Lattice Boltzmann Method, Turbulence and Micromixing

L. Djenidi

Discipline of Mechanical Engineering
School of Engineering
University of Newcastle
Australia

Contents:

- Introduction and Motivation
- The Lattice Boltzmann Method
- Application: Turbulence and Micromixing
- Conclusions

Introduction an Motivation

Lattice Boltzmann Method (LBM)

An alternative to classic CFD method (?)

Big progress made over the last decade

Becomes popular (?)

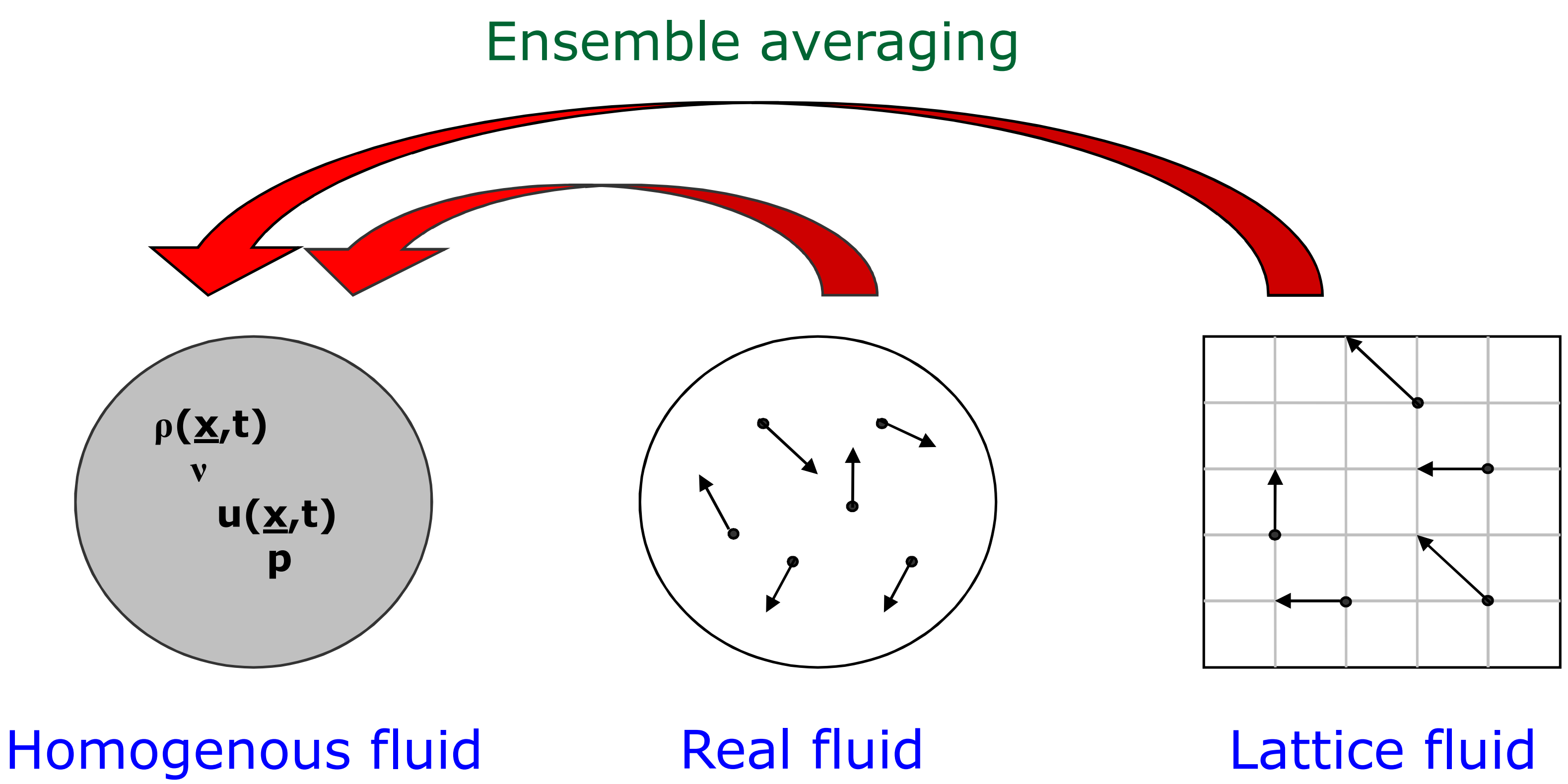
Advantages

Ease of boundary condition implementation

Well adapted for parallel computations

No Poisson equation for the pressure

LBM = Simplified kinetic model to simulate fluid flows



The lattice Boltzmann Method

Boltzmann equation

$$f = f(\vec{r}, \vec{e}, t) \equiv$$ Particle probability density distribution function (PPDF)

