le cnam

Master Structural Mechanics and Coupled Systems

Applied Mathematics

Student presentation 1 (course 06)

- Power method

Let $n \geq 1$ an integer and A a real symmetric positive definite matrix of order n. We consider its eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. We suppose that the greatest eigenvalue λ_1 is simple: $\lambda_1 > \lambda_j$ for all $j \geq 2$. We denote by r_1 a unitary eigenvector associated to this biggest eigenvalue λ_1. We consider the sequence of vectors $x_k \in \mathbb{R}^n$ such that $(x_0, r_1) > 0$ and x_{k+1} is obtained by the action of the matrix A: $x_{k+1} = A\, x_k$.

a) Prove that for each $k \in \mathbb{N}$, the vector x_k is not equal to zero.

We set $y_k = \frac{1}{\|x_k\|} x_k$

b) Prove that the family of vectors y_k converges towards the vector r_1.

c) Prove that the family of numbers $(y_k, A\, y_k)$ converges towards the eigenvalue λ_1.

François Dubois, september 2023.