Number theory – Syllabus

Practical information

- Lectures on fridays 10-12, 13:30-15:30 in Room 1A7 by François Charles – office 3J21, `francois.charles@math.u-psud.fr`. Please do not hesitate to come by and ask questions – it is usually preferable to plan a meeting in advance by email.
- Exercise sessions on wednesdays 16:15-18:15 in Room 0A1 by Olivier Fouquet – office 3P17.
- Lectures are in French. Questions are encouraged in French or English, including basic language questions.
- Attendance at lectures or exercise sessions is not mandatory but obviously more than recommended.

Topics covered (subject to change)

- Central simple algebras and Brauer groups;
- Basic theory of global fields and geometry of numbers;
- Dedekind zeta functions and Artin L-functions;
- Cyclotomic fields and the theorem of Cebotarev;
- Basic theory of local fields;
- The Brauer group of a local field;
- Local class field theory.
- Maybe some ideas on global class field theory.

Grading

There will be a partial exam sometime in november, and a final exam in january. Grade is $\max(\text{final exam}, (\text{final exam} + \text{partial exam})/2)$. Attendance to the partial exam is crucial as it makes it possible to evaluate yourself.

Exams comprise of some direct questions on lectures, and a few exercises or problems. They tend to be too difficult and long to be done entirely in one session but are graded accordingly: I don't expect you to do everything. French and English versions will be available, and you can answer in French or English. Please bring your own paper. Understanding of definitions and proofs will be useful. No notes will be allowed.

Prerequisites

Familiarity with Galois theory is essential, in various formulations. More generally, some familiarity with elementary commutative algebra is expected, including localization and tensor product. The algebra books of Lang and Jacobson are useful references. We will use less commutative algebra than the scheme theory course though. No prior familiarity with number fields is technically assumed, but it is of course extremely useful. The notes at

`http://gaetan.chenevier.perso.math.cnrs.fr/MAT552/TAN_poly_2019.pdf`

are a good read.

The theory of number fields has close ties with that of Riemann surfaces. We will use analogies from algebraic geometry and the theory of holomorphic functions.

The two other classes in the M2 AAG that are most relevant to number theory are Riemann surfaces (as the analogy between number fields and curves runs deep) and scheme theory.

References

Here are a few useful resources:

- Notes for the classes will be posted on my webpage. However, the actual live lecture might contain more material and less mistakes.
- Neukirch, *Algebraic Number Theory* is a basic reference and contains clear proofs and useful exercises.
- Cassels-Fröhlich, *Algebraic Number Theory* is a collection of deep basic texts written by great mathematicians and is essential reading. Exercises at the end are great practice.
- Weil, *Basic Number Theory* is sometimes hard to read but contains important parts on the role of central simple algebras.
- Serre, *Local fields* is the classic reference for local fields.
- Kato-Kurokawa-Saito, *Number Theory 2* contains a beautiful outline of class field theory via central simple algebras.
- More references will be given locally.
- Please read a lot ! Anything you find stimulating is a good read.