- $\vec{e}$: Particle velocity
- $\vec{r}$: Vector position

$$\frac{df}{dt} + \vec{e}.\nabla_r f + \vec{a}.\nabla_e f = (\partial f)_{coll}$$

- $\vec{a}.\nabla_e f$: External force
- $(\partial f)_{coll}$: collision

The lattice Boltzmann Method

Boltzmann's (BGK) equation

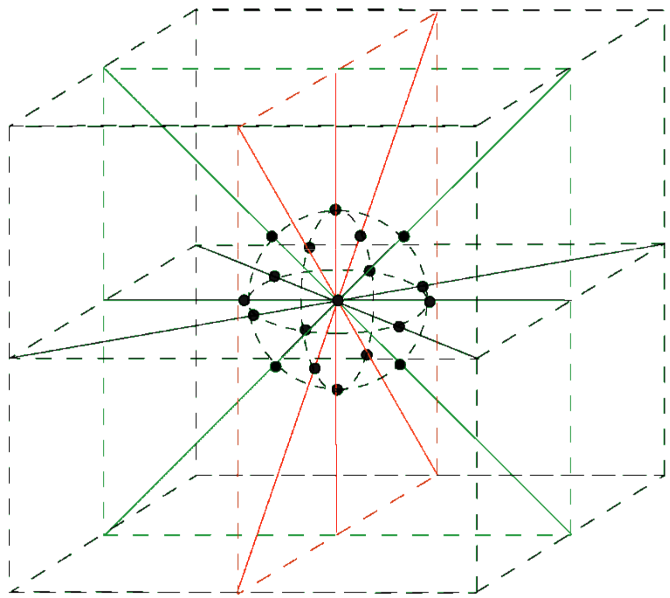
$$\frac{df}{dt} + \vec{e}.\nabla_r f + \vec{a}.\nabla_e f = -\frac{f - f^{eq}}{\tau}$$

If we assume: $\nabla_e f \approx \nabla_e f^{eq}$

$$\frac{df}{dt} + \vec{e}.\nabla_r f = -\frac{f - f^{eq}}{\tau} + \frac{\vec{a}.(\vec{e} - \vec{u})}{RT} f^{eq}$$

The lattice Boltzmann Method

Lattice Boltzmann equation for D3Q19 (no external force)



$$f_a(\vec{r}+\vec{e},t+\Delta t) = f_a(\vec{r},t) - \frac{f_a(\vec{r},t) - f_a^{eq}(\vec{r},t)}{\tau}$$

$$f_a^{eq} = \rho\omega_a(1+3\vec{e}_a.\vec{u}+\frac{9}{2}(\vec{e}_a.\vec{u})^2-\frac{3}{2}\vec{u}^2)$$

$$\rho = \sum_{a=0}^{18} f_a$$

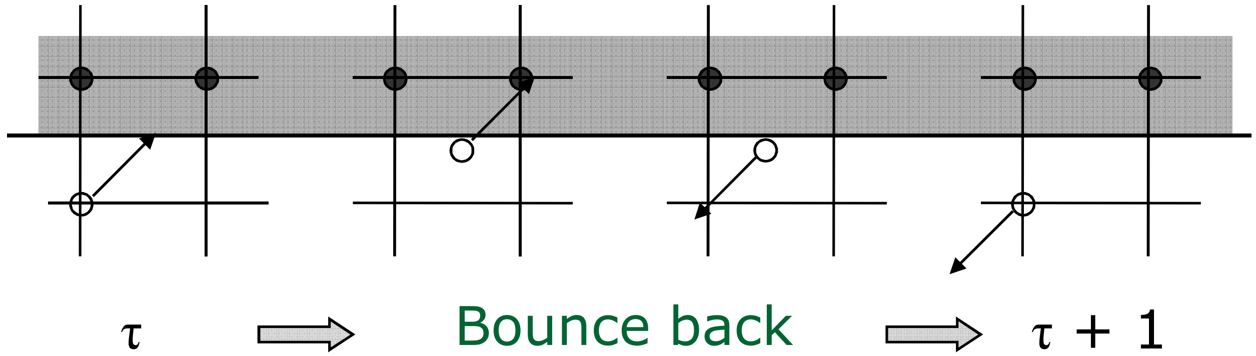
$$\rho\,\mathbf{u} = \sum_{a=0}^{18} f_a \boldsymbol{e_a}$$

