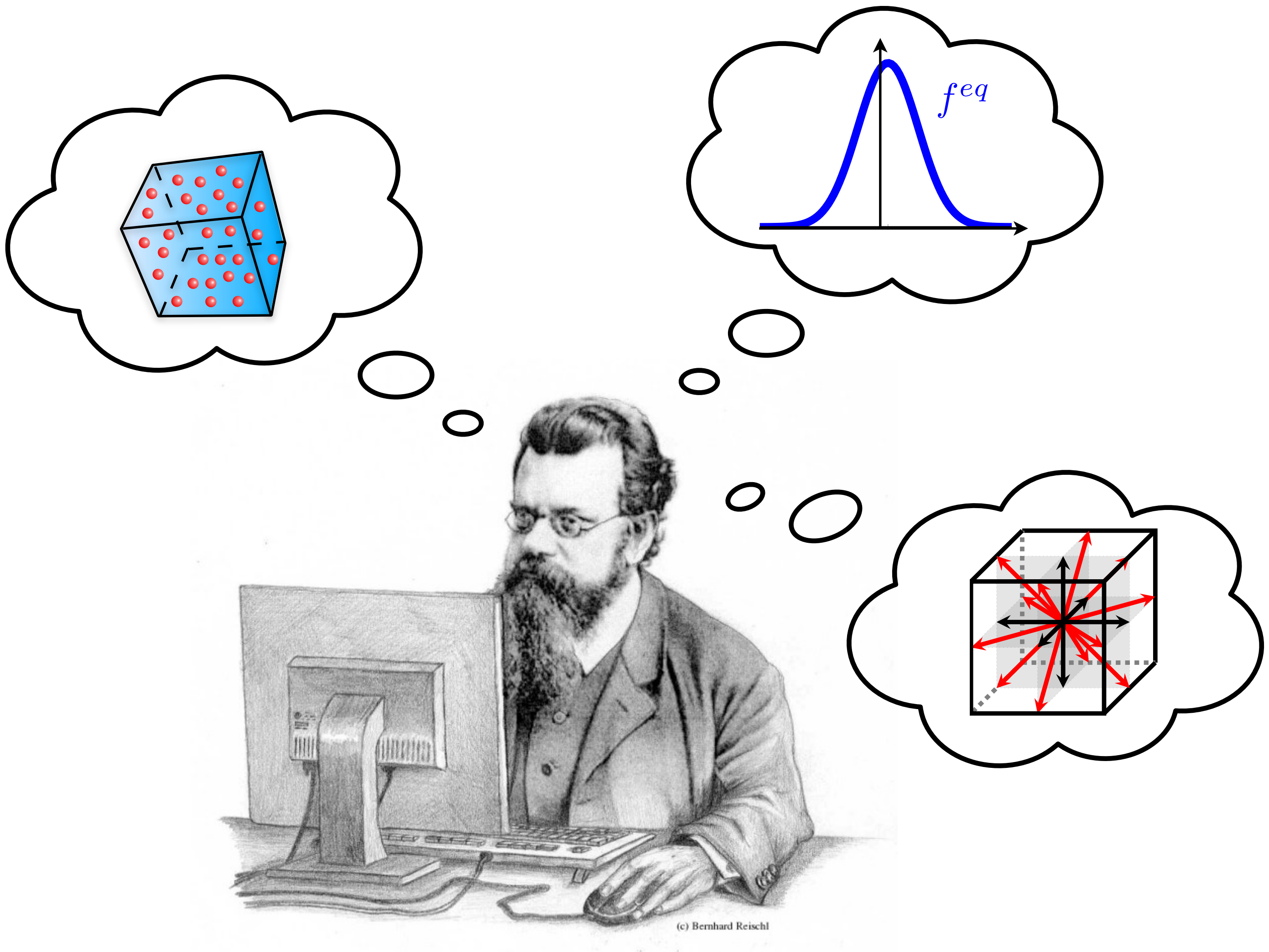


UNIVERSITY OF GENEVA
FACULTY OF SCIENCE
Computer Science Dept

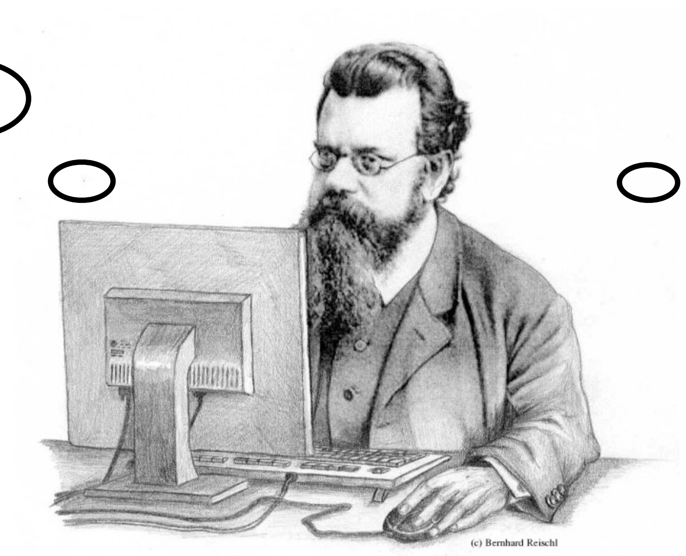
Compressible lattice Boltzmann methods
Overview and recent advances

Christophe Coreixas

Outline (today)

How do we design LBMs ?

Two-equation models



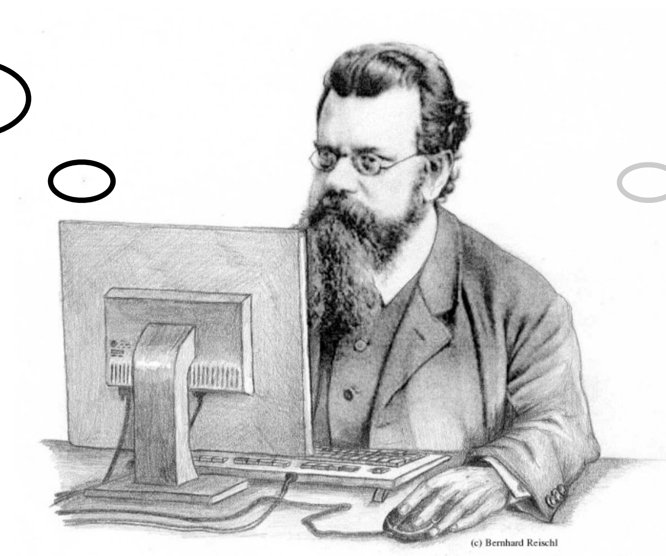
Quadrature free LBMs

Adaptive lattices

Outline (today)

How do we design LBMs ?

Two-equation models



Quadrature free LBMs

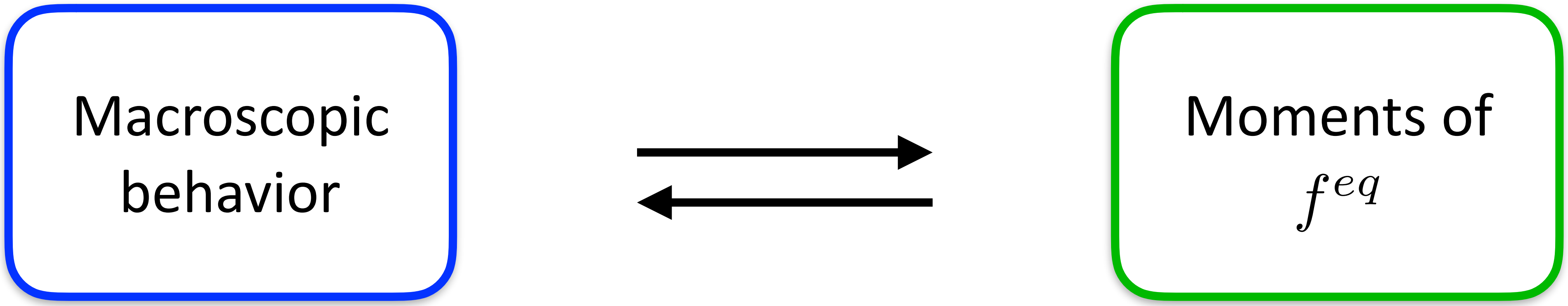
Adaptive lattices

How do we design LBMs?

Macroscopic behavior

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\boldsymbol{\delta}) = \nabla \cdot \mathbf{\Pi} \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{\Pi} \cdot \mathbf{u}) \end{cases}$$

How do we design LBMs?



$$
\begin{cases}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\
\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\boldsymbol{\delta}) = \nabla \cdot \mathbf{\Pi} \\
\partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{\Pi} \cdot \mathbf{u})
\end{cases}
$$

$$
\begin{cases}
\partial_t (M_0^{eq}) + \nabla \cdot (M_1^{eq}) = 0 \\
\partial_t (M_1^{eq}) + \nabla \cdot (M_2^{eq}) \propto \partial_t (M_2^{eq}) + \nabla \cdot (M_3^{eq}) \\
\partial_t (M_{\mathrm{Tr}2}^{eq}) + \nabla \cdot (M_{\mathrm{Tr}3}^{eq}) \propto \partial_t (M_{\mathrm{Tr}3}^{eq}) + \nabla \cdot (M_{\mathrm{Tr}4}^{eq})
\end{cases}
$$

How do we design LBMs?

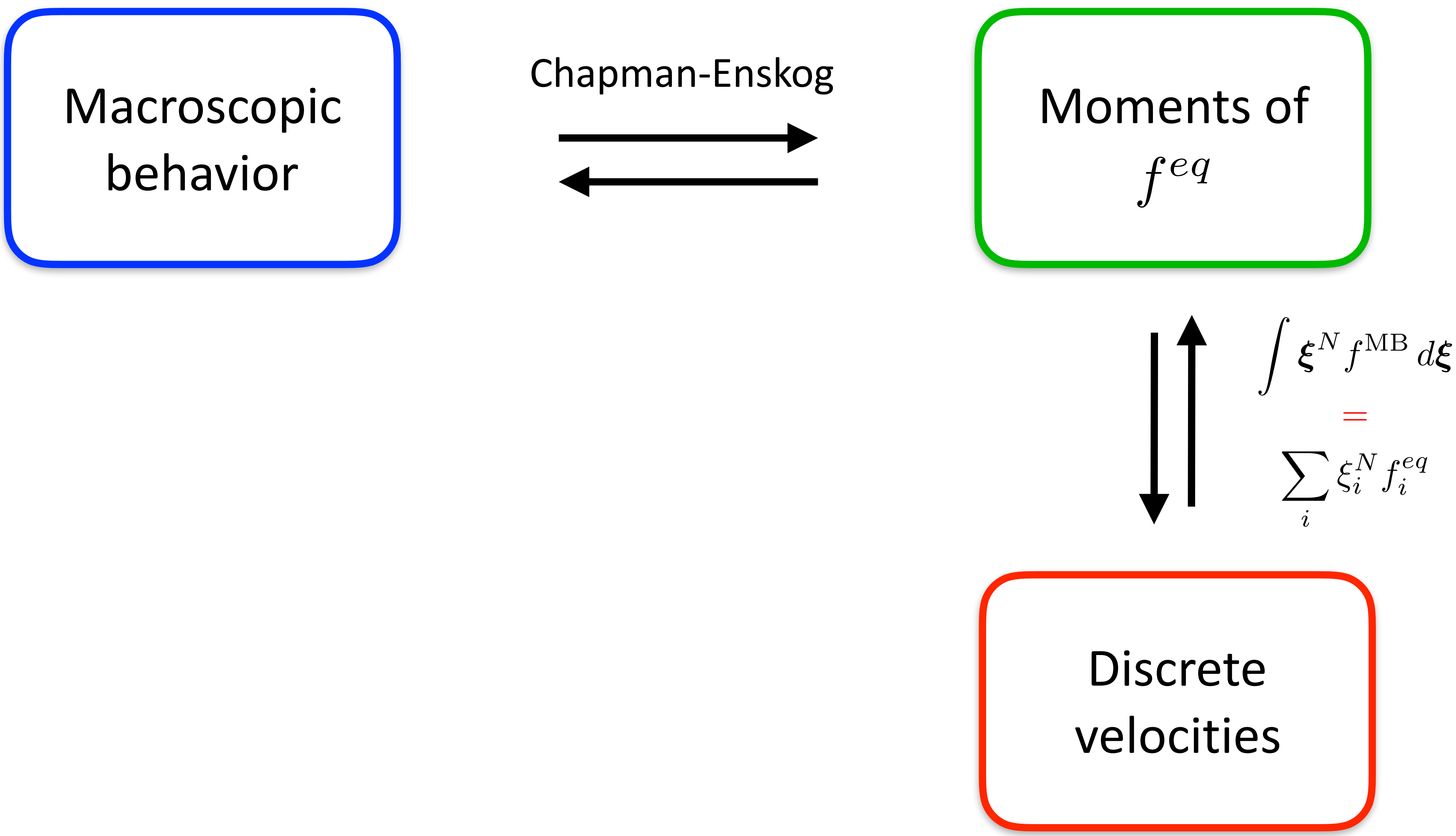
Macroscopic behavior ⇄ Moments of f^{eq}

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + p\boldsymbol{\delta}) = \boxed{\nabla \cdot \boldsymbol{\Pi}} \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \boxed{\nabla \cdot (\lambda \nabla T) + \nabla \cdot (\boldsymbol{\Pi} \cdot \mathbf{u})} \end{cases}$$

$$\begin{cases} \partial_t (M_0^{eq}) + \nabla \cdot (M_1^{eq}) = 0 \\ \partial_t (M_1^{eq}) + \nabla \cdot (M_2^{eq}) \propto \boxed{\partial_t (M_2^{eq}) + \nabla \cdot (M_3^{eq})} \\ \partial_t (M_{\mathrm{Tr}2}^{eq}) + \nabla \cdot (M_{\mathrm{Tr}3}^{eq}) \propto \boxed{\partial_t (M_{\mathrm{Tr}3}^{eq}) + \nabla \cdot (M_{\mathrm{Tr}4}^{eq})} \end{cases}$$

Chapman-Enskog

How do we design LBMs?



How do we design LBMs?

Macroscopic behavior

Chapman-Enskog ⇄

Moments of f^{eq}

$$\int \boldsymbol{\xi}^N f^{\mathrm{MB}} \, d\boldsymbol{\xi} = \sum_i \xi_i^N f_i^{eq}$$

Discrete velocities

- Gauss-Hermite quadrature [1]

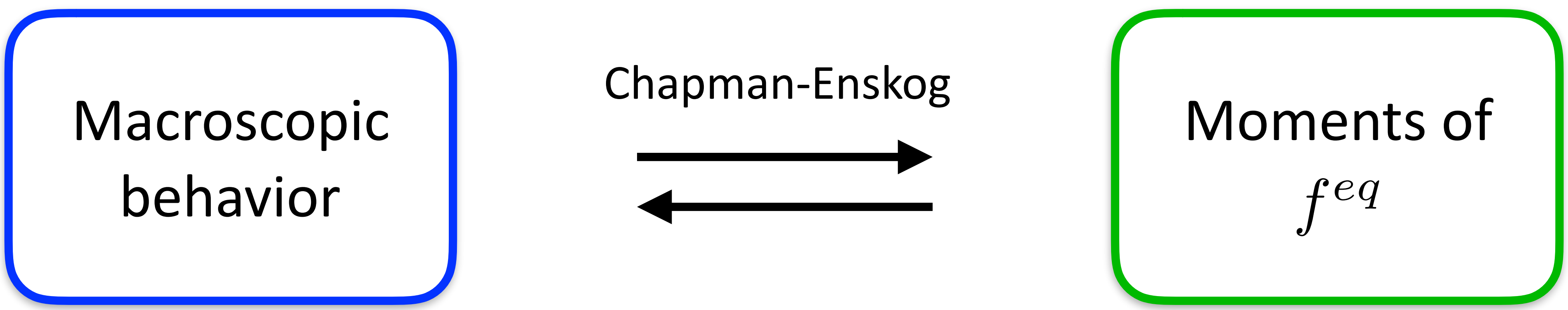
$$f_i^{eq} = \sum_{n=0}^{N} \boldsymbol{a}_n^{eq} : \boldsymbol{\mathcal{H}}_i^{(n)}$$

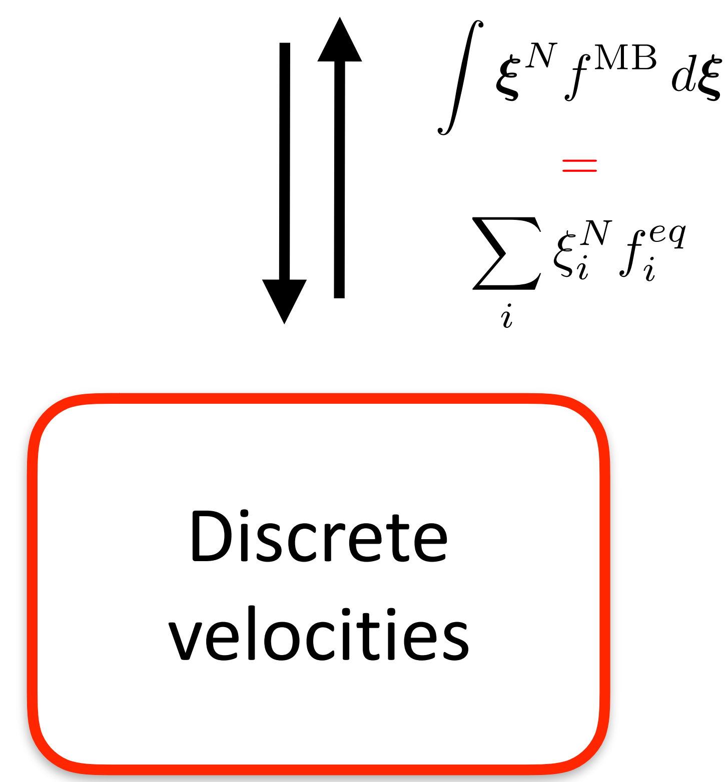
based on orthogonality properties

$$\sum_i w_i \mathcal{H}^{(n)}_{i,\alpha_1\ldots\alpha_n} \mathcal{H}^{(m)}_{i,\beta_1\ldots\beta_m} = \delta_{nm} [\delta_{\alpha_1\beta_1} \ldots \delta_{\alpha_n\beta_m}]_{cyc}$$

[1] **Philippi et al.,** From the continuous to the lattice Boltzmann equation: The discretization problem and thermal models**,** *PRE,* **2006**.

