On Voting Process and Quantum Mechanics

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06 february 2009 *

Abstract
In this communication, we propose a tentative to set the fundamental problem of measuring process done by a large structure on a microscopic one. We consider the example of voting when an entire society tries to measure globally opinions of all social actors in order to elect a delegate. We present a quantum model to interpret an operational voting system and propose an quantum approach for grading step of Range Voting, developed by M. Balinski and R. Laraki in 2007.

Key words: Fractaquantum hypothesis, Range Voting, Information Retrieval, Gleason theorem.

* Presented to *Quantum Interaction QI2009*, Saarbrücken, Germany, 25-27 March 2009. Published in *Lecture Notes in Computer Science*, number 5494 (P. Bruza *et al* Editors), p. 200-210, Springer, 2009.

1 Measure process between different scales

• Matter is constituted by discrete quanta and this fact was empirically put in evidence by E. Rutherford in the beginning of 20th century. Microscopic quanta as classical atoms or photons are not directly perceptible by our senses, as pointed out by M. Mugur-Schächter [MMS08]. In consequence, any possible knowledge for a human observer of a microscopic quantum is founded on experimental protocols. The mathematical framework constructed during the 20th century describes unitary "free evolution" through the Schrödinger equation and "reduction of the wave packet" associated to measure process through a projection operator in Hilbert space. We refer the reader *e.g.* to the book of C. Cohen-Tannoudji *et al* [CDL77]. The philosophical consequences of this new vision of Nature are still under construction; in some sense, an *a priori* or an external description of Nature is not possible at quantum scale. We refer to B. D'Espagnat [DE02] and M. Bitbol [Bi96]. Independently of the development of this renewed physics, the importance of scale invariance have been recognized by various authors as B. Mandelbrot [Ma82] and L. Nottale [No98]. The word "fractal" is devoted to figures and properties that are self-similar whatever the refering scale.

• We have suggested in 2002 the fractaquantum hypothesis [Du02], founded on two remarks: Nature develops a scale invariance and quantum mechanics is completely relevant for small scales. In order to express this hypothesis, we have introduced (see *e.g.* [Du05, Du08a]) the notion of "atom", in fact very similar to the way of vision of Democrite and the ancient Greek philosophers (see *e.g.* J. Salem [Sa97]). To fix the ideas, an "atom" can be a classical atom, or its nucleus, or a molecule, or a micro-organism like a cell, or an entire macro-organism as a human being or till an entire society! If we divide an "atom" into two parts, its qualitative properties change strongly at least in one of these parts. With this framework, elementary components are supposed to exist in Nature at different scales. A classical atom is a "micro state" relative to a Human observer. In this particular case, a ℓittle "atom" ℓ is a classical atom and a Big "atom" B is a human observer. More generally, two "atoms" ℓ and B have different scales when "atom" ℓ is not directly perceptible to "atom" B. In other words, a direct interaction between B and ℓ can not be controlled by B himself. In this case, the direct interaction between little "atom" ℓ and big "atom" B can be neglected as a first order approximation.

• In this contribution, we suggest to revisit this classical quantum formalism when little and big "atoms" are nonclassical ones. In fact, this research program is tremendous! For similar programs, we refer *e.g.* to the works of G. Vitiello [Vi01], P. Bruza *et al* [BKNE08], A. Khrennikov and E. Haven [KH07], P. La Mura *et al* [LMS07]. The phenomenology of possible measurement interactions should be reconstructed. What is a big "atom" B that can measure some quantities on little "atom" ℓ? Does the classical framework of quantum mechanics operates without any modification? Of course all these questions motivate our communication. Due to the lack of knowledge of what can be a measure done by "atoms" at mesoscopic or microscopic scales, we restrict ourselves in this contribution to measures done by human society considered as a whole on individual human beings.

• We consider here a particular example of the measurement process associated with voting. In this case, "atom" ℓ is a social actor and "atom" B is the entire society. We first introduce the scientific problem of voting process and in the following section, we present a preliminary quantum model for voting. In the two following sections we describe with the help of fractaquantum hypothesis the range voting procedure ("vote par valeurs") developed independently by M. Balinski and R. Laraki [BL07a] at Ecole Polytechnique (Paris) and by W.D. Smith [Sm07, RS07] at the "Center of Range Voting" (Stony Brook, New York).