$$\nu = (2\tau - 1)/6$$ (Viscosity)

Applications of LBM

Treatment of the boundary conditions

No slip condition at the wall



Outlet: Convective boundary condition or zero gradient

$$\frac{\partial \mathbf{u}}{\partial t} + U_{mean}\frac{\partial \mathbf{u}}{\partial x} = 0 \quad \text{or} \quad \frac{\partial \mathbf{u}}{\partial x} = 0$$

Inlet: Uniform velocity U_0

Lateral sides: Periodic conditions

Applications of LBM

Turbulent flows

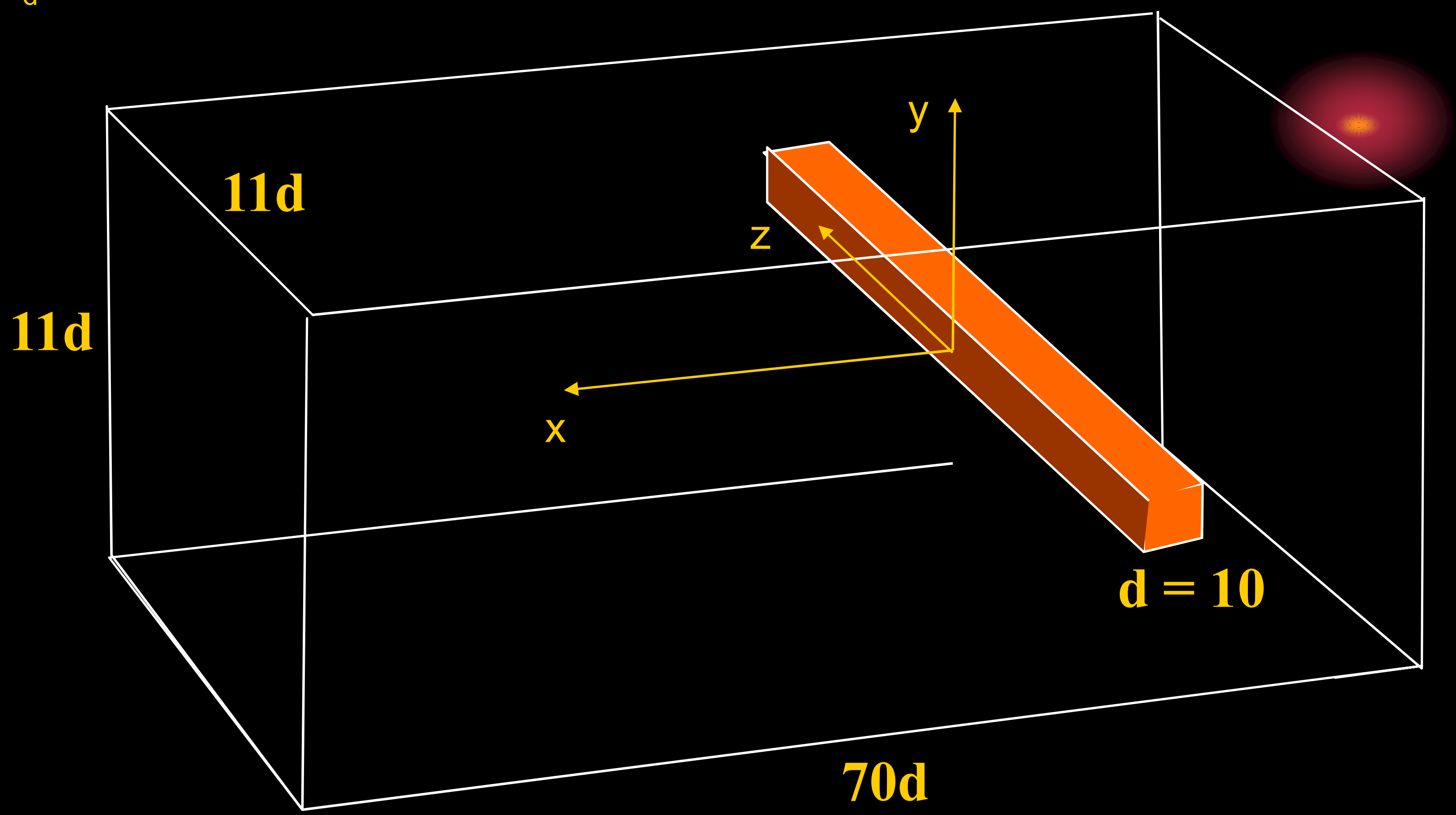
- 3D Transition in a cylinder wake
- Cross-bar wake
- Grid turbulence
- 3D box turbulence
- Urban Flows