How do we design LBMs?





- Gauss-Hermite quadrature [1]

$$f_i^{eq} = \sum_{n=0}^{N} \boldsymbol{a}_n^{eq} : \boldsymbol{\mathcal{H}}_i^{(n)}$$

- Moment matching: analytical [2]

$$f_i^{eq} = \sum_n c_n g_n(\boldsymbol{\xi}_i, \boldsymbol{u}, T, \boldsymbol{u}T)$$

based on isotropy/symmetry rules

[1] **Philippi et al.,** From the continuous to the lattice Boltzmann equation: The discretization problem and thermal models**,** *PRE,* **2006**.
[2] **Chen et al.,** Thermal lattice Bhatnagar-Gross-Krook model without nonlinear deviations in macrodynamic equations, *PRE*, **1994.**

How do we design LBMs?

Macroscopic behavior

Chapman-Enskog ⇄

Moments of f^{eq}

$$\int \boldsymbol{\xi}^N f^{\mathrm{MB}}\, d\boldsymbol{\xi} = \sum_i \xi_i^N f_i^{eq}$$

Discrete velocities

- Gauss-Hermite quadrature [1]

$$f_i^{eq} = \sum_{n=0}^{N} \boldsymbol{a}_n^{eq} : \boldsymbol{\mathcal{H}}_i^{(n)}$$

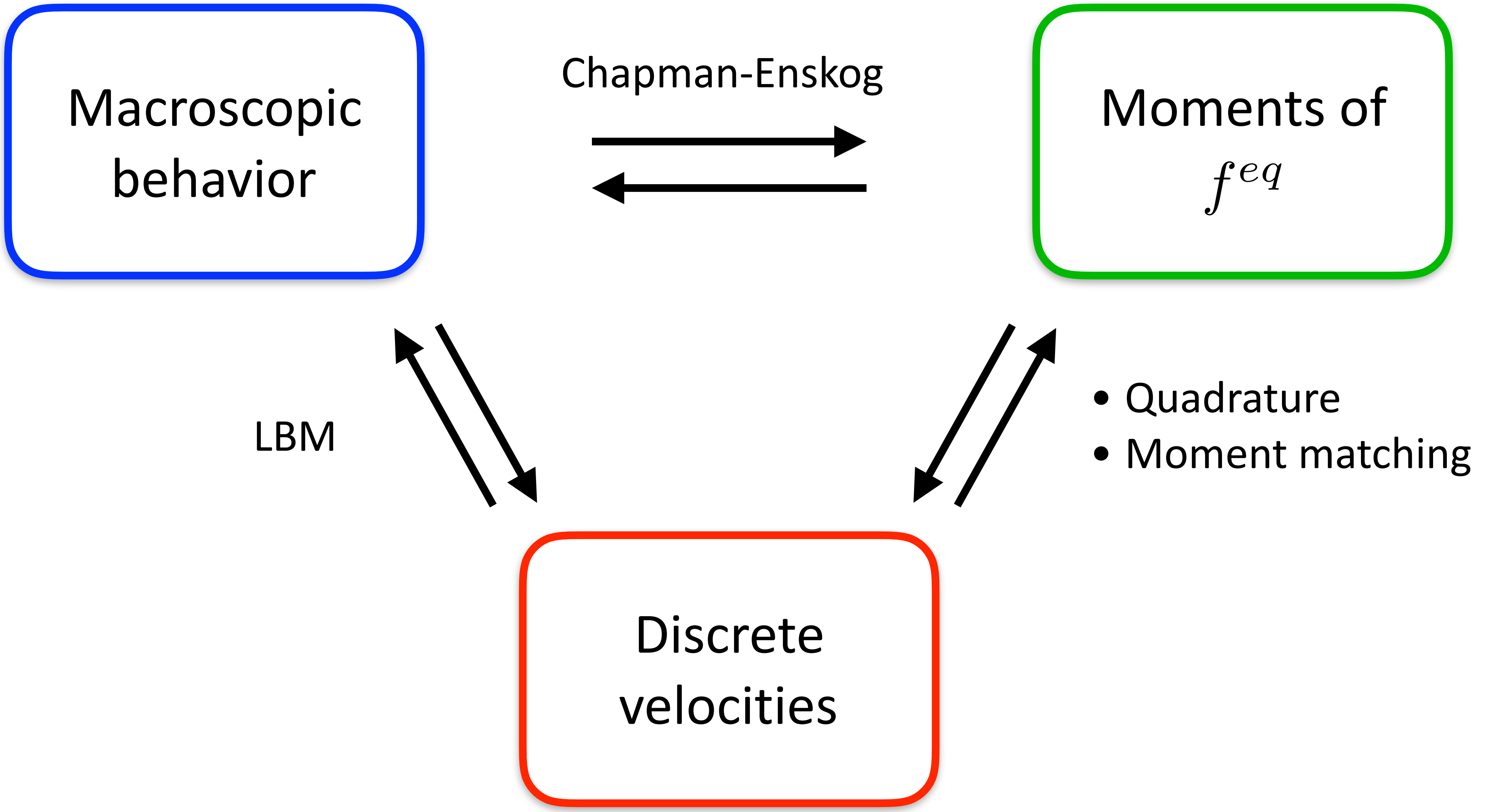
- Moment matching: analytical [2]

$$f_i^{eq} = \sum_n c_n g_n(\boldsymbol{\xi}_i, \boldsymbol{u}, T, \boldsymbol{u}T)$$

- Moment matching: numerical [3]

[1] **Philippi et al.,** From the continuous to the lattice Boltzmann equation: The discretization problem and thermal models**,** *PRE,* **2006**.
[2] **Chen et al.,** Thermal lattice Bhatnagar-Gross-Krook model without nonlinear deviations in macrodynamic equations, *PRE*, **1994.**
[3] **Le Tallec & Perlat,** Numerical Analysis of Levermore's Moment System, *Technical Report*, **1997.**

How do we design LBMs?



Of course, you also need (at least) **two relaxation times** to correctly impose the **Reynolds** and **Prandtl** numbers!

Difficult beginnings…

- <u>Nonlinear deviation in compressible regimes:</u> Qian & Orszag (Europhys. Lett., 1993)

EUROPHYSICS LETTERS 20 January 1993

Europhys. Lett., **21** (3), pp. 255-259 (1993)

Lattice BGK Models for the Navier-Stokes Equation: Nonlinear Deviation in Compressible Regimes.

Y. H. QIAN and S. A. ORSZAG

Program in Applied and Computational Mathematics, Princeton University Princeton, NJ 08544, USA

Model	t_0	t_1	t_2	t_3	t_4	c_s^2
D1Q3	2/3	1/6	0	0	0	1/3
D1Q5	1/2	1/6	0	0	1/12	1
D2Q7	1/2	1/12	0	0	0	1/4
D2Q9	4/9	1/9	1/36	0	0	1/3
D3Q15	2/9	1/9	0	1/72	0	1/3
D3Q19	1/3	1/18	1/36	0	0	1/3
D4Q25	1/3	0	1/36	0	0	1/3

$$N_i^e = \rho t_p \left[1 + \frac{c_{i\alpha} u_\alpha}{c_s^2} + \frac{u_\alpha u_\beta}{2 c_s^4} \left(c_{i\alpha} c_{i\beta} - c_s^2 \delta_{\alpha\beta} \right) \right]$$

2nd-order equilibrium

$$\frac{\text{nonlinear-deviation term}}{\text{linear term}} \sim \frac{\sigma U_0^3}{\nu U_0} \sim M^2$$

Weakly compressible limit of standard lattices

Difficult beginnings…

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)

PHYSICAL REVIEW E VOLUME 47, NUMBER 4 APRIL 1993

Lattice Boltzmann thermohydrodynamics

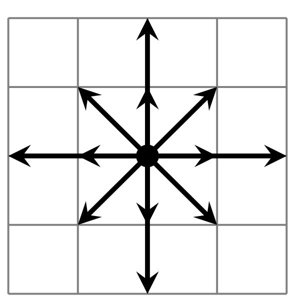
F. J. Alexander, S. Chen, and J. D. Sterling*

Center for Nonlinear Studies and Theoretical Divison, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 23 July 1992)

Simulating Thermohydrodynamics with Lattice BGK Models

Y. H. Qian[1]



$$N_i^e = A_p\rho + B_p c_{i\alpha}\rho u_\alpha + \frac{t_p}{2c_s^4}\left(c_{i\alpha}c_{i\beta} - c_s^2\delta_{\alpha\beta}\right)\rho u_\alpha u_\beta + D_p c_{i\alpha} u_\alpha \rho u^2$$

D2Q13

3rd-order velocity terms

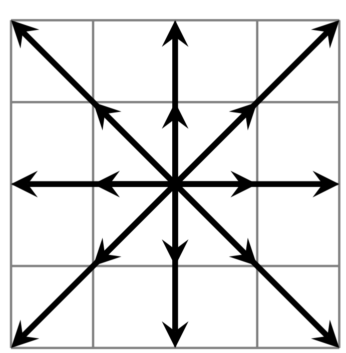
Difficult beginnings…

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)
- Monatomic thermal behavior (but Pr=1): Chen et al. (Phys. Rev. E, 1994)

PHYSICAL REVIEW E VOLUME 50, NUMBER 4 OCTOBER 1994

Thermal lattice Bhatnagar-Gross-Krook model without nonlinear deviations in macrodynamic equations

Y. Chen, H. Ohashi, and M. Akiyama
Department of Quantum Engineering and Systems Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan
(Received 29 December 1993; revised manuscript received 2 May 1994)



D2Q16

$$
\begin{aligned}
N_{pki}^{[eq]} = {} & A_{pk} + M_{pk}(c_{pki\alpha}u_\alpha) + G_{pk}u^2 \\
& + J_{pk}(c_{pki\alpha}u_\alpha)^2 + Q_{pk}(c_{pki\alpha}u_\alpha)u^2 \\
& + H_{pk}(c_{pki\alpha}u_\alpha)^3 + R_{pk}(c_{pki\alpha}u_\alpha)^2u^2 \\
& + S_{pk}u^4 + \mathcal{O}(u^5)\,.
\end{aligned}
$$

4th-order velocity terms

Difficult beginnings…

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)
- Monatomic thermal behavior (but Pr=1): Chen et al. (Phys. Rev. E, 1994)
- Variable Prandtl number: Chen et al. (JSC, 1997), McNamara et al. (J. Stat. Phys, 1997)

Two-Parameter Thermal Lattice BGK Model with a Controllable Prandtl Number

Y. Chen,[1] H. Ohashi,[1] and M. Akiyama[1]

Error in the heat generated by viscous friction

A Hydrodynamically Correct Thermal Lattice Boltzmann Model

Guy R. McNamara,[1] Alejandro L. Garcia,[2,3] and Berni J. Alder[2]

Correct energy equation and minimal lattice!… but unstable…

Difficult beginnings…

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)
- Monatomic thermal behavior (but Pr=1): Chen et al. (Phys. Rev. E, 1994)
- Variable Prandtl number: Chen et al. (JSC, 1997), McNamara et al. (J. Stat. Phys, 1997)
- Polyatomic extension but fixed Prandtl number: Kataoka & Tsutahara (PRE, 2004)

PHYSICAL REVIEW E **69**, 056702 (2004)

Lattice Boltzmann method for the compressible Euler equations

Takeshi Kataoka* and Michihisa Tsutahara
Graduate School of Science and Technology, Kobe University, Rokkodai, Nada, Kobe 657-8501, Japan
(Received 28 November 2003; published 18 May 2004)

PHYSICAL REVIEW E **69**, 035701(R) (2004)

Lattice Boltzmann model for the compressible Navier-Stokes equations with flexible specific-heat ratio

Takeshi Kataoka* and Michihisa Tsutahara
Graduate School of Science and Technology, Kobe University, Rokkodai, Nada, Kobe 657-8501, Japan
(Received 11 November 2003; published 25 March 2004)

Constraints on internal dofs are included in the moment matching approach… but the Prandtl number is fixed and this approach suffer from stability issues

Difficult beginnings...

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)
- Monatomic thermal behavior (but Pr=1): Chen et al. (Phys. Rev. E, 1994)
- Variable Prandtl number: Chen et al. (JSC, 1997), McNamara et al. (J. Stat. Phys, 1997)
- Polyatomic extension but fixed Prandtl number: Kataoka & Tsutahara (PRE, 2004)
- Correct compressible behavior: Li et al. (PRE, 2007)

PHYSICAL REVIEW E **76**, 056705 (2007)

Coupled double-distribution-function lattice Boltzmann method for the compressible Navier-Stokes equations

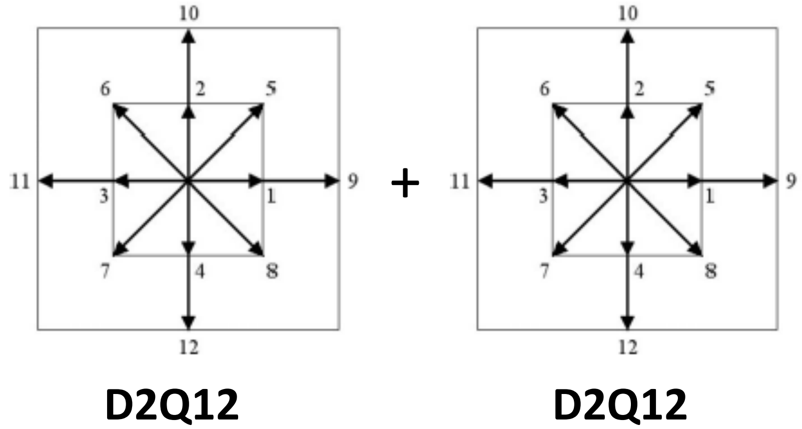
Q. Li, Y. L. He,* Y. Wang, and W. Q. Tao

State Key Laboratory of Multiphase Flow, School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, China

(Received 15 July 2007; revised manuscript received 10 September 2007; published 16 November 2007)

Difficult beginnings…

- Nonlinear deviation in compressible regimes: Qian & Orszag (Europhys. Lett., 1993)
- Correct isothermal behavior: Alexander et al. (1993), Qian (J. Sci. Comput., 1993)
- Monatomic thermal behavior (but Pr=1): Chen et al. (Phys. Rev. E, 1994)
- Variable Prandtl number: Chen et al. (JSC, 1997), McNamara et al. (J. Stat. Phys, 1997)
- Polyatomic extension but fixed Prandtl number: Kataoka & Tsutahara (PRE, 2004)
- Correct compressible behavior: Li et al. (PRE, 2007)



- Double distribution function approach
- Stability issues (requires IMEX + WENO5)

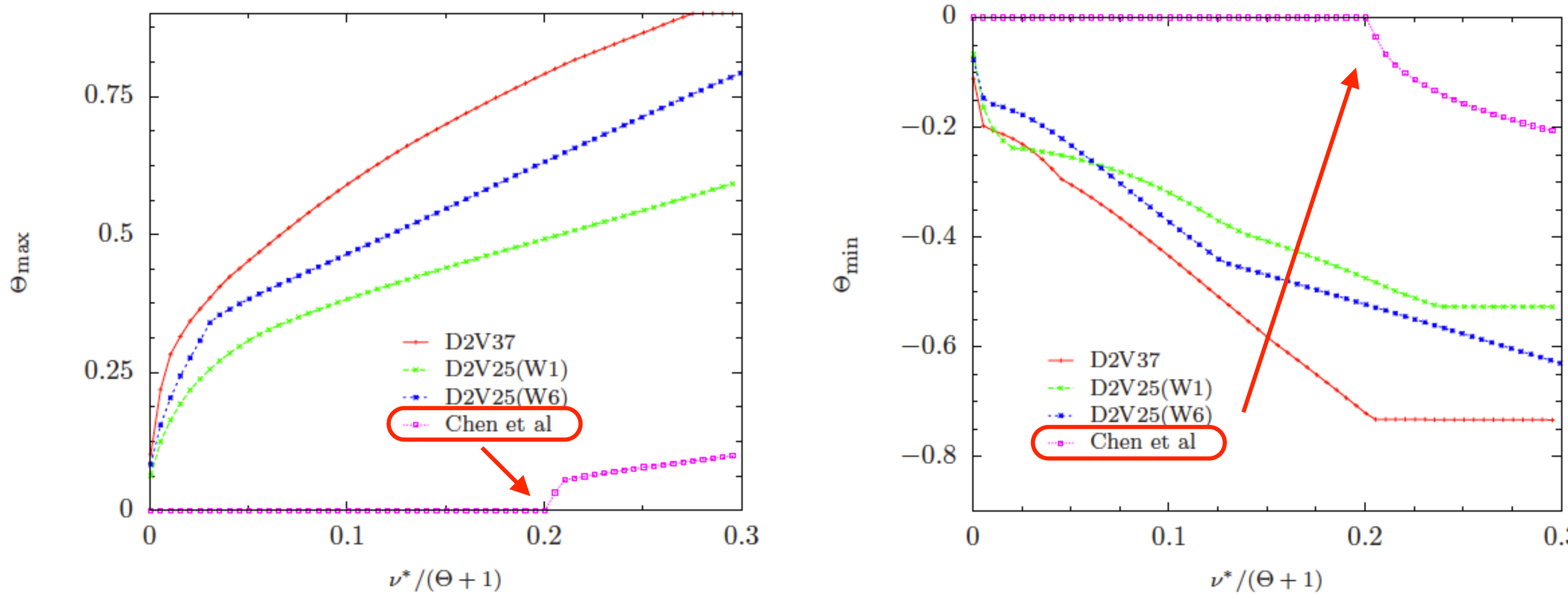
Change the numerical scheme to get stable simulations?

