($\boldsymbol{\Delta}_\omega := e^{-\omega z}\Delta_\omega$) with an index ω running through a discrete set $\Omega_1 \in \mathbb{C}_\bullet$.
- $\mathbf{A}_\omega$ is an ordinary differential operator in z and $\boldsymbol{u}$. Taken together, these $\mathbf{A}_\omega$ amount to a complete system of analytic-holomorphic invariants.

Certain crucial parameters x attached to a 'local object' may also lead to divergent-resurgent expansions, loosely dual to, but more complex than, the z-expansions, and described by two Bridge equations instead of one:

$$\mathbf{BE_2} \qquad \boldsymbol{\Delta}_\omega\, Y(x,\boldsymbol{u}) = \mathbf{B}_\omega(x)\, Y(x,\boldsymbol{u}) \qquad (\omega \in \Omega_2)$$

$$\mathbf{BE_3} \qquad \boldsymbol{\Delta}_\omega\, \mathbf{B}_{\omega_0} = h_{\omega,\omega_0}(\mathbf{B}_{\omega_1}, \mathbf{B}_{\omega_2}, \dots) \qquad (\omega \in \Omega_3)$$