TL;DR – local fields, basic results

Important points

- Local fields, in a rough way, bring the gap between number fields and finite fields (which of course appear as residue fields).
- There are two points of views, which of course end up corresponding: we may complete with respect to valuations/absolute values, or take profinite completions.
- The set of absolute values of a number field is very interesting: we get absolute values coming from p-adic valuations at all primes, and archimedean absolute values corresponding to all embeddings into $\mathbb{R}$ and $\mathbb{C}$.
- Weak approximation theorem: if K is a field, $K_1, \ldots, K_n$ completions of K corresponding to pairwise non-equivalent absolute values, then the diagonal embedding $K \to K_1 \times \ldots K_n$ has dense image.
- You should familiarize yourself with $\mathbb{Q}_p$ and $\mathbb{Z}_p$, and how to compute in it, work with power series, etc. The book of Neukirch contains lots of examples and easy enough exercises.
- The behaviour of extensions is very important: if K is a field with a discrete valuation v and a corresponding absolute value $|.|$, and if L is a finite extensions of K, extensions of $|.|$ to L are in bijections with primes of L above the prime of K corresponding to v. If K is complete, L is complete and the extension is unique.
- In a way, because of the result above, completions "zooms in" enough that it separates the points of $\mathcal{O}_L$ above a given prime of $\mathcal{O}_K$. The Galois groups that appear after completions are the decomposition groups of the original extension.
- Fundamental definition: a local field is a field with a discrete valuation, complete, with finite residue field.
- Hensel's lemma is very useful, and almost built in the definitions: it allows one to lift solutions of polynomial equations, under suitable assumptions, from the residue field to the local field.
- We may use a finer study of ramification in the case of local fields: unramified extensions are in bijection with extensions of the residue field, tamely ramified extensions correspond to taking roots of order prime to the residue characteristic of a uniformizing parameter.

References Serre's book "Local fields" is one of the best references and contains lots of material together with a streamlined exposition. Standard references in algebraic geometry consider completions from the point of view of étale topology.