Change the numerical scheme to get stable simulations?… not necessarily!

PHYSICAL REVIEW E **77**, 026707 (2008)

Lattice Boltzmann equation linear stability analysis: Thermal and athermal models

D. N. Siebert,* L. A. Hegele, Jr.,† and P. C. Philippi‡
LMPT Mechanical Engineering Department, Federal University of Santa Catarina, 88040-900 Florianopolis, SC, Brazil
(Received 1 October 2007; published 26 February 2008)

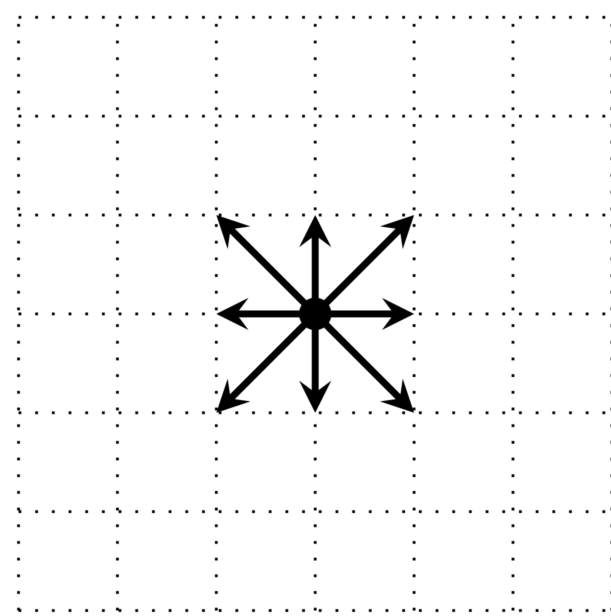


(a) $\Theta > 0$ (b) $\Theta < 0$

Gauss-Hermite quadrature based LBMs seem to be more stable than moment-matching ones!

Gauss-Hermite quadrature based LBMs

✤ **Lattice considered**



D2Q9

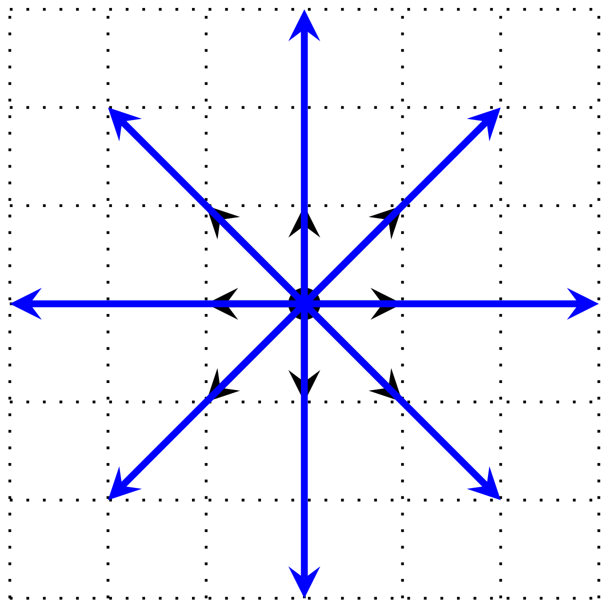
Standard
LBM

✤ **Macroscopic equations (Hermite expansion up to the 2nd order !)**

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} + \mathcal{O}(M^3) \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{\Pi} \cdot \mathbf{u}) \end{cases}$$

Gauss-Hermite quadrature based LBMs

✤ **Lattice considered**



High-Order
LBM

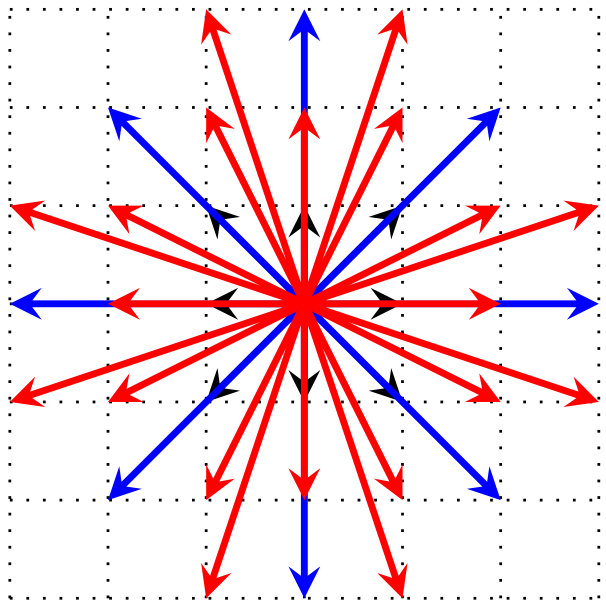
✤ **Macroscopic equations (Hermite expansion up to the 3rd order!)**

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{\Pi} \cdot \mathbf{u}) \\ \qquad + \mathcal{O}\left(M^4, M^2\theta, \theta^2\right) \end{cases}$$

[1] **Philippi *et al.***, *PRE,* **2006**, 73.

Gauss-Hermite quadrature based LBMs

✤ **Lattice considered**



D2V37a [1,2]

High-Order
LBM

✤ **Macroscopic equations (Hermite expansion up to the 4th order!)**

$$
\begin{cases}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\
\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \\
\partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{\Pi} \cdot \mathbf{u})
\end{cases}
$$

[1] Philippi *et al.*, *PRE*, **2006**, 73.

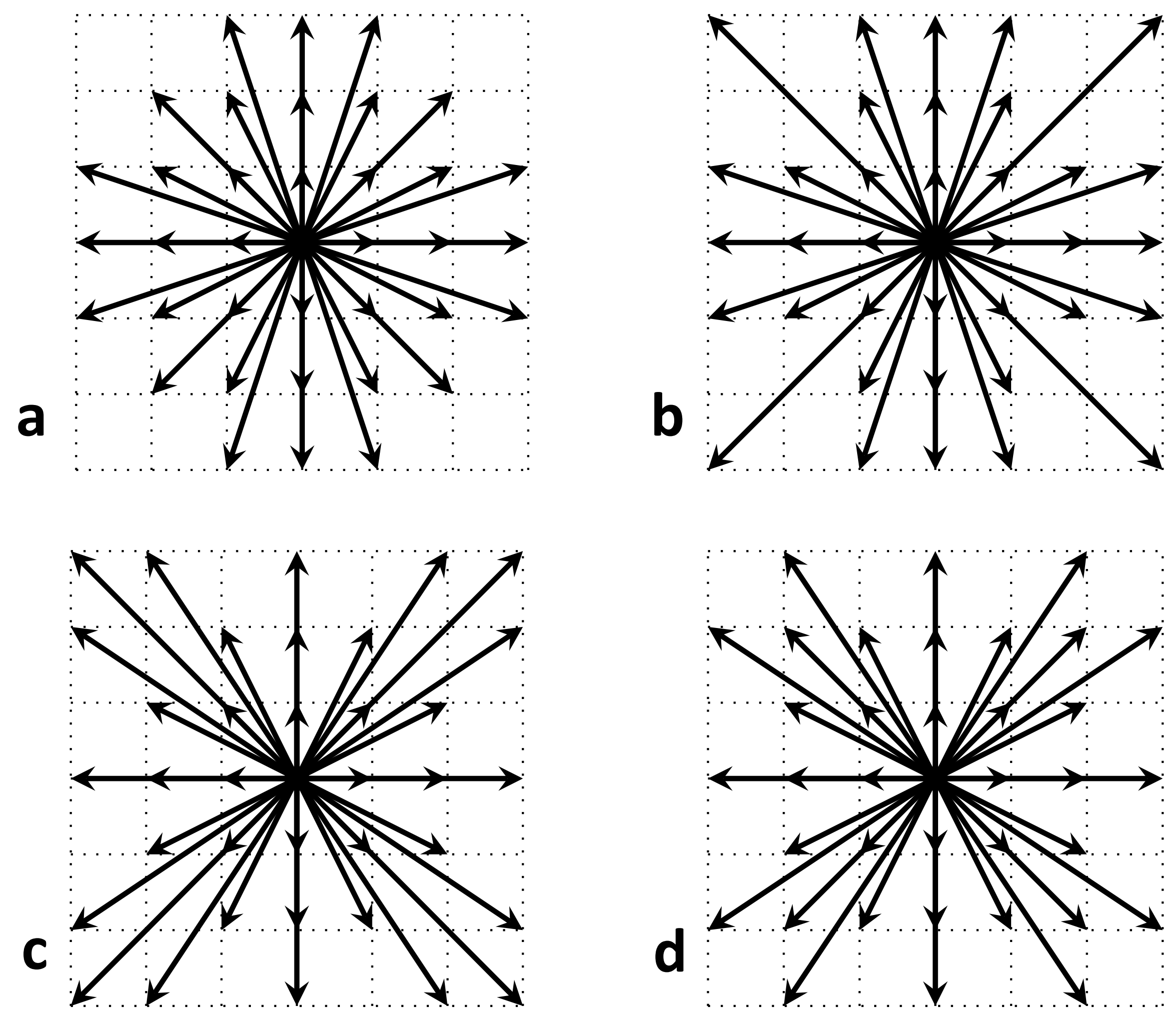
[2] Scagliarini *et al.*, *PoF*, **2010**.

Are they really more stable?

Are they really more stable?

✤ **Linear stability of fourth-order models (isothermal hypothesis)**

D2V37

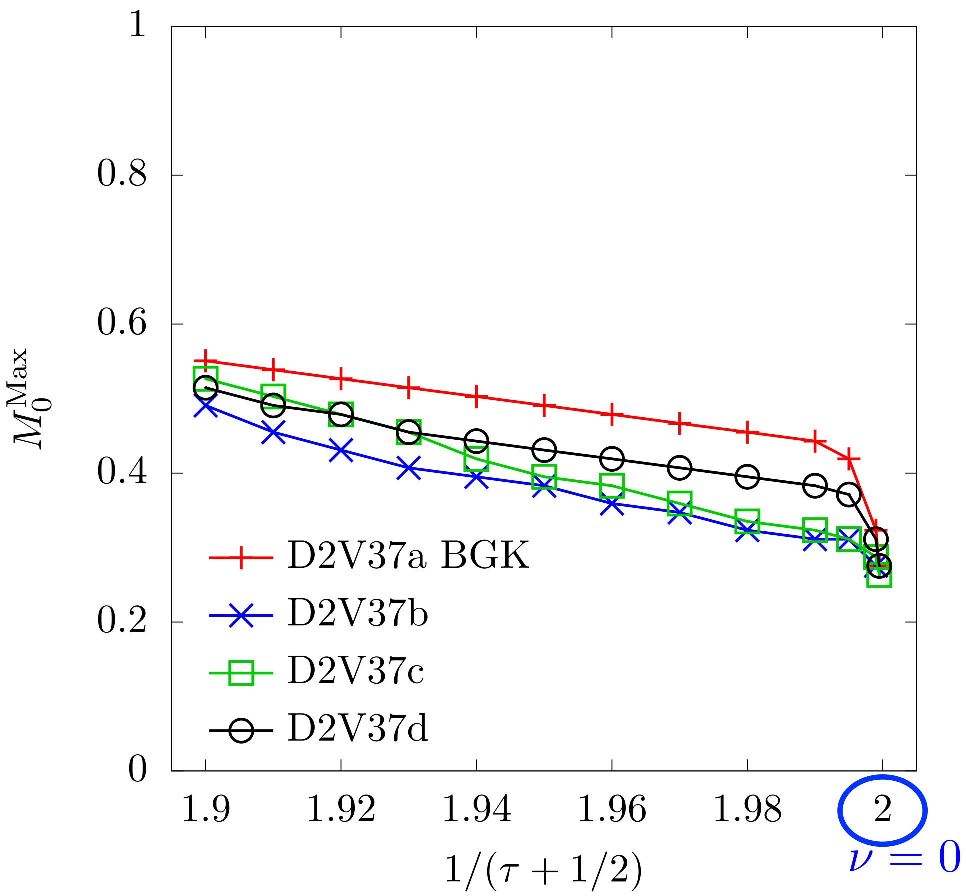


Coreixas, High-order extension of the recursive regularized lattice Boltzmann method, PhD Manuscript, **2018**.

Are they really more stable?

✤ **Linear stability of fourth-order models (isothermal hypothesis)**

BGK



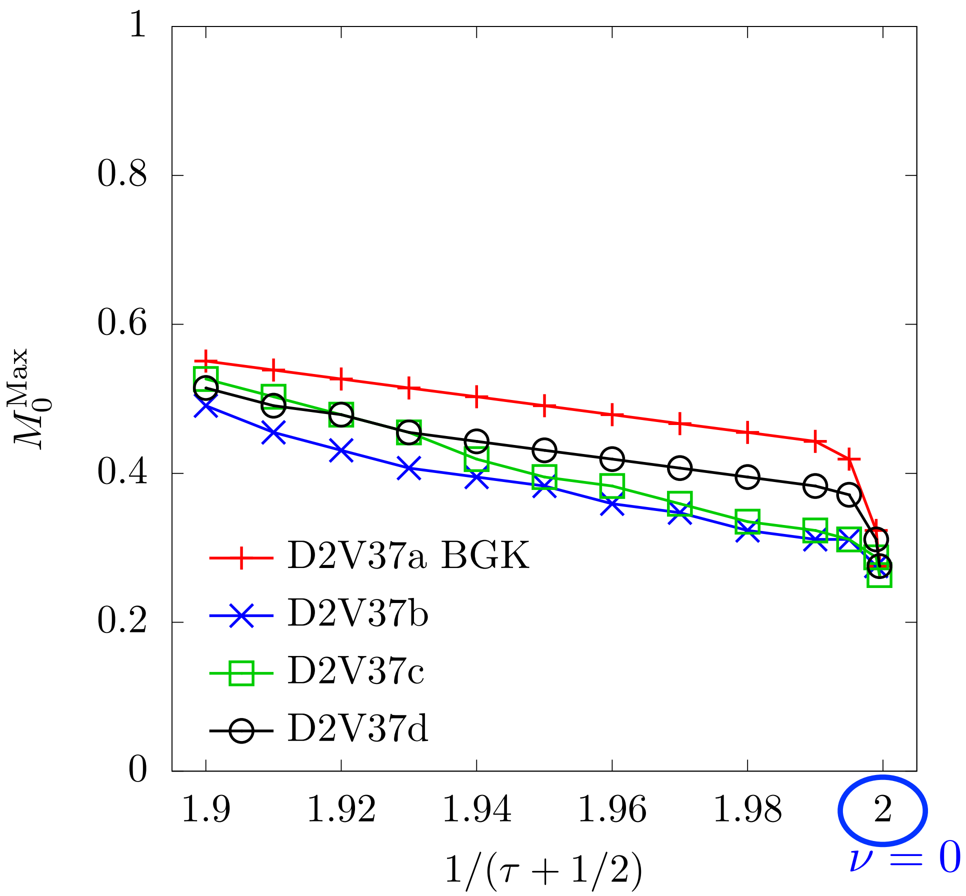
- All BGK-LBMs have **different** stability ranges with a **low maximal Mach number**…

Coreixas, High-order extension of the recursive regularized lattice Boltzmann method, PhD Manuscript, **2018**.

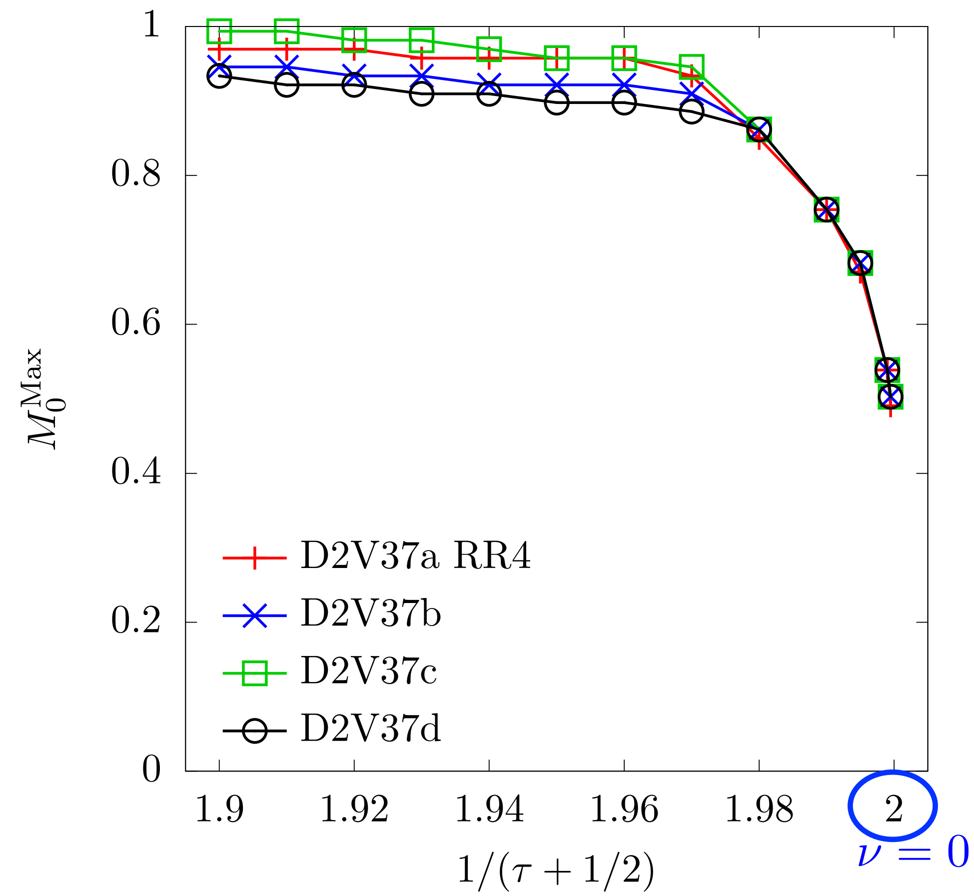
Are they really more stable?