2 On the voting process

• We consider a macroscopic "atom" B composed by an entire social structure. For example, B is a state like France to fix the ideas. The social actors of society B are the little "atoms" ℓ in our model. We write here

$$\ell \in \mathrm{B} \tag{1}$$

even if the expression (1) does not take precisely into account the detailed structure of society B. The numbers of such indistinguable individuals are quite important (10^6 to 10^9 typically). The democratic life in society B suppose that social responsabilities are taken by elected representants of social corpus. Thus a voting process has the objective to determine one particular social actor among all for accepting social responsabilities. This kind of position is supposed to be attractive and a set Γ of candidates γ among the entire set of "atoms" ℓ is supposed to be given in our framework.

• The problem is to determine a single "elected" candidate γ_1 among the family Γ thanks to the synthesis of all opinions of different electors ℓ. The social objective of society B is the determination of one candidate among others through a social process managed by the entire society, modelized here as a macro "atom" B. This problem is highly ill posed and we refer to the pioneering works of J.C. de Borda [1781] and N. de Condorcet [1785] followed more recently by the theorem of non existence of a social welfare function satisfying reasonable hypotheses, proved by K. Arrow [Ar51]. We describe this result in the following of this section.

• With K. Arrow, we suppose that each elector ℓ determines some ordering denoted by $\succ_{\sigma_\ell}$ (or simply by σ_ℓ) among the candidates $\gamma \in \Gamma$:

$$\gamma_{\sigma_l(1)} \succ_{\sigma_\ell} \gamma_{\sigma_l(2)} \succ_{\sigma_\ell} \cdots \gamma_{\sigma_l(i)} \succ_{\sigma_\ell} \gamma_{\sigma_l(i+1)} \ \cdots \succ_{\sigma_\ell} \gamma_{\sigma_l(K)} \, , \quad \ell \in \mathrm{B}.$$

We consider now the set σ of **all** orderings σ_l for all the electors ℓ

$$\sigma = \{\sigma_\ell, \, \sigma_\ell \text{ ordering of candidates } \Gamma, \, \ell \in B\} .$$

A so-called social welfare function f determines a particular social ordering $\sigma^* = f(\sigma)$ as a global synthesis of all orderings σ_ℓ in order to construct a commun and socially coherent position. Some democratic properties are *a priori* required for this function f:

(i) ***Unanimity***

If everybody thinks that candidate γ is better than γ' the social choice must satisfy this property:

$$\text{If } (\forall \ell \in B, \; \gamma \succ_{\sigma_\ell} \gamma') \quad \text{for some } \gamma, \gamma' \in \Gamma, \quad \text{then } (\gamma \succ_{\sigma^*} \gamma') . \tag{2}$$

(ii) ***Independance of irrelevant alternatives***

Consider two orderings σ and τ grading in a similar way the two candidates γ and γ':

$$((\gamma \succ_{\sigma_\ell} \gamma') \text{ and } (\gamma \succ_{\tau_\ell} \gamma')) \text{ or } ((\gamma \prec_{\sigma_\ell} \gamma') \text{ and } (\gamma \prec_{\tau_\ell} \gamma')) , \quad \forall \ell \in B . \tag{3}$$

Then the social orderings $\sigma^* = f(\sigma)$ and $\tau^* = f(\sigma)$ must satisfy the corresponding property:

$$\gamma \succ_{\sigma^*} \gamma \text{ when } ((\gamma \succ_{\sigma_\ell} \gamma') \text{ and } (\gamma \succ_{\tau_\ell} \gamma')) \text{ or } \gamma \prec_{\sigma^*} \gamma \text{ when } ((\gamma \prec_{\sigma_\ell} \gamma') \text{ and } (\gamma \prec_{\tau_\ell} \gamma')) . \tag{4}$$

The social welfare function depends only on the relative ranking and not on the intermediate candidates.

• Then the Arrow impossibility theorem (proven elegantly by J. Geanakoplos in [Ge01]) implies that under conditions (2) of unanimity and (3)-(4) of independance of irrelevant alternatives, the social welfare function is simply a constant:

(iii) ***Dictatorship***

$$\exists \, d \in \Gamma , \quad f(\{\sigma_\ell, \, \ell \in B\}) \equiv \sigma_d \tag{5}$$

and the result is a dictature! In other terms, it is impossible to construct a social welfare function that has the two first properties of unanimity and independance of irrelevant alternatives and the non-dictatorship property, obtained by negation of (5).

