le cnam

Master Structural Mechanics and Coupled Systems

Applied Mathematics

Student presentation 5 (course 10)

- Solving a linear system by a minimization process

Let $n \geq 1$ an integer and A a real symmetric positive definite matrix of order n. We introduce also a given vector $b \in \mathbb{R}^n$ and the functional J defined from $\mathbb{R}^n$ to the set of real numbers by the relation $J(x) = \frac{1}{2}(x, Ax) - (b, x)$, with $x, y) \equiv \sum_j x_j y_j$ the scalar product of two vectors in $\mathbb{R}^n$. We admit that there exists some $\overline{x} \in \mathbb{R}^n$ such that for all $x \in \mathbb{R}^n$, $J(x) \geq J(\overline{x})$.

a) Prove that the functional J is differentiable in $\mathbb{R}^n$.

b) What is the action $\mathrm{d}J(x).h$ of the differential of the functional J on a arbitrary vector h?

c) Show that the point of minimum introduced previously satisfy to the relation $\mathrm{d}J(\overline{x}).h = 0$ for every vector $h \in \mathbb{R}^n$.

d) Explicit a simple equation satisfied by the vector $\overline{x}$.

e) Solve completely the following example with $n = 2$, $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

François Dubois, september 2023.