✤ **Linear stability of fourth-order models (isothermal hypothesis)**

BGK



Recursive regularization

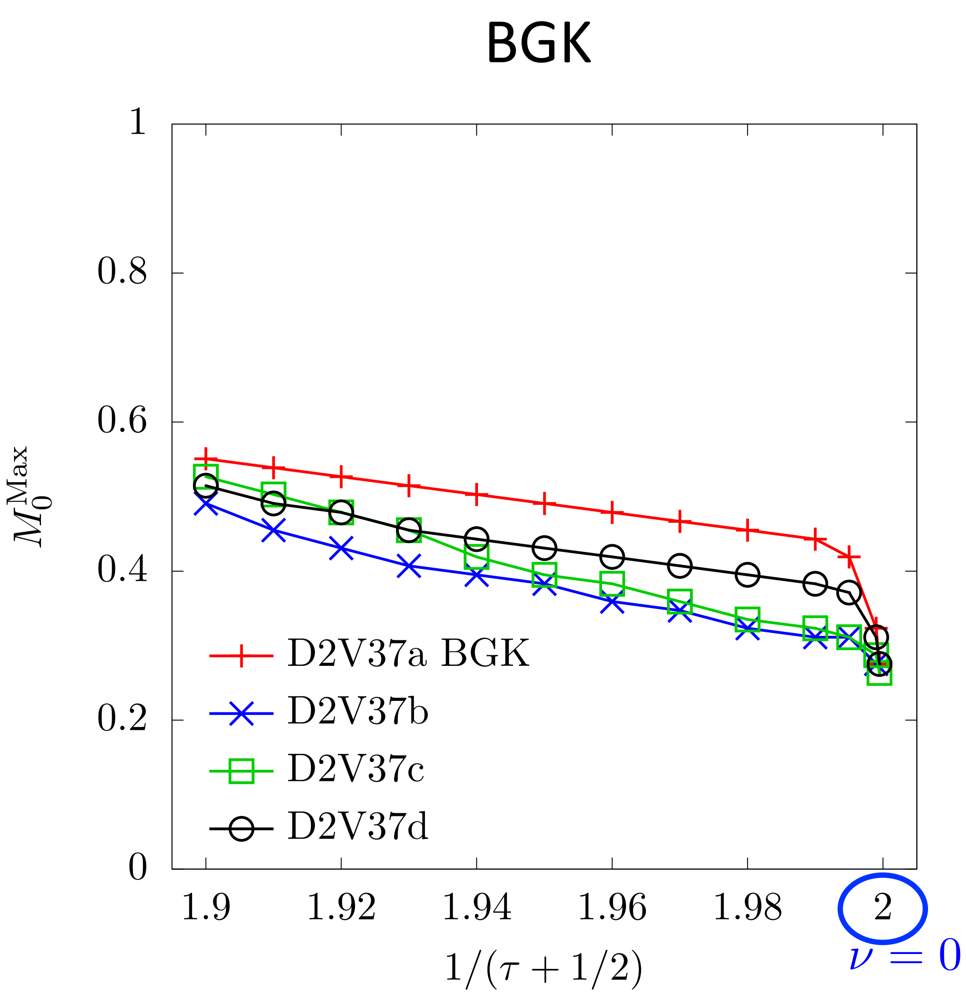


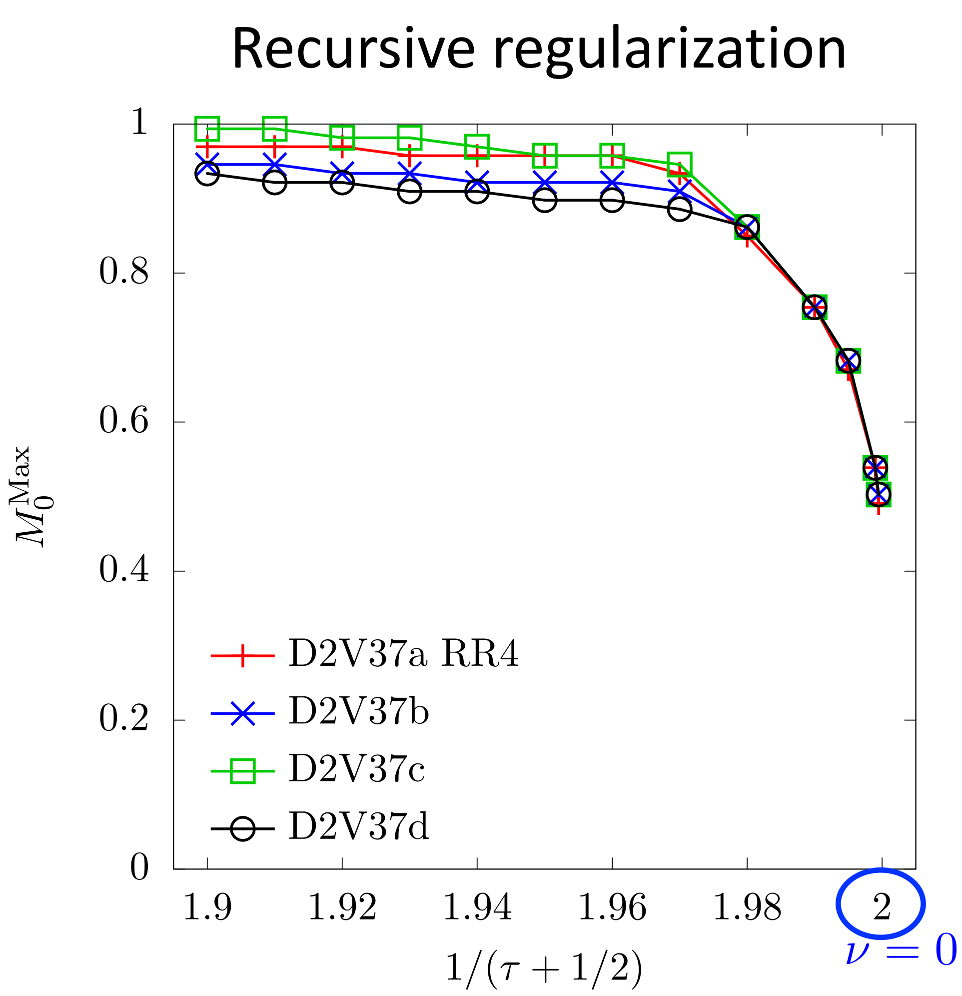
- All BGK-LBMs have **different** stability ranges with a **low maximal Mach number**…
- But this **can be improved by changing the collision model without changing the numerical scheme!**

Coreixas, High-order extension of the recursive regularized lattice Boltzmann method, PhD Manuscript, **2018**.

Are they really more stable?

✤ **Linear stability of fourth-order models (isothermal hypothesis)**



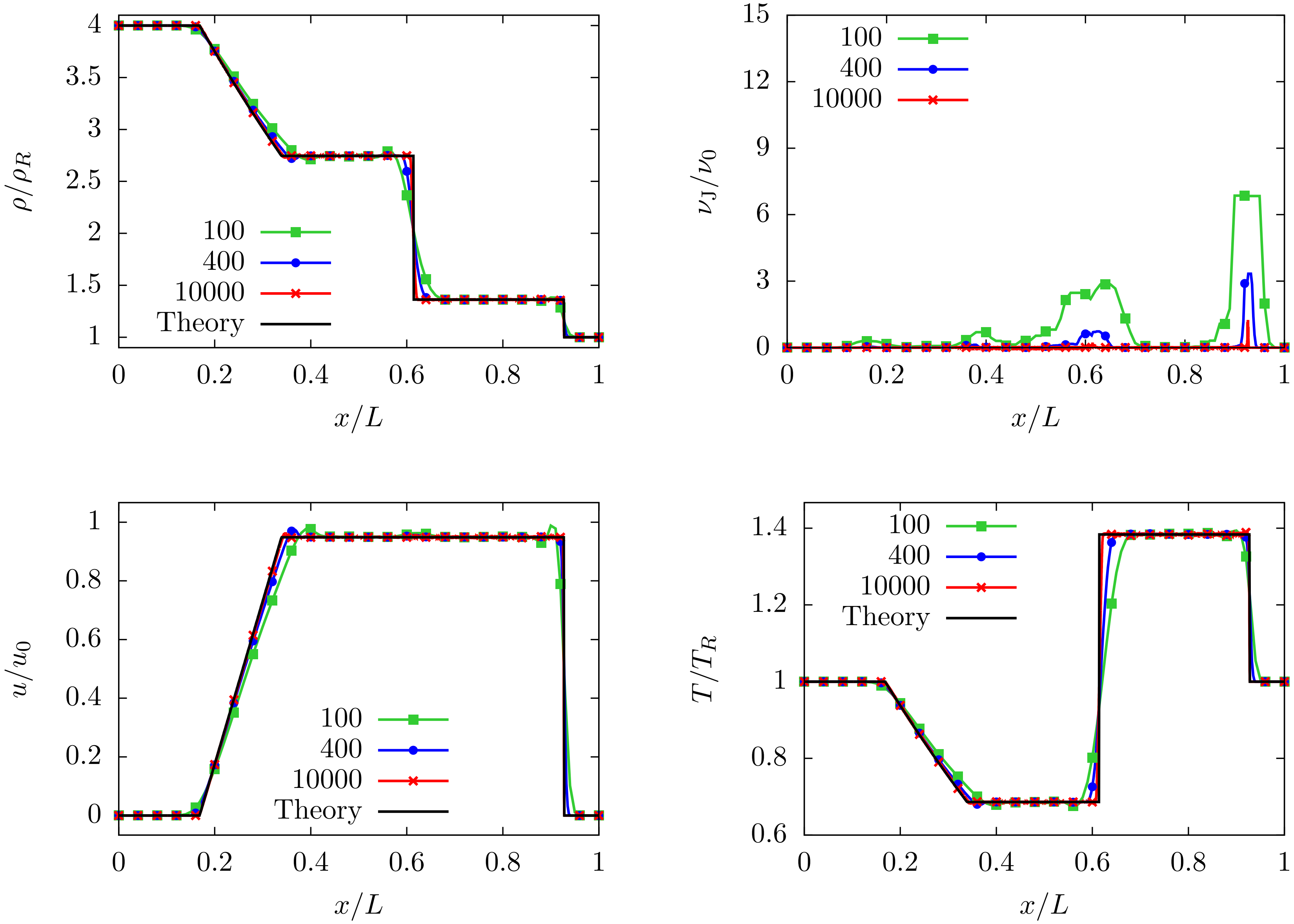


- All BGK-LBMs have **different** stability ranges with a **low maximal Mach number**…
- But this **can be improved by changing the collision model without changing the numerical scheme!**
- More info regarding the stabilization properties (check papers below)

Brogi et al., Hermite regularization of the lattice Boltzmann method for open source computational aeroacoustics, *JASA*, **2017.**
Coreixas et al., Recursive regularization step for high-order lattice Boltzmann methods, *PRE*, **2017**.
Coreixas et al., Impact of collision models on the physical properties and the stability of lattice Boltzmann methods, *PTRSA*, **2020**.
Wissocq et al., Linear stability and isotropy properties of athermal regularized lattice Boltzmann methods, *PRE*, **2020**.

Are they really more stable?

✤ **Not bad in the fully compressible case (D2V37a + RR + Jameson-like sensor)**



$$T_L/T_R = 1$$

$$\rho_L/\rho_R = 4$$

$$u_L = u_R = 0$$

Very good agreement wrt the theory.

Coreixas, High-order extension of the recursive regularized lattice Boltzmann method, PhD Manuscript, **2018**.

Ok, but the lattice is too large in 3D!

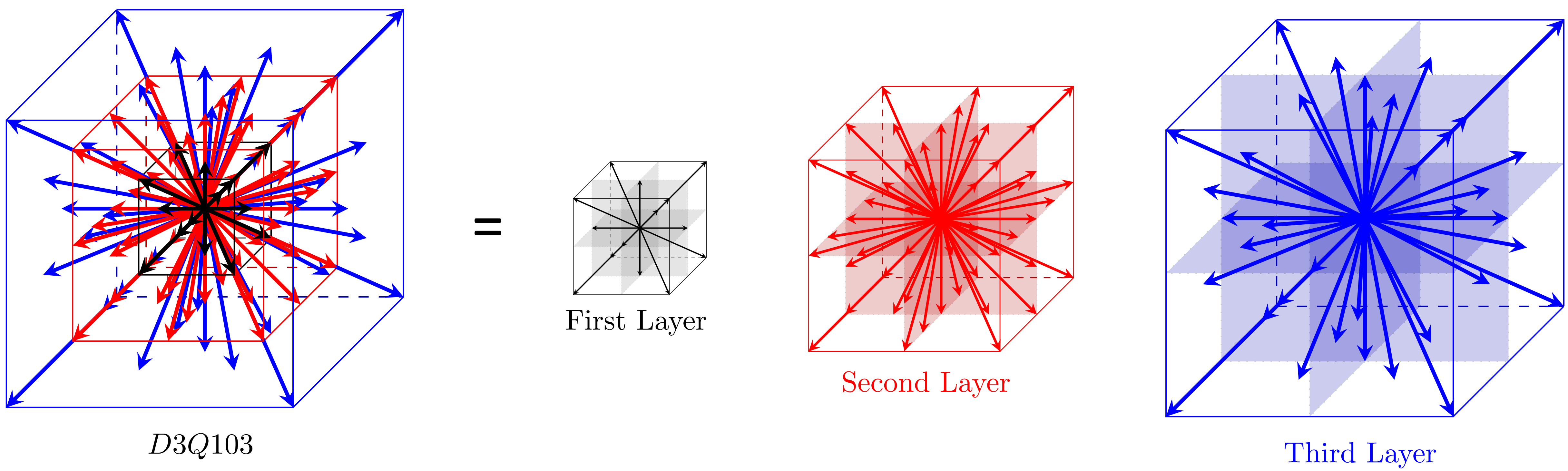
Shan, The mathematical structure of the lattices of the lattice Boltzmann method**,** *JCS,* **2016**.

Ok, but the lattice is too large in 3D!

D3Q103 ⟶ Eq Order = 4 ⟶

Thermal and fully compressible

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{u} \cdot \mathbf{\Pi}) \end{cases}$$



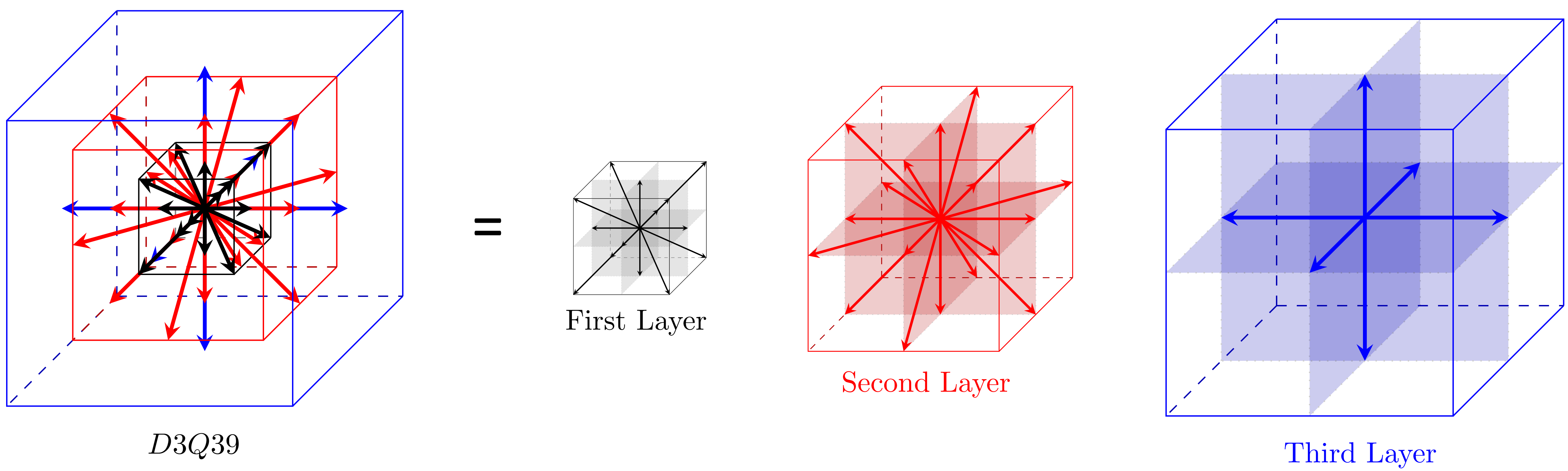
Shan, The mathematical structure of the lattices of the lattice Boltzmann method, *JCS,* **2016**.

Good compromise (PowerFLOW)

Isothermal (no compressibility restriction)

D3Q39 ⟶ Eq Order = 3 ⟶

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \\ \partial_t (\rho E) + \nabla \cdot ([\rho E + p]\, \mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{u} \cdot \mathbf{\Pi}) + \mathcal{O}(M^4, M^2\theta, \theta^2) \end{cases}$$



Shan *et al.*, Kinetic theory representation of hydrodynamics: A way beyond the Navier–Stokes equation, *JFM,* **2006**.