3 A preliminary quantum model for voting

• We describe in this Section a quantum model presented in [Du08b]. We restrict here to the so-called "first tour" process as implemented in a lot of situations. In this process, each elector ℓ has to transmit the name of at most **one** candidate γ. Then an ordered list of candidates is obtained by counting the number of expressed votes for each candidate.

Introduce the space H_Γ of candidates generated formally by the finite family Γ of all candidates:

$$H_\Gamma = \bigoplus_{\gamma \in \Gamma} \mathbb{C} \, | \, \gamma > \tag{6}$$

where $\mathbb{C}$ denotes the field of complex numbers. This decomposition (6) is supposed to be orthogonal:

$$< \gamma \, | \, \gamma' > = \begin{cases} 0 & \text{if} \quad \gamma \neq \gamma' \\ 1 & \text{if} \quad \gamma = \gamma', \end{cases} \quad , \qquad \gamma, \gamma' \in \Gamma.$$

The "wave function" associated with an elector ℓ is represented by a state denoted by $| \, \ell >$ in this space H_Γ:

$$| \, \ell > = \sum_{\gamma \in \Gamma} | \, \gamma > < \ell \, | \, \gamma > \quad . \tag{7}$$

The scalar product $< \ell \, | \, \gamma >$ in relation (7) is the component of elector ℓ relative to each candidate γ. This number represents the political sympathy of elector ℓ relative to the candidate γ. We suppose here that the norm $\| \, \ell \, \|$ of state $| \, \ell >$ *id est*

$$\| \, \ell \, \| \equiv \sqrt{\sum_{\gamma \in \Gamma} | < \ell \, | \, \gamma > |^2}$$

is **inferior or equal** to unity. We follow the Born rule and suggest that the probability for elector ℓ to give its vote to candidate γ is equal to $| < \ell \, | \, \gamma > |^2$. We suggest also that the probability to unswer by a vote "blank or null" is $1 - \| \, \ell \, \|^2$ in this framework.

- The interpretation of the projection process in the quantum measurement for such a first tour of election process is quite clear. During the election, *id est* the particular day where the measure process occurs, the elector ℓ is **obliged** to choose at most one candidate γ_0. In consequence, all his political sensibility is socially "reduced" to this particular candidate. We can write:

$$| \, \ell > = \, | \gamma_0 >$$

to express the wave function collapse. This quantum interpretation of such voting process clearly shows the **violence** of such king of decision making. Of course, no elector has political opinions that are identical to one precise candidate and this measurement process is a true mathematical projection. Nevertheless, the operational social voting process imposes this projection in order to construct a social choice. The disadvantage and dangers of such process have been clearly demonstrated in France during the presidential election process in 2002 (see *e.g.* [wiki]).

4 Range Voting (i): quantum approach for grading step

• The voting process suggested by M. Balinski and R. Laraki [BL07a] is more complex than the one studied in the previous section. The key point in order to overcome the Arrow impossibility theorem is the fact that in this framework the opinion of electors among the candidates are **codified** by society B through a given set of so-called "grades". These grades are *a priori* very similar to the ones given by the scolar system, as integers between 0 and 20 in France with an associated order

$$0 \prec 1 \prec \ldots \prec j \prec j+1 \prec \ldots \prec 19 \prec 20 \,,$$

letters from A to F in the United States with an order

$$\mathrm{A} \succ \mathrm{B} \succ \mathrm{C} \succ \mathrm{D} \succ \mathrm{E} \succ \mathrm{F} \,,$$

or numbers from 1 to 6 in Germany with the following (mathematically unusual!) order

$$1 \succ 2 \succ 3 \succ 4 \succ 5 \succ 6 \,.$$

These grades can be also an ordered list of given words

"very good" $\succ$ "good" $\succ$ "not so bad" $\succ$ "passable" $\succ$ "insufficient" $\succ$ "to be rejected"

as proposed by the previous authors [BL07b] in Orsay experiment for French presidential election in 2007. These grades define an elementary **common language** that is supposed to be endowed by all social actors ℓ of society B. In other terms, a common ordered set G of grades ν is supposed to be given:

$$\nu_1 \succ \nu_2 \succ \ldots \succ \nu_K \,, \quad \nu_j \in G \,. \tag{8}$$

As a consequence, an ordering of opinions explicitly refer to this particular set of given grades and to an explicit ordering between these grades like in (8). Remind that in Balinski-Laraki process [BL07a], the society B imposes a commun grading referential to all electors.