Microfluidics

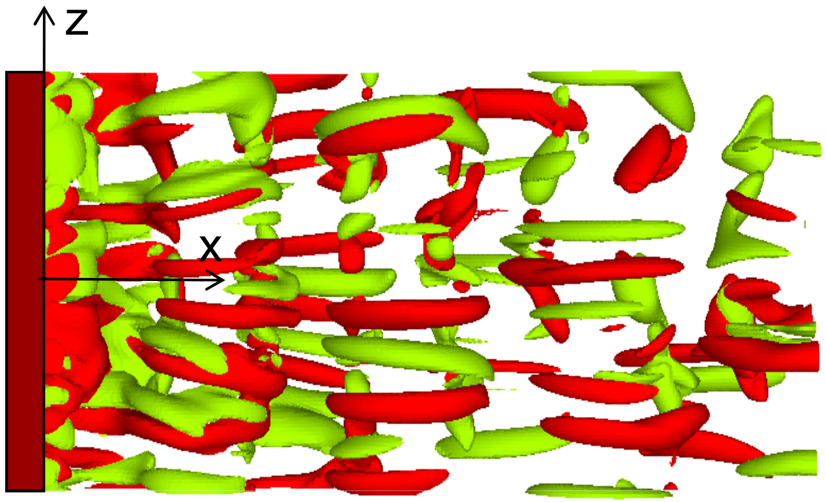
- Micromixer
  - Passive mixer
  - Active mixer