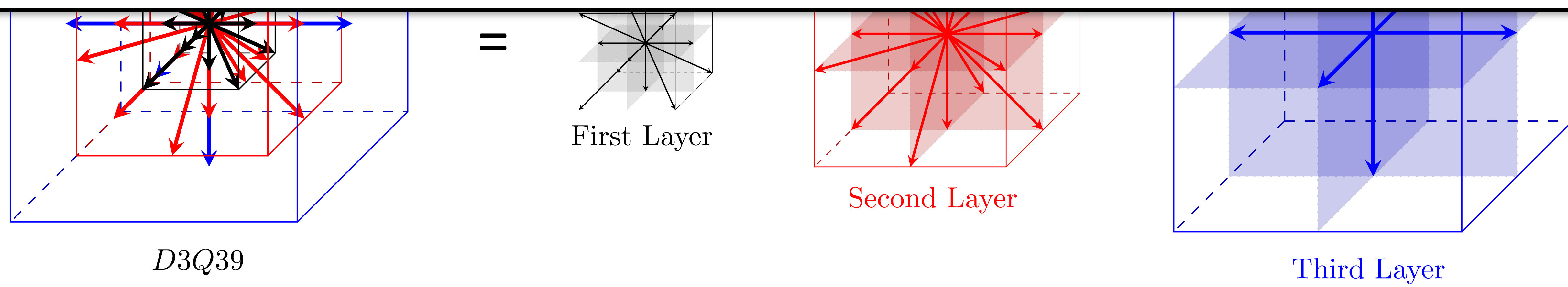
Good compromise (PowerFLOW)

D3Q39 ⟶ Eq Order = 3 ⟶

Isothermal (no compressibility restriction)

$$
\begin{cases}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\
\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \boldsymbol{\Pi} \\
\partial_t (\rho E) + \nabla \cdot ([\rho E + p]\,\mathbf{u}) = \nabla \cdot (\lambda \nabla T) + \nabla \cdot (\mathbf{u} \cdot \boldsymbol{\Pi}) + \mathcal{O}(M^4, M^2\theta, \theta^2)
\end{cases}
$$

How do we get the correct physics with that lattice?

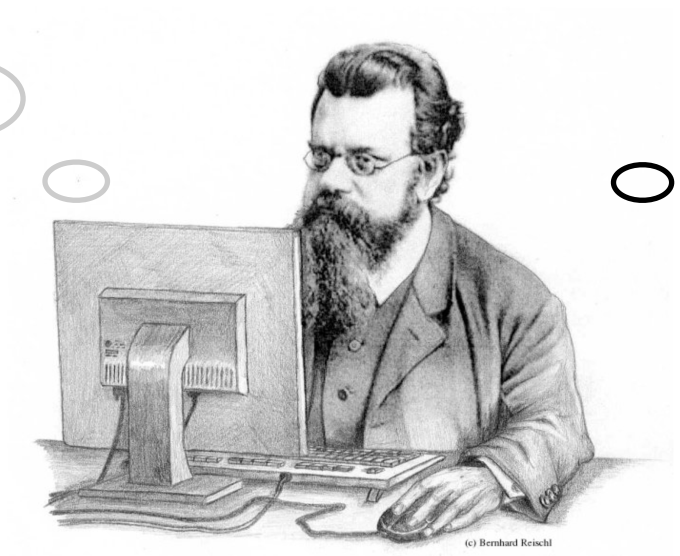


Shan *et al.*, Kinetic theory representation of hydrodynamics: A way beyond the Navier–Stokes equation, *JFM,* **2006**.

Outline

How do we design LBMs ?

Two-equation models



Quadrature free LBMs

Adaptive lattices

CPU Time and Memory Reduction Strategy

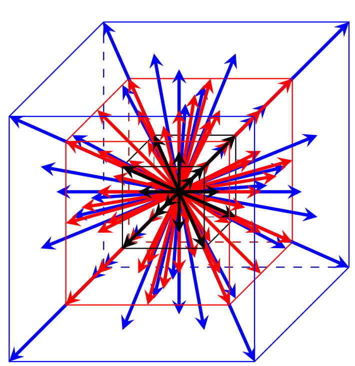
Compressible LBMs

↓

Single Distribution

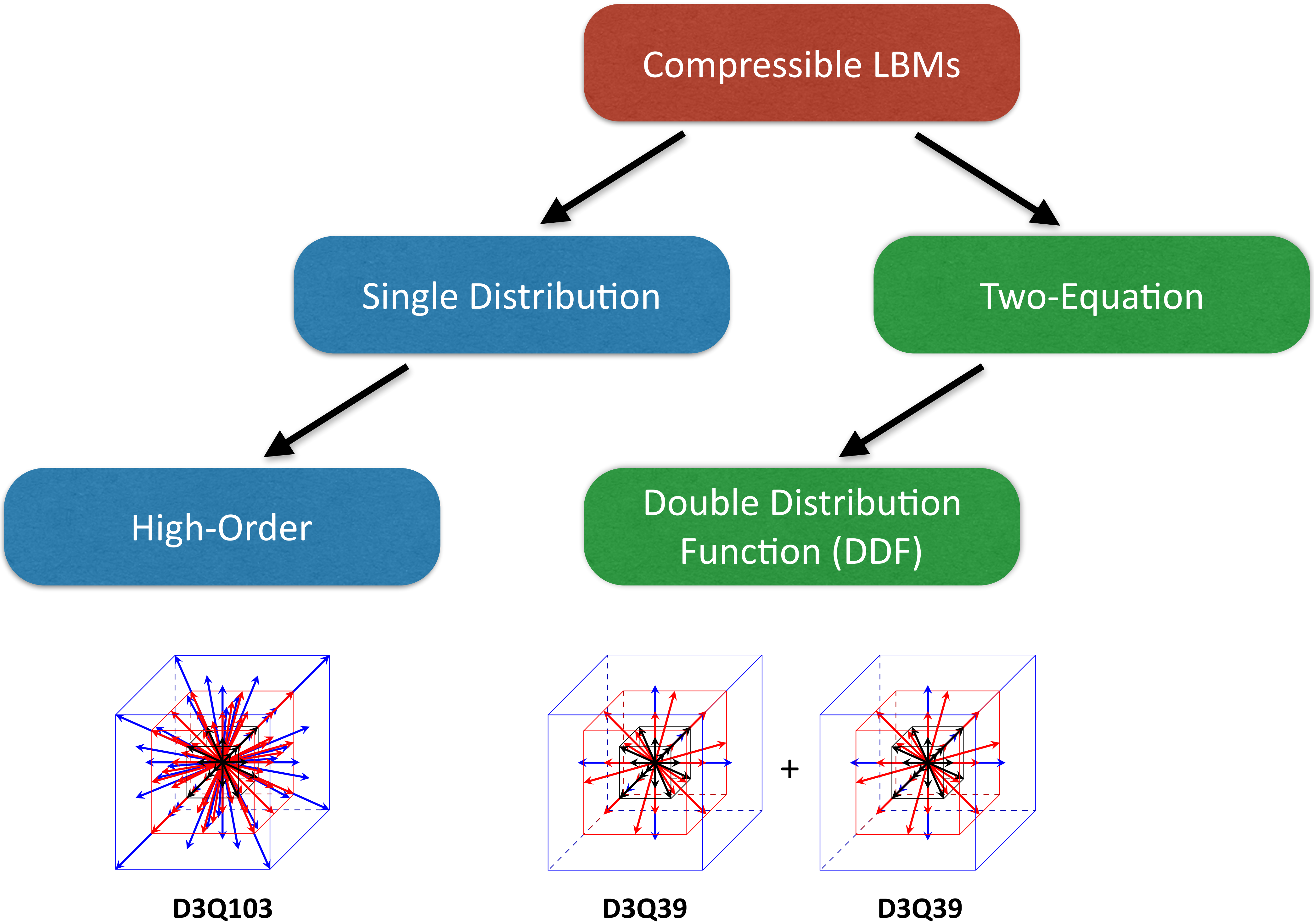
↓

High-Order

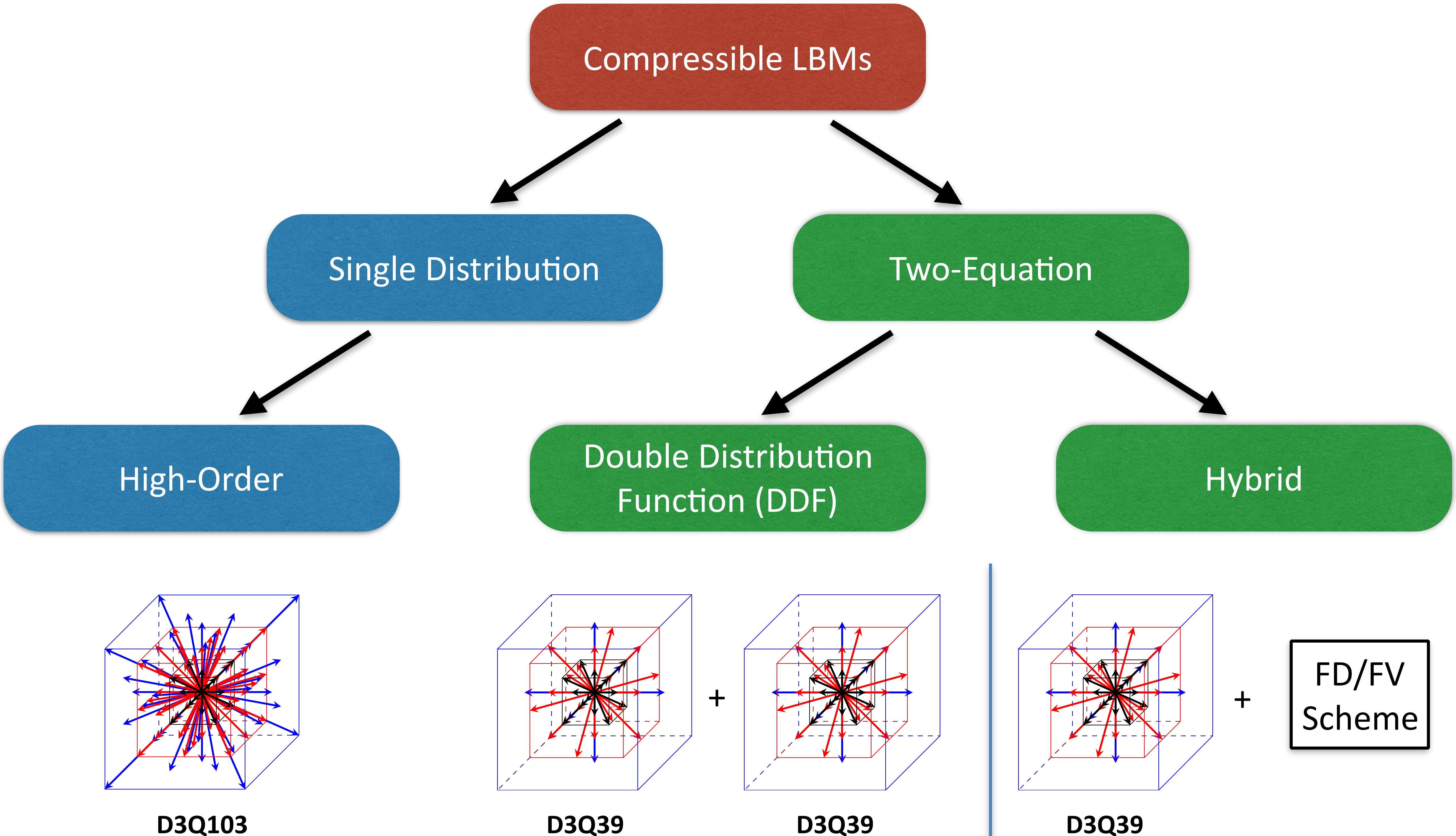


D3Q103

CPU Time and Memory Reduction Strategy



CPU Time and Memory Reduction Strategy



Compressible physics at lower cost

Double Distribution Function (DDF)

$$\frac{\partial f_i}{\partial t} + \xi_{i\alpha}\frac{\partial f_i}{\partial x_\alpha} = -\frac{1}{\tau_f}\left[f_i - f_i^{eq}\right]$$

LBM

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \end{cases}$$

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$(\rho, \mathbf{u})$

$$\frac{\partial h_i}{\partial t} + \xi_{i\alpha}\frac{\partial h_i}{\partial x_\alpha} = -\frac{1}{\tau_h}\left[h_i - h_i^{eq}\right] + H_i$$

$$h_i^{eq} = \frac{1}{2}\left[\xi_i^2 + \left(\frac{2}{\gamma - 1} - D\right)T\right] f_i^{eq}$$

LBM

LBM

$$\begin{cases} \partial_t \rho + \nabla\cdot(\rho\mathbf{u}) = 0 \\ \partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = -\nabla p + \nabla\cdot\mathbf{\Pi} \end{cases}$$

$$\partial_t(\rho E) + \nabla\cdot([\rho E + p]\,\mathbf{u}) = \nabla\cdot(\lambda\nabla T) + \nabla\cdot(\mathbf{u}\cdot\mathbf{\Pi})$$

Compressible physics at lower cost

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Implicit coupling

$$\frac{\partial h_i}{\partial t} + \xi_{i\alpha}\frac{\partial h_i}{\partial x_\alpha} = -\frac{1}{\tau_h}\left[h_i - h_i^{eq}\right] + H_i$$

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$(\rho, \mathbf{u})$

LBM → LBM

$p = \rho T$

$\nu = \nu(T)$

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \end{cases}$$

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Li *et al.*,Coupled double-distribution-function lattice Boltzmann method for the compressible Navier-Stokes equations**,** *PRE,* **2007**.

Compressible physics at lower cost

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Guo *et al.*,Thermal lattice Boltzmann equation for low Mach number flows: Decoupling model**,** *PRE,* **2007**.

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Double Distribution Function (DDF)

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$$\frac{\partial h_i}{\partial t} + \xi_{i\alpha}\frac{\partial h_i}{\partial x_\alpha} = -\frac{1}{\boxed{\tau_h}}[h_i - h_i^{eq}] + \boxed{H_i}$$

$$h_i^{eq} = \frac{1}{2}\left[\xi_i^2 + \boxed{\left(\frac{2}{\gamma - 1} - D\right)}T\right] f_i^{eq}$$

LBM $\xrightarrow{(\rho, \mathbf{u})}$ LBM

LBM $\xleftarrow[\nu = \nu(T)]{p = \rho T}$ LBM

$$\begin{cases} \partial_t \rho + \nabla\cdot(\rho\mathbf{u}) = 0 \\ \partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) = -\nabla p + \nabla\cdot\mathbf{\Pi} \end{cases}$$

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- Flexible **Pr** and γ
- These models allow for the **reduction of CPU and memory consumptions**

Compressible physics at lower cost

Hybrid LBM

$$\frac{\partial f_i}{\partial t} + \xi_{i\alpha}\frac{\partial f_i}{\partial x_\alpha} = -\frac{1}{\tau_f}\left[f_i - f_i^{eq}\right] + F_i$$

$(\rho, \mathbf{u})$

LBM

FD or FV discretization of the energy equation [1,2]

$p = \rho T$

$\nu = \nu(T)$

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \\ \partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\Pi} \end{cases}$$

These models allow us to **further reduce CPU and memory consumptions**…
BUT it is **difficult** to get **accurate AND stable** hybrid schemes!

[1] **Filippova and Hänel**, A Novel Lattice BGK Approach for Low Mach Number Combustion, *JCP*, **2000.**
[2] **Zhang and Chen,** Lattice Boltzmann method for simulations of liquid-vapor thermal flows, *PRE,* **2003**.

Compressible physics at lower cost
Stability of hybrid LBMs

✤ **Hybrid LBMs based on the space-time evolution of** $\phi = e,\ E,\ s$

Conservative form

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho u_\alpha \phi)}{\partial x_\alpha} = \mathcal{P}_\phi + \mathcal{F}_\phi + \mathcal{V}_\phi$$

Primitive form

$$\frac{\partial\phi}{\partial t} + u_\alpha \frac{\partial\phi}{\partial x_\alpha} = \frac{1}{\rho}\left(\mathcal{P}_\phi + \mathcal{F}_\phi + \mathcal{V}_\phi\right)$$

Renard et al., A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**

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$\mathcal{P}_\phi$ pressure work (heat production by compressibility effects)

$\mathcal{F}_\phi$ Fourier heat flux (heat loss by diffusion)

$\mathcal{V}_\phi$ Viscous production (heat source by friction)

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terms \ variables	E	e	s
T	$\frac{1}{c_v}\left(E - \frac{1}{2}u_\alpha^2\right)$	$\frac{e}{c_v}$	$\rho^{\gamma_g - 1}\exp\left(\frac{s}{c_v}\right)$
$\mathcal{P}_\phi$	$-\frac{\partial(u_\alpha p)}{\partial x_\alpha}$	$-p\frac{\partial u_\alpha}{\partial x_\alpha}$	$\varnothing$
$\mathcal{F}_\phi$	$\frac{\partial}{\partial x_\alpha}\left(\lambda \frac{\partial T}{\partial x_\alpha}\right)$	$\frac{\partial}{\partial x_\alpha}\left(\lambda \frac{\partial T}{\partial x_\alpha}\right)$	$\frac{1}{T}\frac{\partial}{\partial x_\alpha}\left(\lambda \frac{\partial T}{\partial x_\alpha}\right)$
$\mathcal{V}_\phi$	$-\frac{\partial}{\partial x_\beta}\left(a_{\alpha\beta}^{(1)} u_\alpha\right)$	$-a_{\alpha\beta}^{(1)}\frac{\partial u_\alpha}{\partial x_\beta}$	$-\frac{a_{\alpha\beta}^{(1)}}{T}\frac{\partial u_\alpha}{\partial x_\beta}$

The entropy formulation is the only one with an implicit pressure work term!