• The ranking process between the candidates proceeds by two steps. First each elector gives a grade to each candidate. Secondly the candidates are arranged in order through "majority ranking". Each elector ℓ has to express an opinion relative to each candidate $\gamma \in \Gamma$ through a grade $g(\gamma, \ell) \in G$. During the day of the election as in [BL07b], each elector grades each candidate. We propose in this section a quantum model for the first step of this processus. This first step is a measure done by society B on each little "atom" ℓ which constitutes it, as suggested by relation (1). Observe now that each candidate γ has a published political program, is giving radio and television interviews, has a blog, *etc.* We introduce a "political Hilbert space" H_P that refer to **all** this set of political information, following modern approaches for Information Retrieval as suggested by K. von Rijsbergen [vR04]. The family G of grades is imposed by the general laws of society B. Nevertheless, the evaluation of the political program of all candidates is done by the elector ℓ himself in such a process! We suggest that each elector ℓ decomposes this Hilbert space H_P into "grading" orthogonal components E^ℓ_ν through his own **internal** process:

$$H_P = \bigoplus_{\nu \in G} E^\ell_\nu \,, \quad \ell \in \mathrm{B}\,. \tag{9}$$

The subspace E^ℓ_ν is the eigenspace giving the grade ν relative to the opinion of elector ℓ. If we denote by A^ℓ the quantum self-adjoint operator associated with the grading process done by elector ℓ, we have

$$A^\ell \bullet |\, \xi > = \nu \, |\, \xi > \,, \quad |\, \xi > \in E^\ell_\nu \subset H_P \,, \quad \nu \in G\,. \tag{10}$$

In other words, we introduce the orthogonal projector P^ℓ_ν onto the closed space E^ℓ_ν. Then these projectors commute

$$P^\ell_\nu \, P^\ell_{\nu'} = P^\ell_{\nu'} \, P^\ell_\nu \,, \qquad \nu \,, \nu' \in G \,, \;\; \ell \in \mathrm{B}$$

and generate a decomposition of the identity operator $\mathrm{Id}(H_P)$ in the political Hilbert space H_P:

$$\sum_{\nu \in G} P^\ell_\nu \equiv \mathrm{Id}(H_P) \,, \quad \ell \in \mathrm{B}\,. \tag{11}$$

On a very concrete point of view, in front of each political idea, each elector has the capability to give an opinion in the language suggested *a priori* by the set G of grades. The examples of such sets given above show also that the way of decomposition of political space H_P through the grades is strongly influenced by the social choice of the family G.

• In some sense, *via* a particular choice of grading, the society B imposes some filtering of space H_P of all political data. Note that the precise way this filter is done depends on each citizen ℓ. In this model, society B imposes the set G of eigenvalues and each elector ℓ fixes the eigenvectors as in (10). After the elector has interpreted the grades ν in his own vocabulary, *id est* once he has decomposed the space H_P into orthogonal components, we suppose that the grading process, *id est* the result of the measure is *a priori* obtained according to the Born rule. Precisely, we introduce the "perception" ρ^ℓ_γ of political opinion of candidate γ by the elector ℓ. Mathematically speaking, the elector ℓ measurates the political ideas of the candidate γ in a quantum way relatively to the Hilbert space H_P. According to Gleason theorem [Gl57], such a quantum probability is defined by a density matrix, *id est* a positive self-adjoint operator of unity-trace that we denotes also by ρ^ℓ_γ:

$$\rho^\ell_\gamma \text{ positive self-adjoint operator } H_P \longrightarrow H_P \,, \quad \mathrm{tr}\,\big(\rho^\ell_\gamma\big) = 1\,.$$