Lattice Boltzmann simulation of transitional cylinder wake

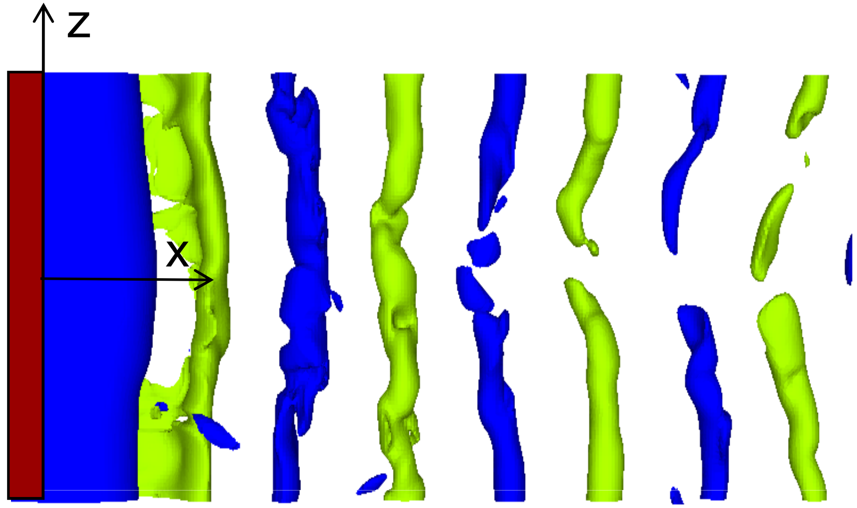
$R_d = 200$



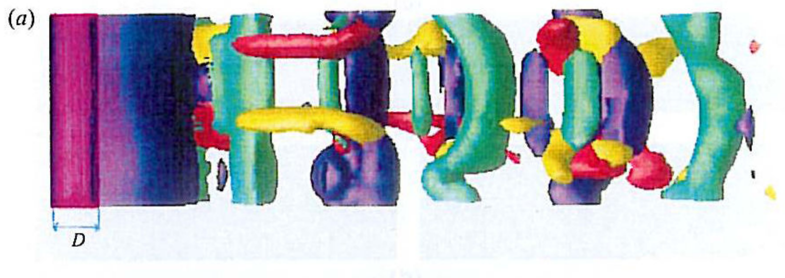
Results – 3D transitional wake (Continued)



$\omega_x / \omega_{x\,max} = +/- 0.1$



$\omega_z / \omega_{z\,max} = +/- 0.1$



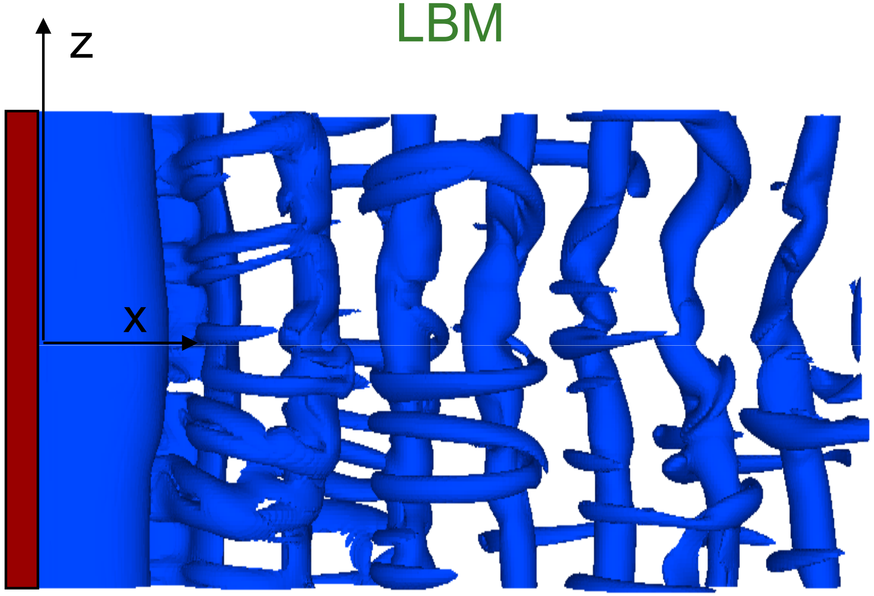
DNS, ω_x = +/- 0.25, ω_z = +/- 0.25

Persillon and Braza, JFM, 1998, **365**,23-88

Vorticity iso-contours

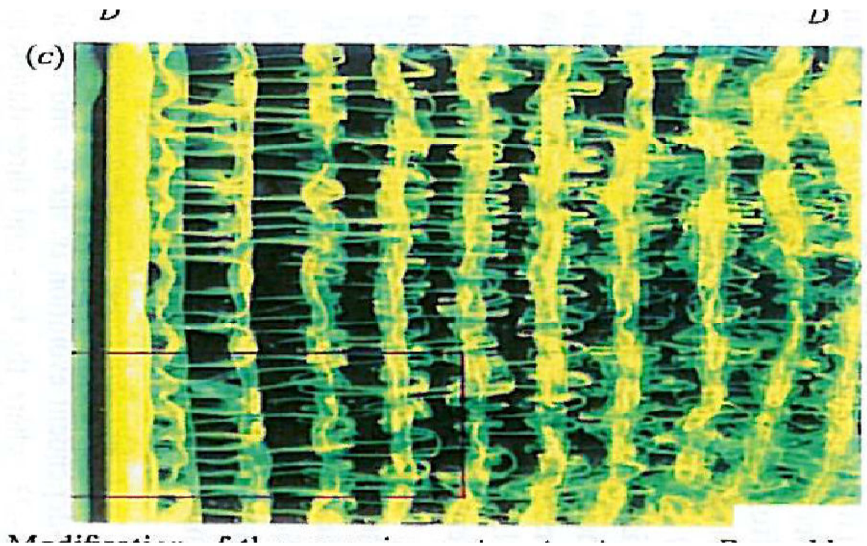
Results - 3D transitional wake (Continued)