Renard et al., A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**

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✤ **Two numerical discretizations**

	$\frac{\partial(\rho\phi)}{\partial t}$	$\frac{\partial(\rho u_\alpha \phi)}{\partial x_\alpha}$	$\mathcal{P}_\phi$	$\mathcal{F}_\phi$	$\mathcal{V}_\phi$
RK1UPO1	RK1	D1UPO1	D1CO2	D2CO2	D2CO2
RK4CO2	RK4	D1CO2	D1CO2	D2CO2	D2CO2

Renard et al., A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**

Compressible physics at lower cost
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RK1UPO1	RK1	D1UPO1	D1CO2	D2CO2	D2CO2
RK4CO2	RK4	D1CO2	D1CO2	D2CO2	D2CO2

The study focuses on errors related to time and convective terms

Renard et al., A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**

Compressible physics at lower cost

Stability of hybrid LBMs

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RK4CO2	RK4	D1CO2	D1CO2	D2CO2	D2CO2

The study focuses on errors related to time and convective terms

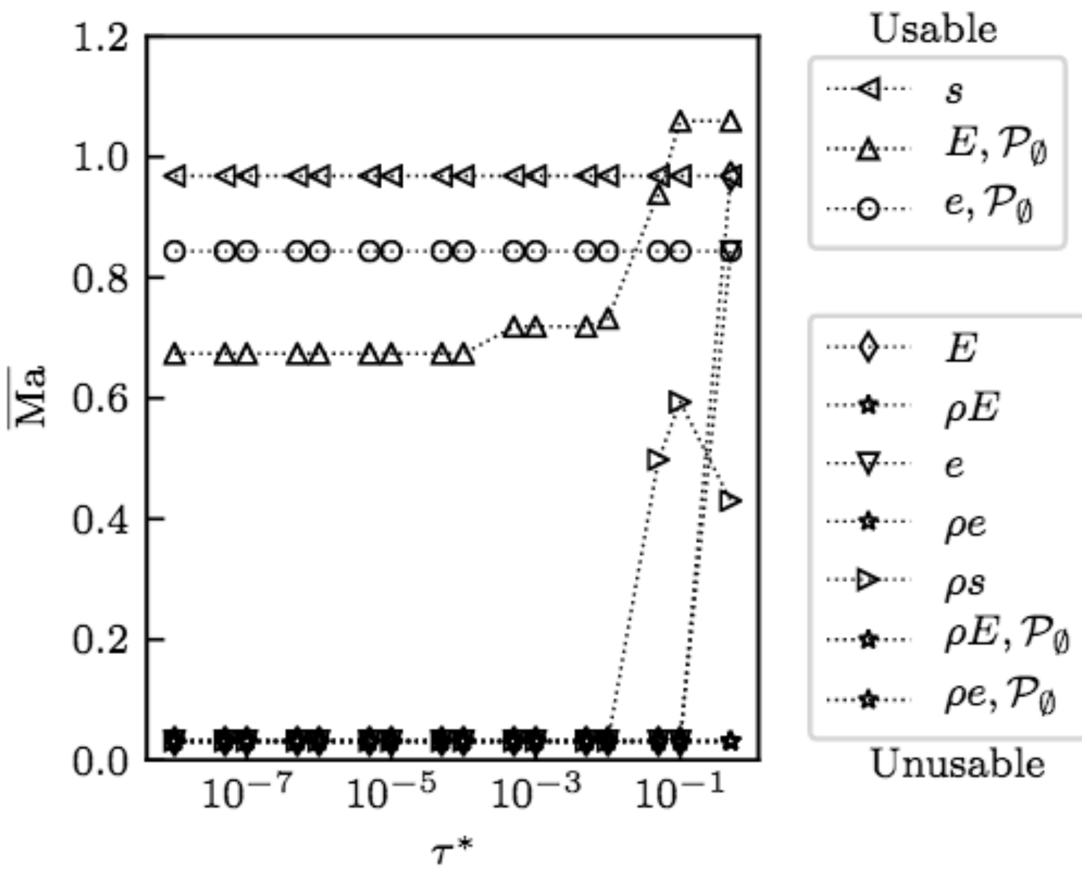
Central scheme for source/sink terms

Renard et al., A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**

Compressible physics at lower cost
Stability of hybrid LBMs

✤ **Linear stability domain of hybrid D2Q9-LBMs (implicit coupling + HRR collision)**

RK1UPO1



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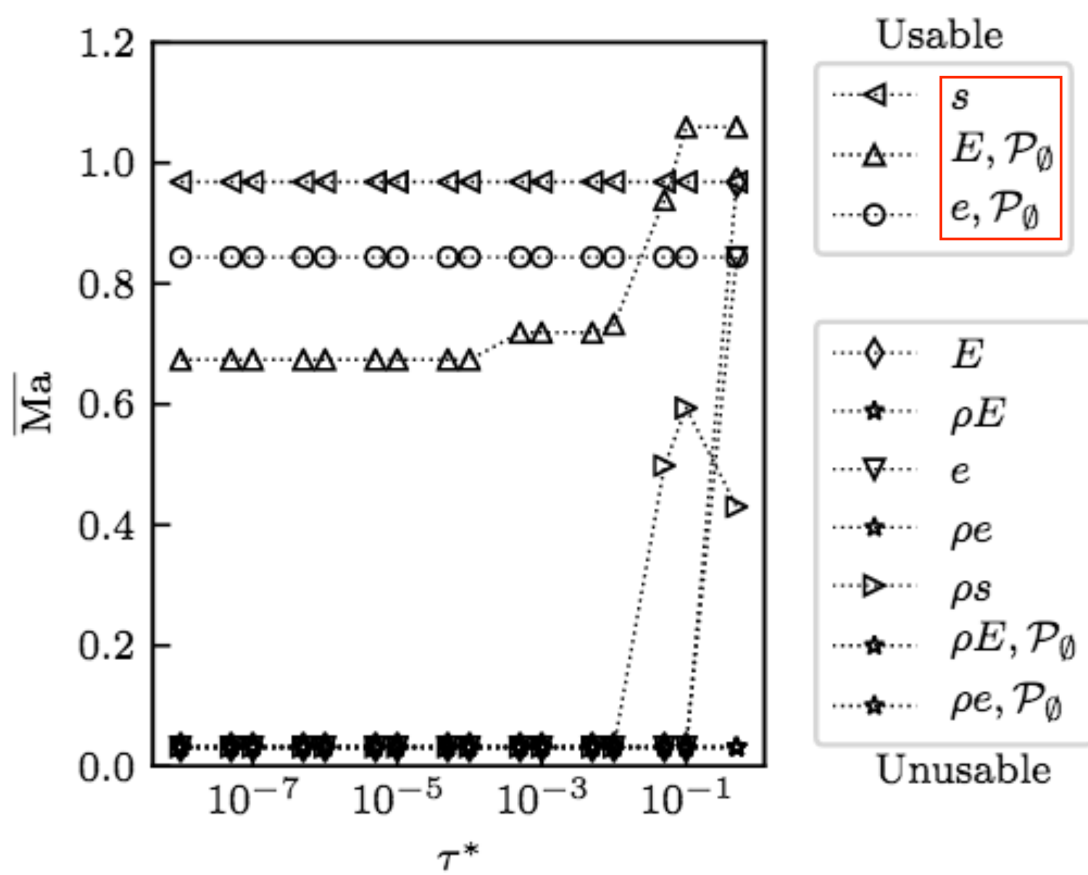

- For this **low**-order formulation, **only primitive forms are stable**
- For internal and total energy, the **pressure work** leads to **stability issues**

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Compressible physics at lower cost

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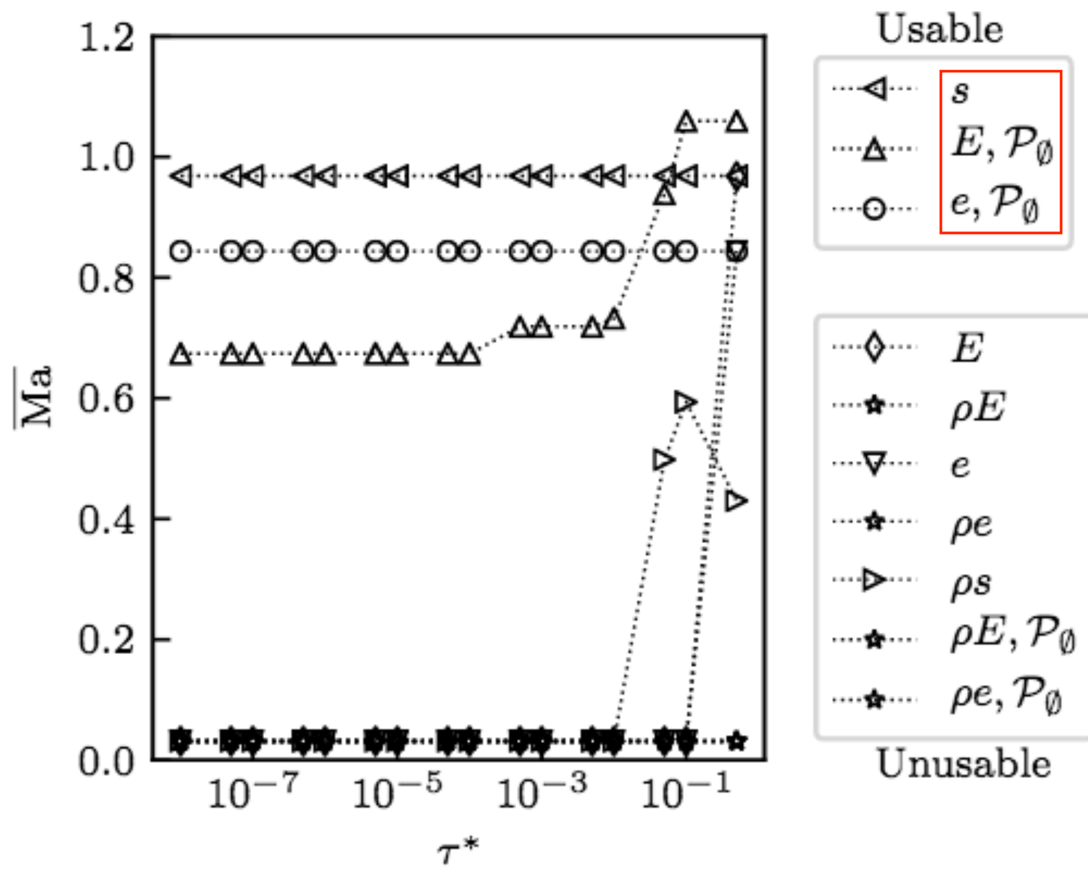
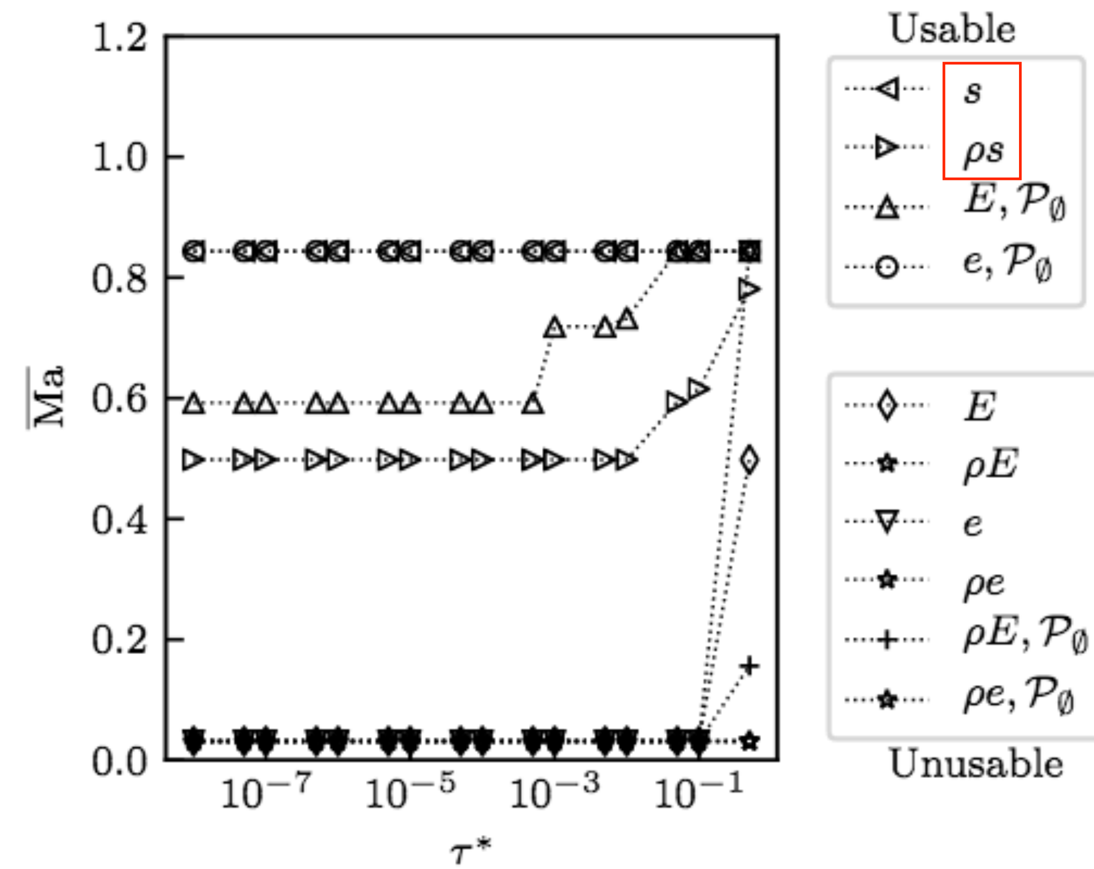


RK4CO2



- For this **low**-order formulation, **only primitive forms are stable**
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- For the **high**-order formulation, the **conservative form** of the **entropy** equation is also **stable**

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Compressible physics at lower cost

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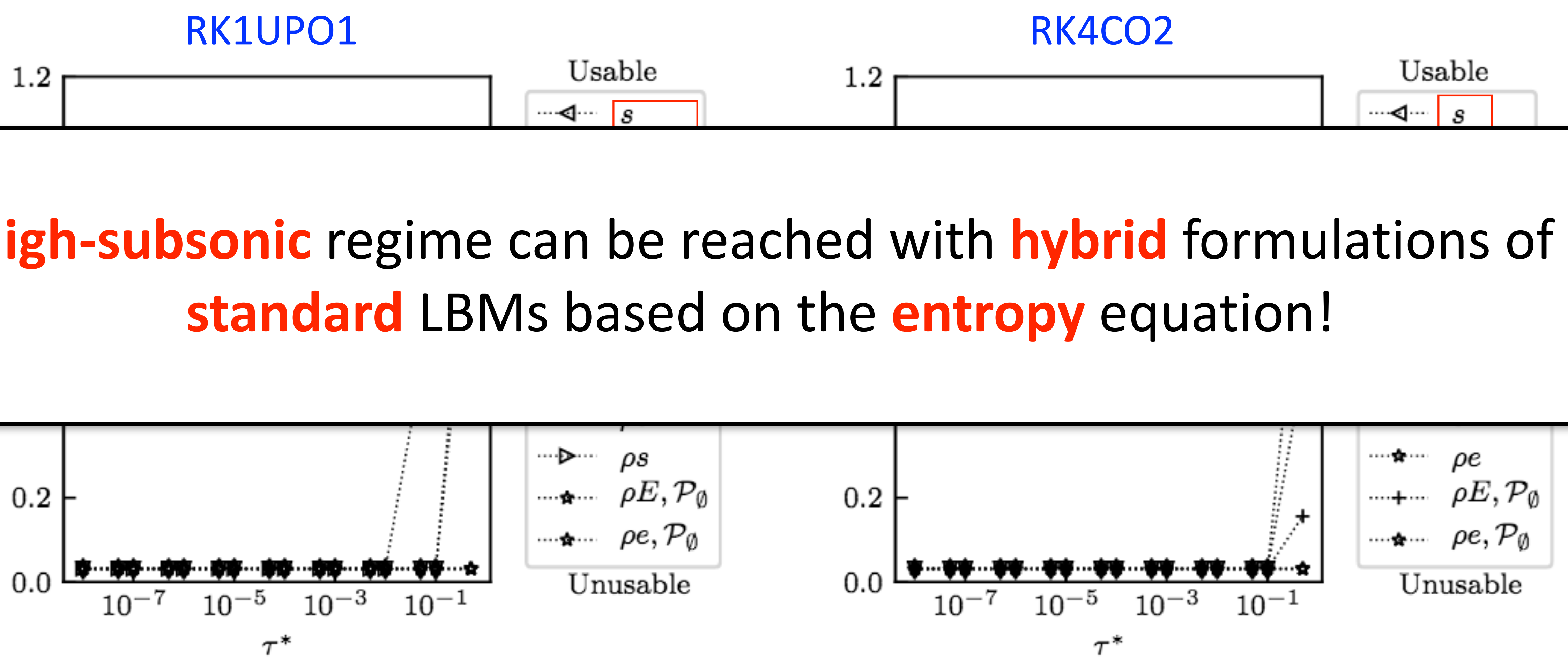

High-subsonic regime can be reached with **hybrid** formulations of **standard** LBMs based on the **entropy** equation!

