TL;DR – 15/11/19

Important points

- A Dedekind A ring is a noetherian integral domain where every nonzero prime ideal is maximal, and which is integrally closed.
- Schematically, this means that Spec$A$ is a noetherian, integral one-dimensional normal scheme.
- If K is a number field, then its ring of integers $\mathcal{O}_K$ is a Dedekind ring.
- In Dedekind rings $\mathcal{O}$ with fraction field K, we introduce fractional ideals (analogous to divisors on normal schemes). These are the nonzero $\mathcal{O}$-submodules of K.
- Fractional ideals form a group for multiplication. This is actually the free group on nonzero prime ideals. This does not hold for arbitrary orders in number fields.
- Fractional ideals modulo principal ideals form a group: the ideal class group. By Jordan-Zassenhaus, it is finite.
- Local Dedekind rings are quite simple: these are discrete valuation rings. In particular, they are principal.

References

All the material is covered in the general references for the class.