# Vessel pressurization

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## Introduction

The purpose of this test-case is to check the ability of an Euler solver to compute a slow pressurization of a closed cavity filled with a non-reactive gas.

## 1 The dimensional problem

We consider the 2D rectangular cavity of figure 1 ($H = L = 10$ m). We suppose that it is initially filled with a calorically perfect gas such as air ($\gamma = 1.4$, $R = 288$ J/kg/K) at rest and in uniform conditions, with initial temperature $T_I = 300$ K and initial pressure $P_I = 1$ bar $= 10^5$ Pa. We

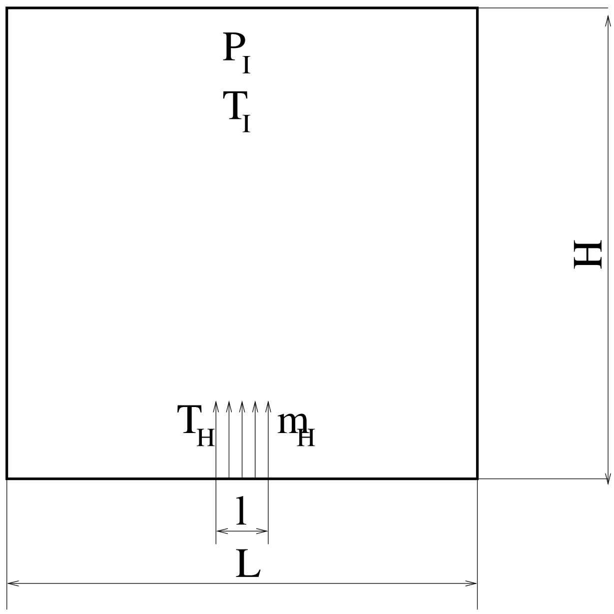


*Figure 1:* Rectangular cavity initially filled with air at rest. On the bottom part we have hot air injection.

inject the same gas from the bottom part of the cavity. The cavity wall is supposed to be impermeable and adiabatic. The injected flow temperature is $T_H = 600$ K; the mass flux $\dot{m}_H$ is constant along the injection surface ($l = 2$ m). Numerical experiments are performed by considering two values of the mass flow rate $\dot{m}_H$. We consider $\dot{m}_H = 1.0$ kg/m$^2$/s. The flow is subsonic and in low Mach number regime. Indeed, the order of magnitude of the flow velocity in the vessel is the same as in the injection region:

$$w_H = \frac{\dot{m}_H}{\rho_H} = \frac{\dot{m}_H}{p_H} R T_H$$

Since the injection pressure is larger than $P_I$, it is

$$w_H < \frac{\dot{m}_H}{P_I} R T_H$$

and it is about 1.7 m/s; the sound speed is initially about $\sqrt{\gamma R T_I} = 350$ m/s in the containment and 500 m/s at the injection, i.e. much bigger that the flow speed.
We compute the solution at $t = 6$, 12, 18 s.

## 2 Non-dimensional problem

For the sake of simplicity, let us consider the 1D Euler equations (non-dimensional 2D or 3D Euler equations can be obtained in the same way).

$$\begin{cases} \frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho w) = 0 \\ \frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}\left(\rho w^2 + P\right) = 0 \\ \frac{\partial}{\partial t}\left(\rho e + \frac{1}{2}\rho w^2\right) + \frac{\partial}{\partial x}\left(w(\rho e + P + \frac{1}{2}\rho w^2)\right) = 0 \end{cases}$$

subjected to the Equations of State (EOS)

$$\begin{cases} P = \rho R T \\ e = \frac{1}{\gamma - 1} R T \end{cases}$$

We consider as independent reference values $x^*$, $\rho^*$, $P^*$ and we define

$$\begin{aligned} w^* &= \sqrt{\frac{P^*}{\rho^*}} \\ e^* &= \frac{P^*}{\rho^*} \\ T^* &= \frac{P^*}{\rho^* R} \\ t^* &= \frac{x^*}{w^*} \end{aligned}$$

