Summary of the work
of
Jean-Marc Fontaine

Jean-Marc Fontaine is one of the founders of p-adic Hodge theory which is now one of the main tools of Arithmetic Geometry.

After proving a conjecture of Serre on Artin's representations, Fontaine solves Grothendieck's question about the "mysterious functor", i.e. explains how to construct the Tate module of a p-divisible group over the ring of integers of a p-adic field from its filtered Dieudonné module and vice-versa.

He constructs the field B_{dR} of p-adic periods which is, in some sense, the p-adic analogue of the field of complex numbers and introduces the hierarchy of p-adic representations of the absolute Galois group of a p-adic field. He states the main conjectures of the subject and various technics of approaching them: theory of norm fields (with Wintenberger), Fontaine-Laffaille theory which has been the key of studying deformations of crystalline representations, classification of p-adic representations via (ϕ, Γ)-modules, construction (with William Messing) of sheaves for the syntomic topology,...

With William Messing, he obtains one of the first comparison theorem between the different p-adic cohomologies. With Pierre Colmez, he obtains the first proof of one of the main conjectures of the subject giving a complete classification of potentially semi-stable Galois representations via their filtered (φ, N, G)-modules.

More recently, with Laurent Fargues, he constructs the fundamental curve of p-adic Hodge theory. They use it to get a new apporach of the arithmetic part of p-adic Hodge theory and new proofs of the main conjectures.

Fontaine is also interested by arithmetic applications of p-adic Hodge theory:

– he obtains the first proof of the fact that there is no non zero abelian variety over $\mathbb{Q}$ with good reduction everywhere,

– with Bernadette Perrin-Riou, he develops some useful formalism for the study of Bloch-Kato's conjectures on special values of L-functions of motives over number fields,

– with Barry Mazur, he introduces the notion of geometric ℓ-adic representations of the absolute Galois group of a number field. The Fontaine-Mazur's conjecture which said that these are precisely the Galois representations "coming from algebraic geometry" is at the heart of an impressive number of papers in Arithmetic Geometry.