LBM



$\omega/\omega_{max} = 0.07$

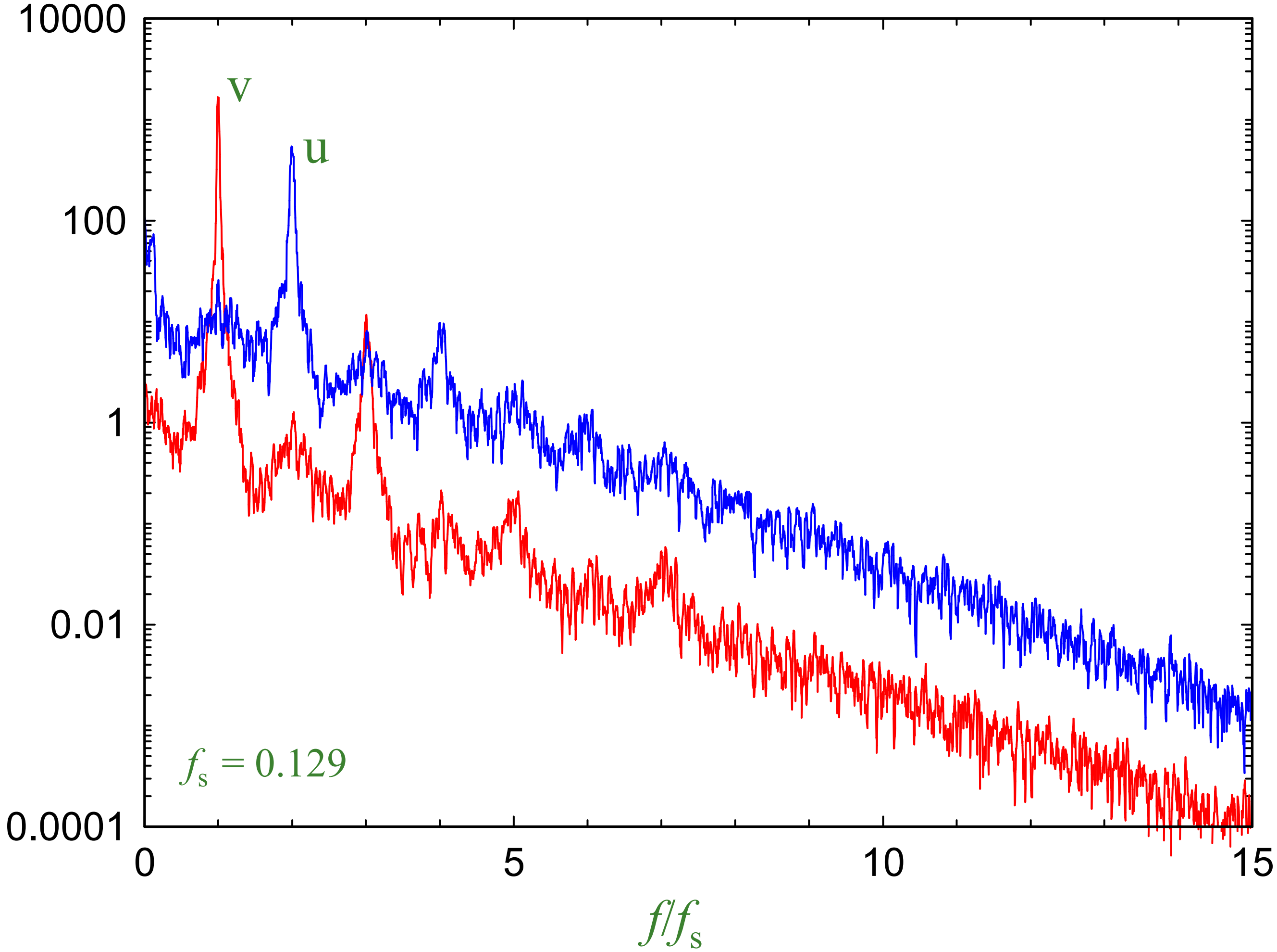
Flow visualization



Williamson, *Ann. Rev. Fluid Mech.* 1996, **28**,477-539