Then, following A. Gleason [Gl57] and K. von Rijsbergen [vR04], the measure μ^ℓ_γ associated with elector ℓ and candidate γ of any closed subspace $E \subset H_P$ is given in all generality according to

$$\mu^\ell_\gamma(E) \,=\, \mathrm{tr}\,\big(\rho^\ell_\gamma \, P_E\big)\,, \quad E \subset H_P\,,\;\; \ell \in \mathrm{B}\,, \tag{12}$$

where P_E is the orthogonal projector onto space E. Consider now the space $E = E^\ell_\nu$ introduced in (9). Then the (real!) number $\mu^\ell_{\gamma,\nu}$ defined by

$$\mu^\ell_{\gamma,\nu} \,=\, \mu^\ell_\gamma(E^\ell_\nu) \,=\, \mathrm{tr}\,\big(\rho^\ell_\gamma \, P^\ell_\nu\big) \tag{13}$$

represents the quantum probability for elector ℓ to give the grade ν to candidate γ. Of course, if we insert the identity operator $\mathrm{Id}(H_P)$ decomposed in (11) inside relation (12), we have due to (13)

$$\sum_{\nu \in G} \mu^\ell_{\gamma,\nu} \,=\, 1\,, \qquad \ell \in \mathrm{B},\; \gamma \in \Gamma, \tag{14}$$

and the sum of probabilities for all different grades is equal to unity.

• Remark that two different ingredients are necessary to determine the previous probability $\mu^\ell_{\gamma,\nu}$ in (13). First the decomposition (9) of the political space through the grades G. As usual in quantum mechanics, no detailed structure of "atom" ℓ is transmitted through the measure process. In this case, the orthogonal decomposition (9) is not known by the society. Second the "perception operator" ρ^ℓ_γ which represents in some sense the particular "political knowledge" that the elector ℓ has constructed for himself about the candidate γ. Remark that no direct interaction between the candidates occurs in the model. According to Condorcet's ideas [1795], each citizen is adult has make his own opinion through his own way of thinking!

5 Range Voting (ii): majority ranking

• After this first step of grading, the result of the vote of elector ℓ is a list

$$g(\gamma,\,\ell) \,\in G\,, \quad \gamma \in \Gamma\,, \quad \ell \in \mathrm{B}$$

of grades $\nu = g(\gamma,\,\ell)$ given by elector ℓ to **each** candidate γ. We give in this section the major points introduced By Balinski and Laraki [BL07a] without any modification. After summation, each candidate γ has a certain number $n^\gamma_\nu \in \mathbb{N}$ of opinions transmitted by the electors:

$$n^\gamma_\nu \,=\, \; \mathrm{Card}\;\{\, \ell \in B\,,\; g(\gamma,\,\ell) = \nu\,\} \,\in \mathbb{N}\,, \quad \gamma \in \Gamma\,, \quad \nu \in G. \tag{15}$$

The way of ranking such a list

$$n^\gamma \equiv \left(n^\gamma_{\nu_1}, n^\gamma_{\nu_2}, \dots n^\gamma_{\nu_K}\right) \in \mathbb{N}^K, \quad \gamma \in \Gamma \tag{16}$$

when the grades $\nu \in G$ are arranged in order without ambiguity by (8) can be explicited with the so-called "majority ranking" introduced by Balinski and Laraki [BL07a]. We give here some details of the algorithm, based on a successive extraction of a **median value** from a list as the one described in (16) and refer to [BL07a], [BL07b] and [PB06].

- From an algorithmic point of view, the list n^γ can also be written as a list m^γ of grades written in decreasing order to fix the ideas:

$$m^\gamma = \Big(\underbrace{\nu_1, \nu_1, \dots, \nu_1}_{n^\gamma_{\nu_1} \text{ times}}, \underbrace{\nu_2, \nu_2, \dots, \nu_2}_{n^\gamma_{\nu_2} \text{ times}}, \dots, \underbrace{\nu_K, \nu_K, \dots, \nu_K}_{n^\gamma_{\nu_K} \text{ times}} \Big) \in \mathbb{N}^{|B|} \tag{17}$$

where $|B| = \mathrm{Card}(B)$ is the number of electors. Then a list m^γ_1 can be constructed by omitting the grade $\nu^\gamma_{j_1}$ at the **median** position $\frac{|B|}{2}$ inside the list (17). We obtain in this way a new list extracted from (17)