If we define the generic non-dimensional variable $q' = q/q^*$, then we can rewrite the 1D Euler equations in their non-dimensional form

$$\begin{cases} \frac{\partial}{\partial t'}(\rho') + \frac{\partial}{\partial x'}(\rho' w') = 0 \\ \frac{\partial}{\partial t'}(\rho' w') + \frac{\partial}{\partial x'}\left(\rho' w'^2 + P'\right) = 0 \\ \frac{\partial}{\partial t'}\left(\rho' e' + \frac{1}{2}\rho' w'^2\right) + \frac{\partial}{\partial x'}\left(w'(\rho' e' + P' + \frac{1}{2}\rho' w'^2)\right) = 0 \end{cases}$$

subjected to the EOS

$$\begin{cases} P' = \rho' T' \\ e' = \frac{1}{\gamma - 1} T' \end{cases}$$

We chose as reference values

$$x^* = l = 2 \text{ m}$$

$$\rho^* = \frac{P_I}{RT_I} = \frac{10^5}{288 \cdot 300} \frac{\text{kg}}{\text{m}^3} \approx 1.157 \frac{\text{kg}}{\text{m}^3}$$

$$P^* = P_I = 10^5 \frac{\text{N}}{\text{m}^2}$$

It follows that the reference velocity, internal energy, temperature and time are given by

$$w^* = \sqrt{\frac{P^*}{\rho^*}} = \sqrt{RT_I} \approx 293.9 \frac{\text{m}}{\text{s}}$$

$$e^* = \frac{P^*}{\rho^*} = RT_I = 86400 \frac{\text{m}^2}{\text{s}^2}$$

$$T^* = \frac{P^*}{\rho^* R} = T_I = 300K$$

$$t^* = \frac{x^*}{w^*} \approx 6.805 \cdot 10^{-3} s$$

We can now compute the non-dimensional inputs of our problem:

$$l' = 1, H' = L' = 5$$

$$P_I' = 1, T_I' = 1$$

$$T_H' = 2$$

$$\dot{m'}_H = \frac{\dot{m}_H}{\rho^* w^*} \approx 2.946 \cdot 10^{-3}$$

We compute the solution at $t' = 881.7$, 1763., 2645..
Before concluding, we emphasize that

- $\dot{m'}_H << 1$ is linked to the fact there is a big difference between the speed of sound and the injection velocity. Moreover we have a characteristic time which is linked to the propagation of the acoustic waves (i.e. $t^*$) and a characteristic time which is linked to the injection of the mass into the cavity (much bigger than the "acoustic" one). Then we compute the solution at $t' >> 1$.
- In the imput of this problem the temperature appears. The temperature is not an usual unknown of the Euler Equations. However the problem can be easily reformulated by replacing the temperature by the internal energy, since they are linked by the relationship

$$e' = \frac{1}{\gamma - 1} T'$$

# 3 Conservation properties

The conservation laws allow us to compute the average density and average pressure variations as function of time. Indeed the total mass variation in the containment is given by

$$\int_V \rho(\vec{r}, t) \mathrm{d}V = \int_V \rho(\vec{r}, 0) \mathrm{d}V + \dot{m}_H l t \tag{1}$$

As far as the total energy is concerned, at low Mach number regime we have

$$\rho e_t = \frac{1}{\gamma - 1} P + 0.5 \rho \vec{w} \cdot \vec{w} \approx \frac{1}{\gamma - 1} P$$

and the total energy flux is given by

$$\dot{m}_H h_t = \dot{m}_H \frac{\gamma}{\gamma - 1} R T_H + \dot{m}_H 0.5 \vec{w} \cdot \vec{w} \approx \dot{m}_H \frac{\gamma}{\gamma - 1} R T_H$$

Then, the average pressure is about

$$\int_V P(\vec{r}, t) \mathrm{d}V = \int_V P(\vec{r}, 0) \mathrm{d}V + \gamma \dot{m}_H R T_H \frac{h}{HL} t \tag{2}$$