- For this **low**-order formulation, **only primitive forms are stable**
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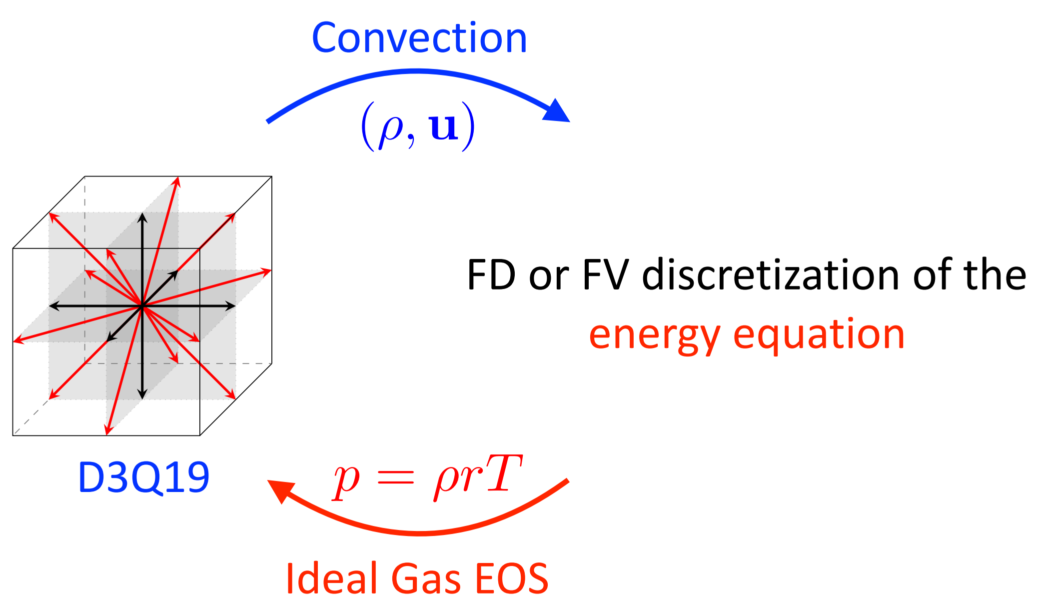
This is in agreement with PowerFLOW's methodology



High-subsonic LBM [1]

- D3Q19 + Mach correction
- **Entropy** equation
- Limitation: **Mach < 0.9**

[1] Casalino et al., Towards lattice-boltzmann prediction of turbofan engine noise. AIAA Paper, **2014-3101.**

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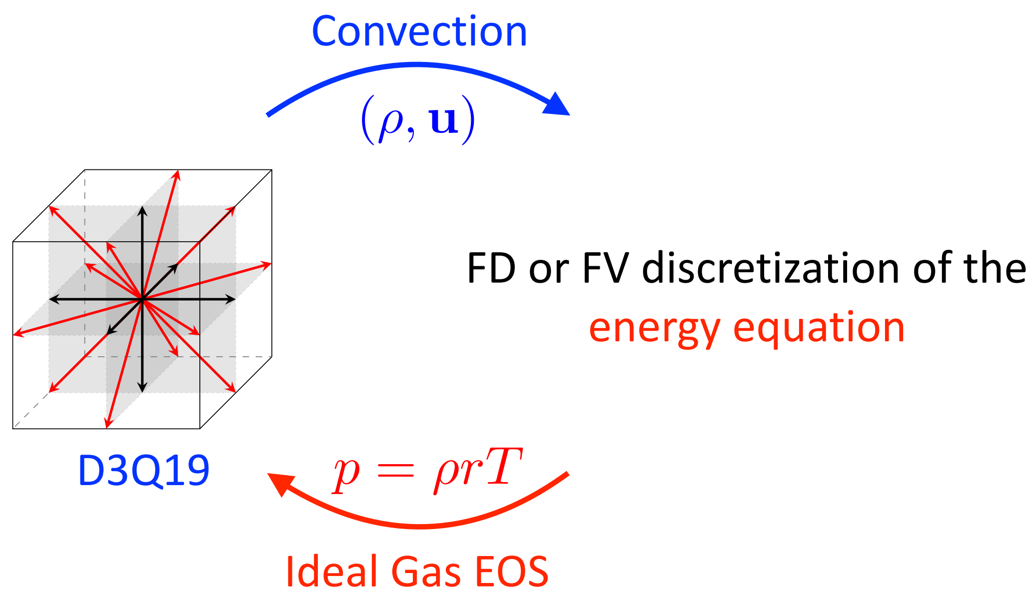

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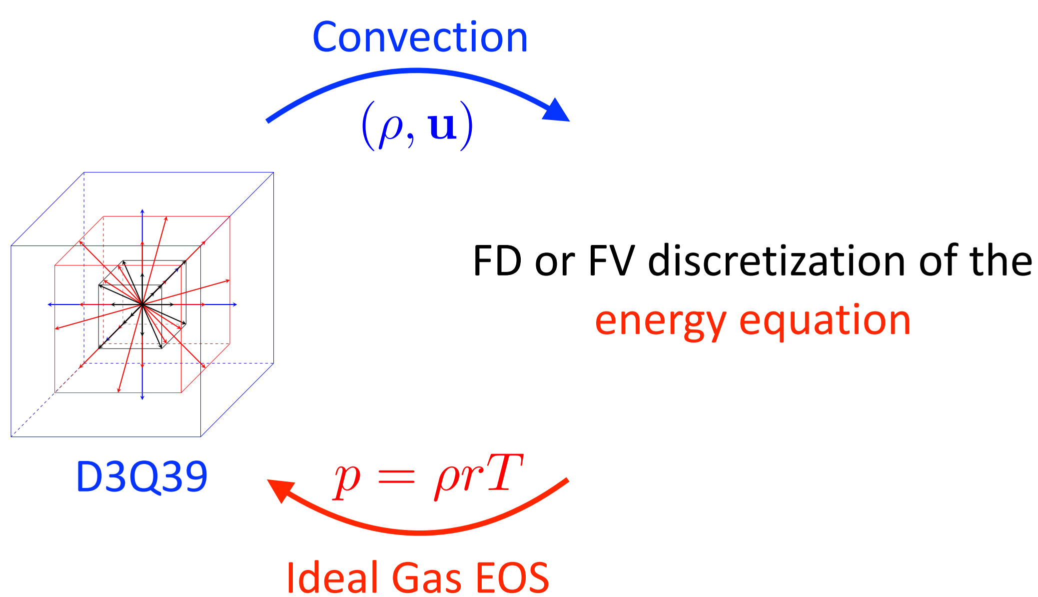

Supersonic LBM [2]

- D3Q39
- **Entropy** equation
- Limitation: **Mach < 2**

[1] Casalino et al., Towards lattice-boltzmann prediction of turbofan engine noise. AIAA Paper, **2014-3101.**
[2] Fares *et al.*, Validation of a Lattice-Boltzmann Approach for Transonic and Supersonic Flow Simulations, AIAA Paper, **2014-0952.**

But it seems that we can do better…

✤ **Supersonic regime via tailoring of corrections with the D2Q9 (HRR collision)**

Renard et al., Improved compressible Hybrid Lattice Boltzmann Method on standard lattice for subsonic and supersonic flows, *arXiv*, **2020.**

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$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

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↑

Compressibility error correction

$$E_{1,\alpha\beta} = \frac{\partial}{\partial x_\alpha} \underbrace{\left(\rho u_\alpha \left(1 - \theta - u_\alpha^2\right)\right)}_{=-a^{eq}_{\alpha\alpha\alpha}} \delta_{\alpha\beta}$$

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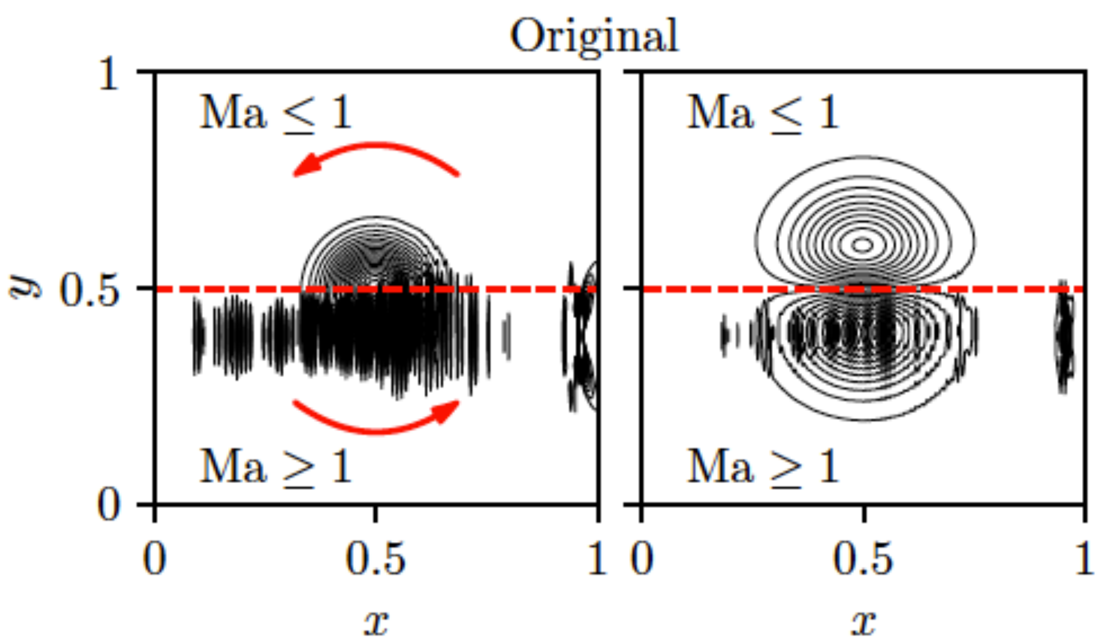

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2nd-order centered

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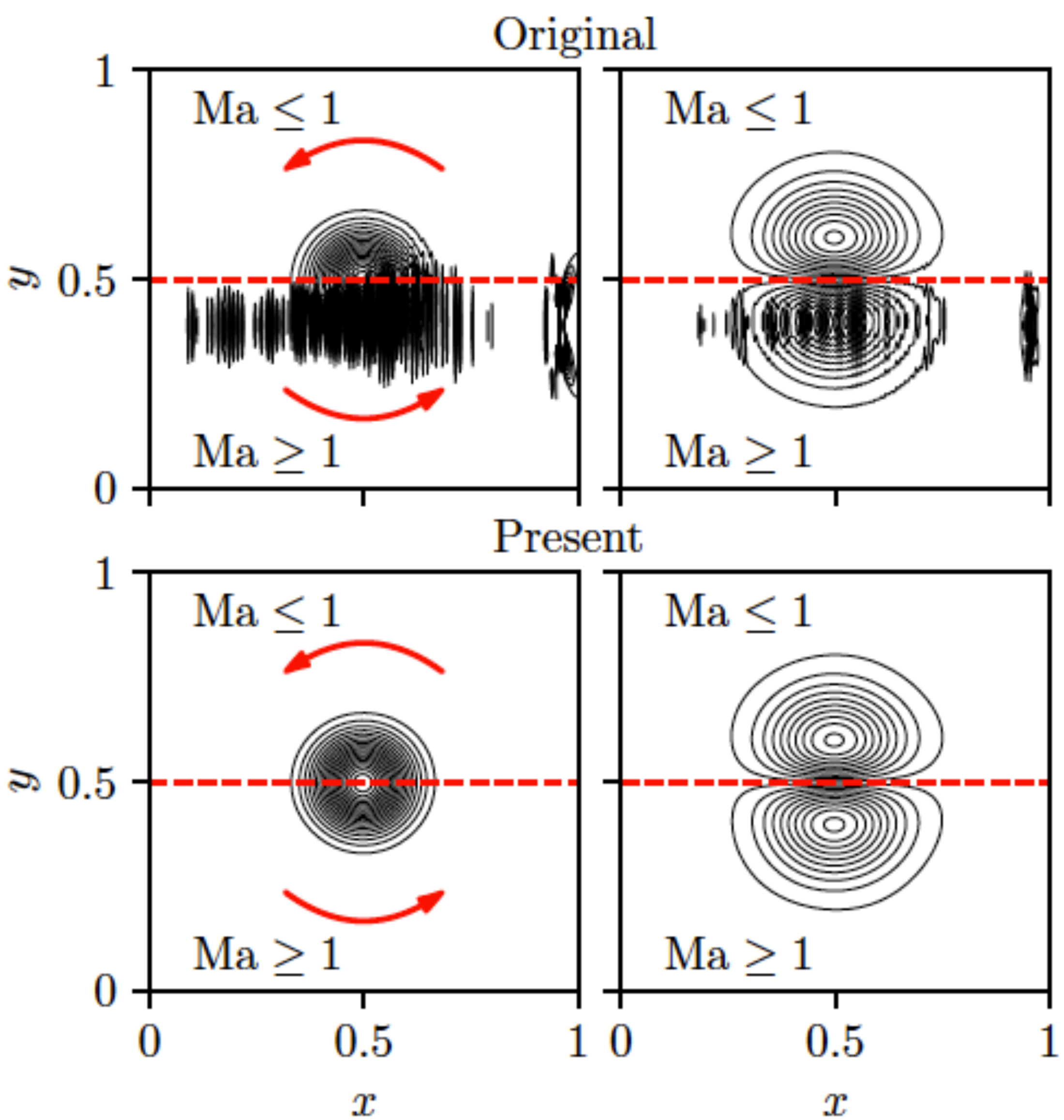

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1st-order upwind

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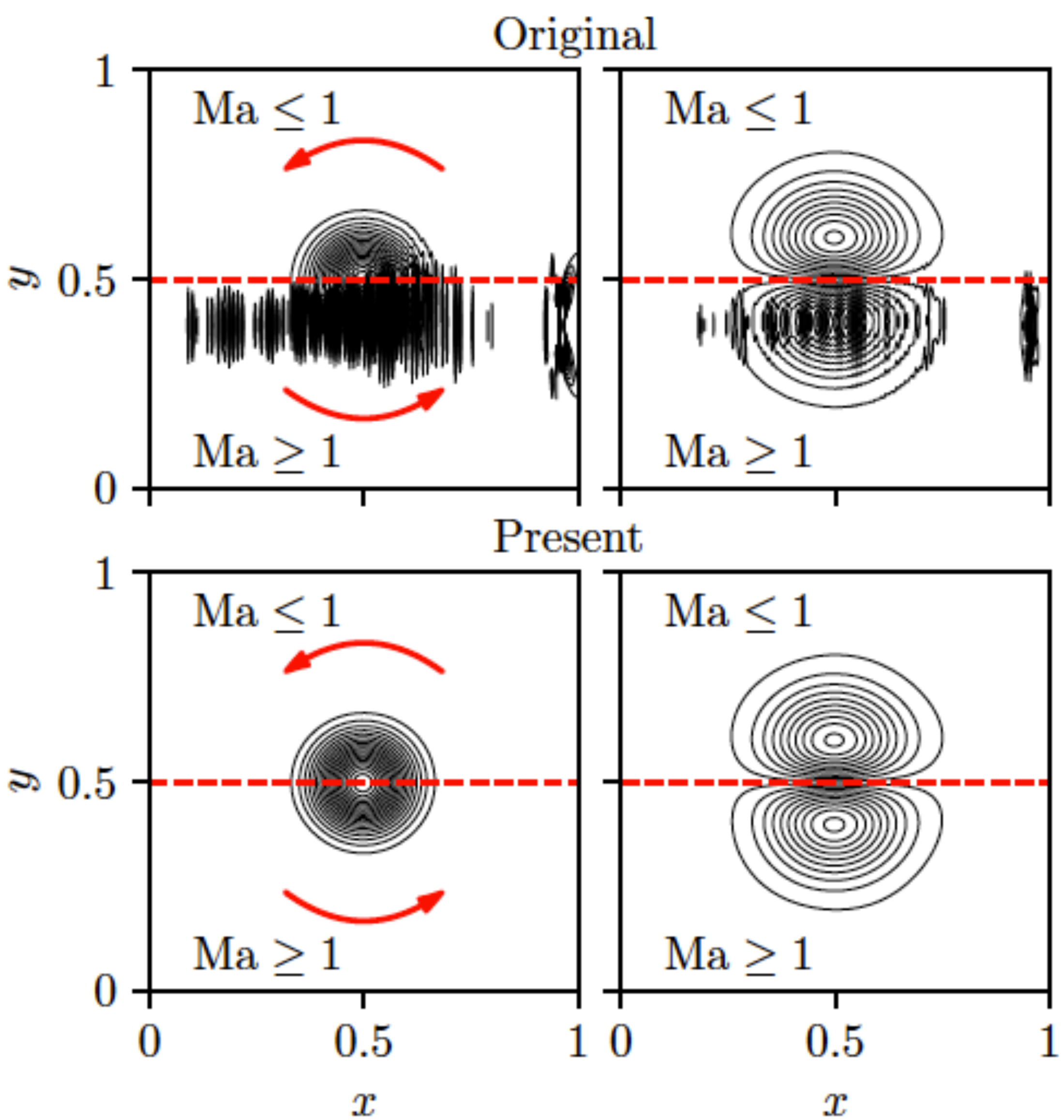

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1st-order upwind

It is worth noting that the **collision model** plays an important role in the **stabilization process**!

Renard et al., Improved compressible Hybrid Lattice Boltzmann Method on standard lattice for subsonic and supersonic flows, *arXiv*, **2020.**

But it seems that we can do better…

✤ **Supersonic regime via tailoring of corrections with the D2Q9 (HRR collision)**

✤ **This can be rigorously proven through linear stability analyses**

A linear stability analysis of compressible hybrid lattice Boltzmann methods

Florian Renard[a], Gauthier Wissocq[a], Jean-François Boussuge[a], Pierre Sagaut[b],

[a]CERFACS, 42 Avenue G. Coriolis, 31057 Toulouse cedex, France
[b]Aix Marseille Univ, CNRS, Centrale Marseille, M2P2, 13451 Marseille, France.