$$m^\gamma_1 = \Big(\underbrace{\nu_1, \nu_1, \dots, \nu_1}_{n^\gamma_{1,\nu_1} \text{ times}}, \underbrace{\nu_2, \nu_2, \dots, \nu_2}_{n^\gamma_{1,\nu_2} \text{ times}}, \dots, \underbrace{\nu_K, \nu_K, \dots, \nu_K}_{n^\gamma_{1,\nu_K} \text{ times}} \Big) \in \mathbb{N}^{|B|-1} \tag{18}$$

and the integers n^γ_{1,ν_i} are equal to the $n^\gamma_{\nu_i}$ except for index j_1 for which we have

$$n^\gamma_{1,\nu^\gamma_{j_1}} = n^\gamma_{\nu^\gamma_{j_1}} - 1\,.$$

The grade $\nu^\gamma_{j_1}$ is the first "majority grade" of candidate γ in the majority ranking algorithm of Balinski and Laraki. If $\nu^\gamma_{j_1} \succ \nu^{\gamma'}_{j_1}$ then we have the relative final position $\gamma \succ \gamma'$ between the candidates γ and γ'. If $\nu^\gamma_{j_1} = \nu^{\gamma'}_{j_1}$ we apply the same step from (17) to (18) except that we start with the list (18). Doing this, we extract a second grade $\nu^\gamma_{j_2}$ for each candidate γ. If $\nu^\gamma_{j_2} \succ \nu^{\gamma'}_{j_2}$ or $\nu^\gamma_{j_2} \prec \nu^{\gamma'}_{j_2}$, the conclusion is established. Otherwise the process is carried on until the two majority grades at a certain step are distinct.

- It is a main contribution of M. Balinski and R. Laraki [BL07a] to extract an intrinsic order

$$\gamma_1 \succ \gamma_2 \succ \dots \gamma_j \succ \gamma_{j+1} \succ \dots, \quad \gamma_j \in \Gamma$$

among the candidates Γ from the given double list (16) of integers n^γ. The important social fact is that the overdetermination of a favorite candidate essentially does **not** influence the final majoritary ranking with this grading method! The proof of this important fact is omitted here and we refer to [BL07a]. We could also think that there is a contradiction between this positive result and the Arrow impossibility theorem. In fact, as pointed in [BL07a], the hypotheses of Arrow theorem are qualitative: each elector consider some ordering of the candidates with his own sensibility. As we have intensively explained with the orthogonal decomposition (9), the social choice of a **given** family of grades is essential for the grading step and the majority ranking.

6 Conclusion

• The very elaborated process initialized by M. Balinski and R. Laraki [BL07a] for range voting has been studied in this contribution. The second step of "majority ranking" has been described without adding any new idea to this beautiful article. Concerning the first step of the algorithm devoted to the grading of each candidate by each elector with a given list of grades, we have proposed a quantum algorithm essentially based on modern quantum approaches for Information Retrieval presented in K. von Rijsbergen's book [vR04]. First an orthogonal decomposition of the political Hilbert space supposes that each elector has the capability to have a precise opinion for each political subject. Second, following Gleason theorem [Gl57], we have introduced a "perception operator" that describes mathematically the way a given candidate is politically understood by a given elector. In some sense, a psychological model is incorporated with this description.

• With these two ingredients, the computation of the probability for an elector to give a particular grade to each candidate can be evaluated as a result of the model. Of course, it is not actually clear which precise practical advantages has this quantum approach in the description of the voting process. Moreover, we want to find in future works some previsions of the quantum model, and try to compare it with the previsions of a classic model.

• In this contribution, we have also presented a first quantum model of a classical election. In this framework, the big scale (the society) imposes a direct generalization of the measure process in quantum mechanics. All the characteristics of the mathematical measure operator are controlled by the large scale. We have noticed the violence of the multiscale interaction through such a the measuring process.

• Last but not least, this work is motivated by the fractaquatum hypothesis [Du02]. The case of a voting process is an example of measuring process between two different scales in Nature. If we suppose that the general concepts of quantum mechanics have an extension to all "atoms" in Nature, the process of measuring has to be re-visited to all pairs of "atoms" with different scales. This contribution is a small step in this direction!

Acknowlegments

The author thanks the referees who pointed clearly the importance of Information Retrieval framework for this work and proposed a list of very interesting remarks and an important number of which have been incorporated in the present writing!

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