Implicit coupling
\+
Thermal LSA
\+
HRR

But it seems that we can do better…

✤ **Supersonic regime via tailoring of corrections with the D2Q9 (HRR collision)**

✤ **This can be rigorously proven through linear stability analyses**

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Implicit coupling
+
Thermal LSA
+
HRR

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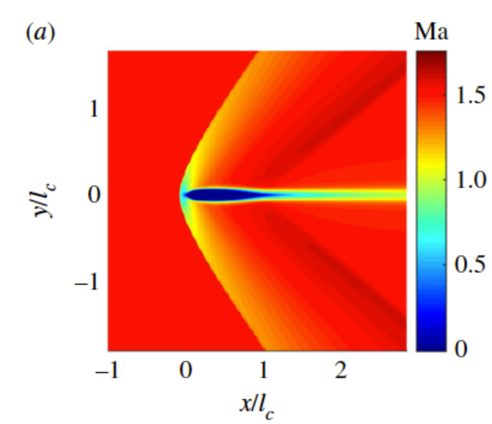
Research

Check for updates

Compressibility in lattice Boltzmann on standard stencils: effects of deviation from reference temperature

S. A. Hosseini[1,2,3], N. Darabiha[2] and D. Thévenin[1]



Explicit coupling
+
Isothermal LSA
(reference temperature as a free parameter)
+
T-normalized CHM-**REG**
(equivalent to RR)

Other recent hybrid LBMs for supersonic flows

D3Q19 formulation + Mass conservation improvement

An efficient lattice Boltzmann method for compressible aerodynamics on D3Q19 lattice

S. Guo [a], Y. Feng [a,*], J. Jacob [a], F. Renard [b], P. Sagaut [a]

[a] *Aix Marseille Univ, CNRS, Centrale Marseille, M2P2 UMR 7340, 13451 Marseille, France*
[b] *CERFACS, Toulouse, France*

Unsteady boundary conditions

Solid wall and open boundary conditions in hybrid recursive regularized lattice Boltzmann method for compressible flows

Cite as: Phys. Fluids 31, 126103 (2019); doi: 10.1063/1.5129138
Submitted: 25 September 2019 • Accepted: 20 November 2019 •
Published Online: 12 December 2019

Y. Feng (封永亮), S. Guo (郭少龙),[a)] J. Jacob, and P. Sagaut

Grid refinement

Grid refinement in the three-dimensional hybrid recursive regularized lattice Boltzmann method for compressible aerodynamics

Y. Feng, S. Guo,[*] J. Jacob, and P. Sagaut
Aix Marseille Univ, CNRS, Centrale Marseille, M2P2, Marseille, France

(Received 23 October 2019; accepted 5 May 2020; published 4 June 2020)

Alternative formulation to prevent mode couplings [1,2]

A pressure-based regularized lattice-Boltzmann method for the simulation of compressible flows

Cite as: Phys. Fluids 32, 066106 (2020); doi: 10.1063/5.0011839
Submitted: 25 April 2020 • Accepted: 5 June 2020 •
Published Online: 23 June 2020

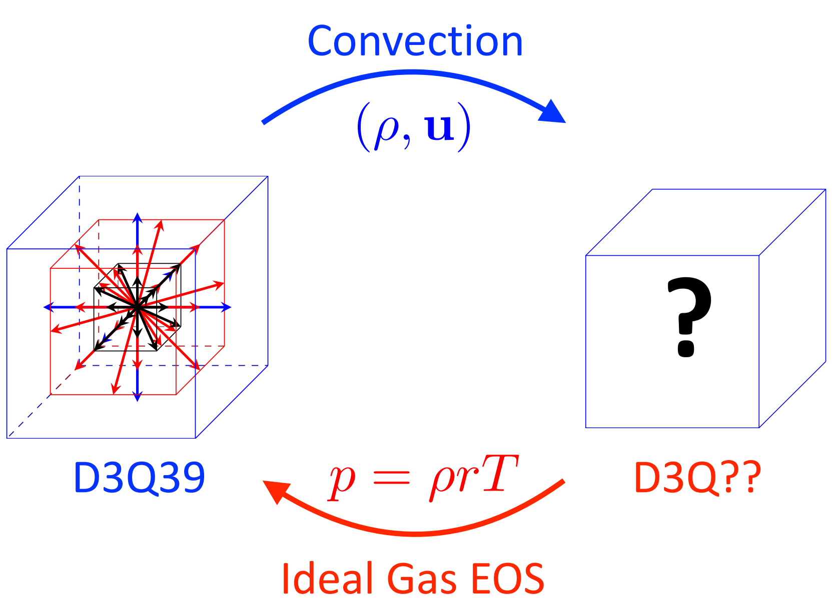
G. Farag, S. Zhao, T. Coratger, P. Boivin,[a)] G. Chiavassa, and P. Sagaut

[1] **Renard et al.**, A linear stability analysis of compressible hybrid lattice Boltzmann methods, *arXiv*, **2020.**
[2] **Zhao et al.**, A pressure-based hybrid regularized Lattice-Boltzmann method for numerical combustion, **2020.**
(see https://pierre-boivin.cnrs.fr/wp-content/uploads/2020/05/article.pdf)

PowerFLOW is coming back to DDF-LBMs due to conservation issues of the entropy formulation…

Lattice-Boltzmann Very Large Eddy Simulations of Fluidic Thrust Vectoring in a Converging/Diverging Nozzle

Avinash Jammalamadaka [*], Gregory Laskowski [†], Yanbing Li[‡]
James Kopriva [§] Pradeep Gopalakrishnan [¶] Raoyang Zhang [‖], and Hudong Chen [**]
Dassault Systemes SIMULIA Corp, Waltham, MA, 02451, U.S.A.



Regularized collision models

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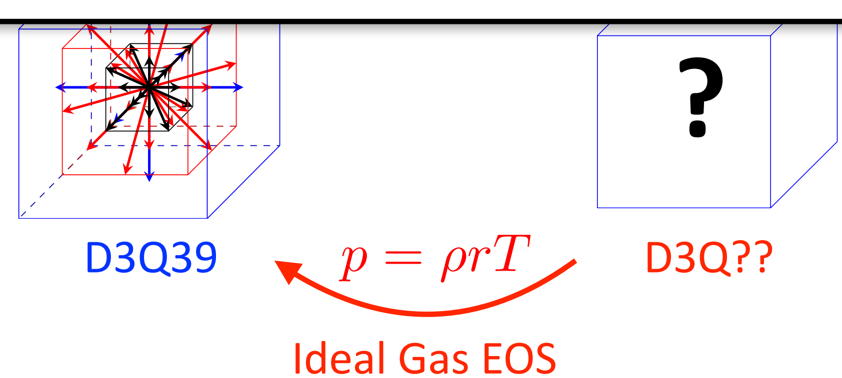
What does that imply for the development of future hybrid LBMs?



Regularized collision models

Further reading

✤ **Compressible flows are not all about high-speed conditions and shockwaves…**

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Go check papers about combustion simulations!

Hybrid LBMs

Low-Mach hybrid lattice Boltzmann-finite difference solver for combustion in complex flows

Cite as: Phys. Fluids 32, 077105 (2020); https://doi.org/10.1063/5.0015034
Submitted: 25 May 2020 . Accepted: 25 June 2020 . Published Online: 20 July 2020

S. A. Hosseini, A. Abdelsamie, N. Darabiha, and D. Thévenin

A pressure-based hybrid regularized Lattice-Boltzmann method for numerical combustion

S. Zhao[a,b], G. Farag[a], M. Tayyab[a], P. Boivin[1a]

[a] *Aix Marseille Univ, CNRS, Centrale Marseille, M2P2, Marseille, France*
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[b] *CNES Launchers Directorate, Paris, France*

Discrete Boltzmann methods (DBMs)

Discrete Boltzmann modeling of unsteady reactive flows with nonequilibrium effects

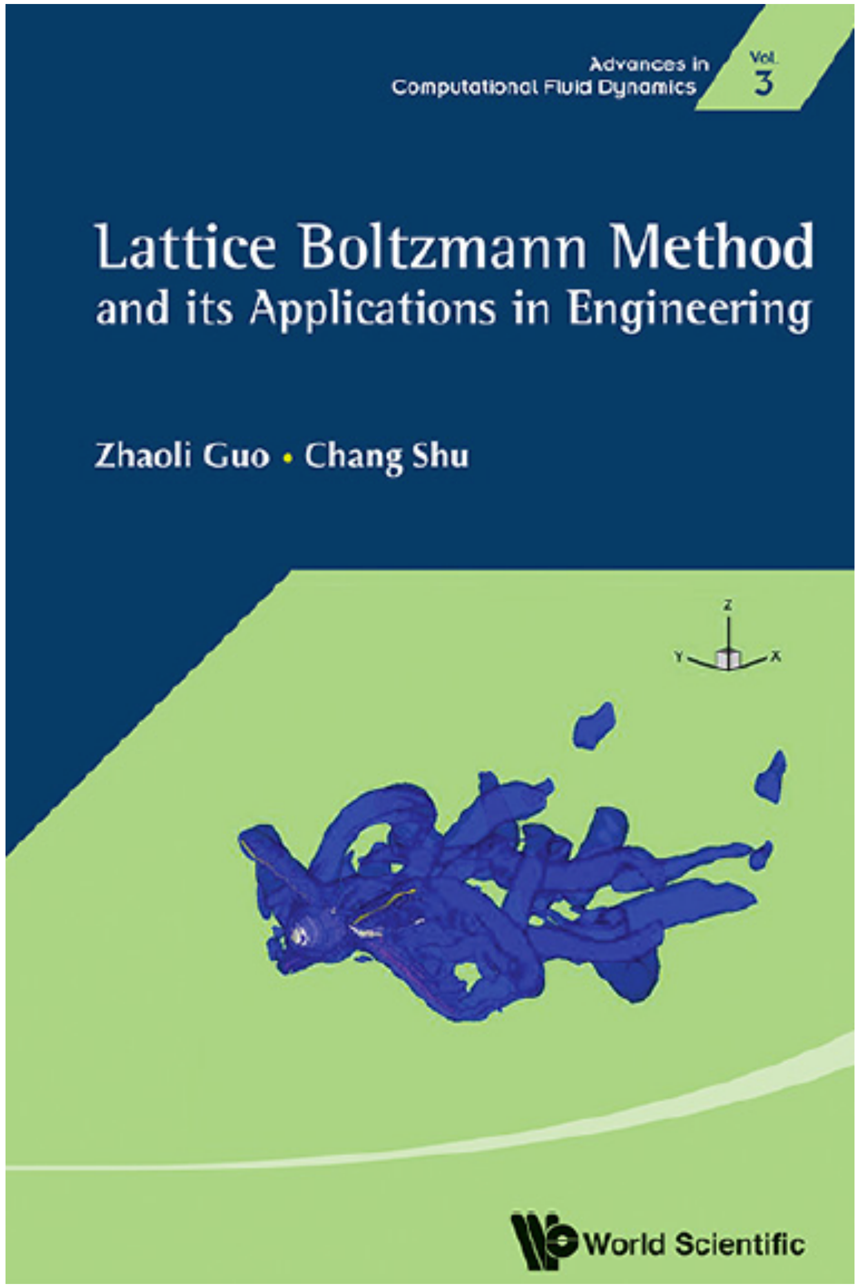
Chuandong Lin[1,*] and Kai H. Luo[1,2,†]
[1] *Center for Combustion Energy; Key Laboratory for Thermal Science and Power Engineering of Ministry of Education, Department of Energy and Power Engineering, Tsinghua University, Beijing 100084, China*
[2] *Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, United Kingdom*

(Received 12 June 2018; published 29 January 2019)

Kinetic Simulation of Unsteady Detonation with Thermodynamic Nonequilibrium Effects

C. Lin[a] and K. H. Luo[b]

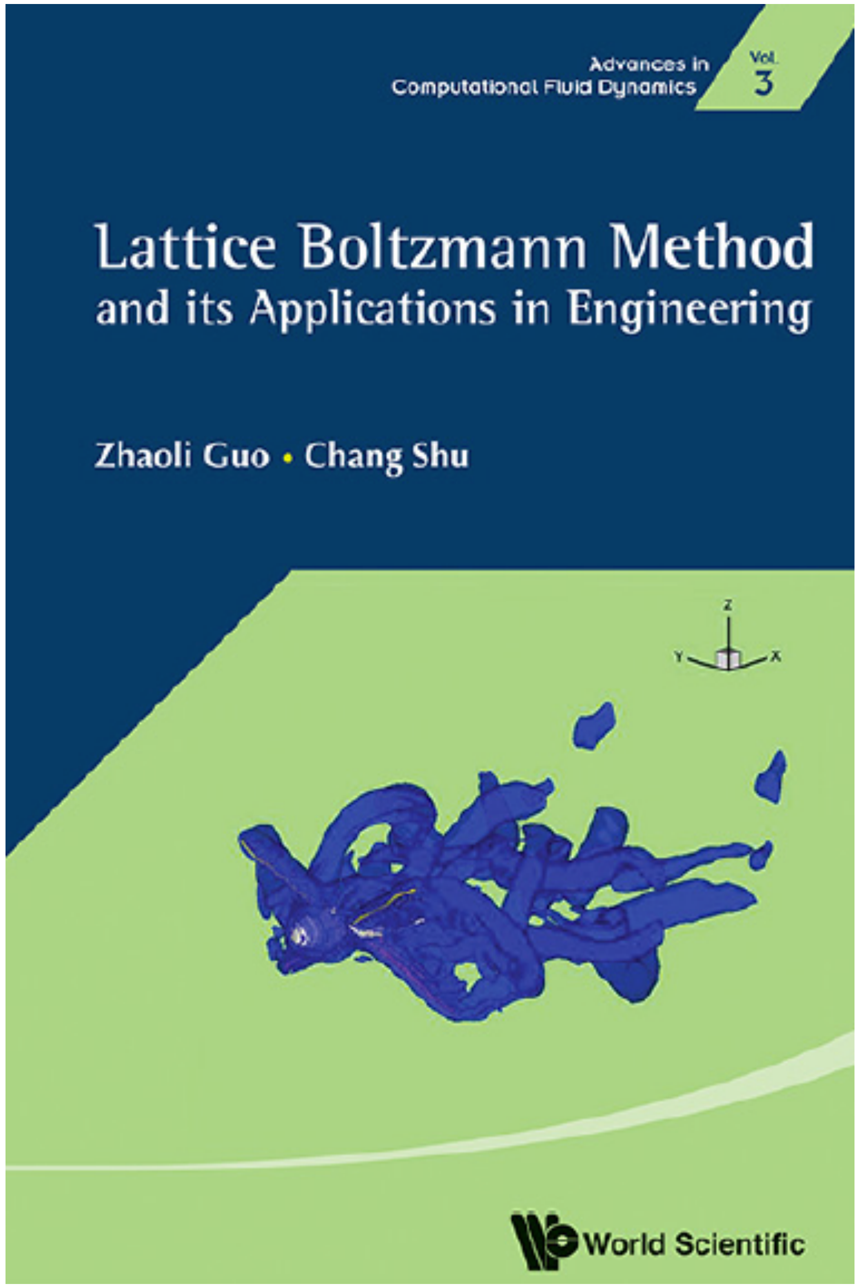
Further reading



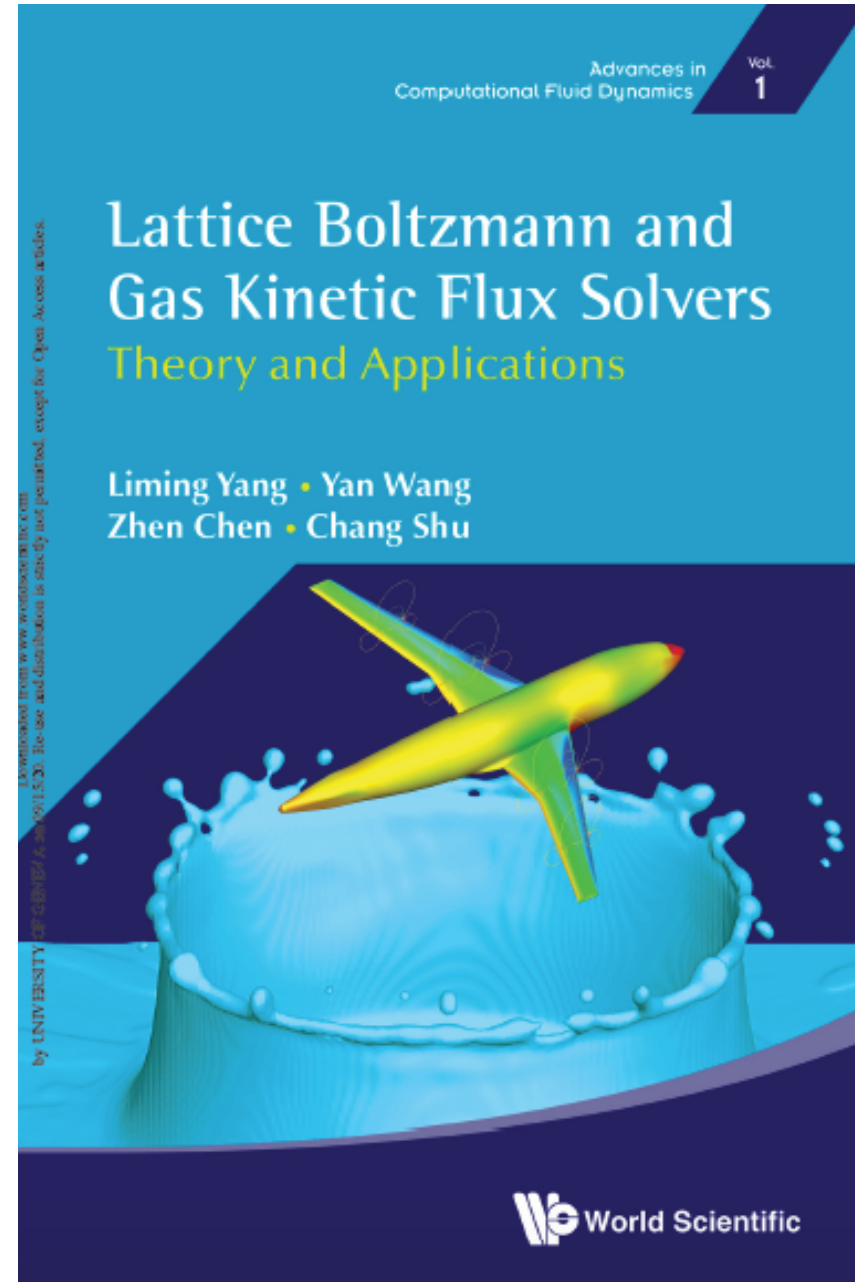
Guo & Shu, *Lattice Boltzmann Method and Its Applications in Engineering*, World Scientific, **2013.**

- Other types of equilibria (circular, spherical, etc)
- Other numerical discretizations (TVD, IMEX, etc)

Further reading



Guo & Shu, *Lattice Boltzmann Method and Its Applications in Engineering*, World Scientific, **2013.**



Yang et al., *Lattice Boltzmann and Gas Kinetic Flux Solvers,* World Scientific, **2020.**

- Other types of equilibria (circular, spherical, etc)
- Other numerical discretizations (TVD, IMEX, etc)
- Lattice Boltzmann / gas kinetic flux solvers
- Go and check papers about DUGKS and DBM (not shown here)

Thank you for your attention!
Questions?

UNIVERSITY OF GENEVA
FACULTY OF SCIENCE