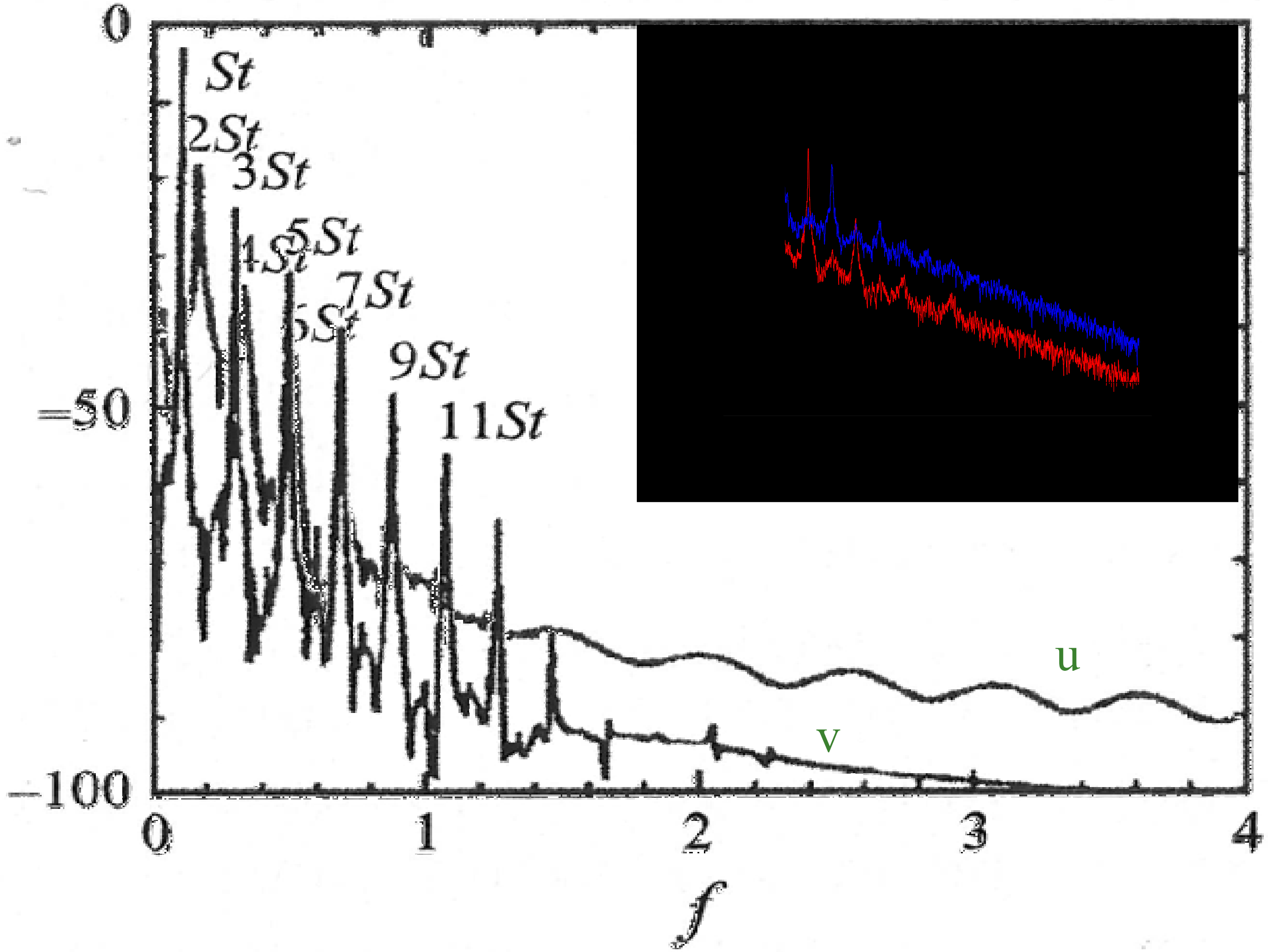
Vorticity iso-contours

Results - 3D transitional wake (Continued)



Velocity spectra at x/D = 2.5, y/D = 0

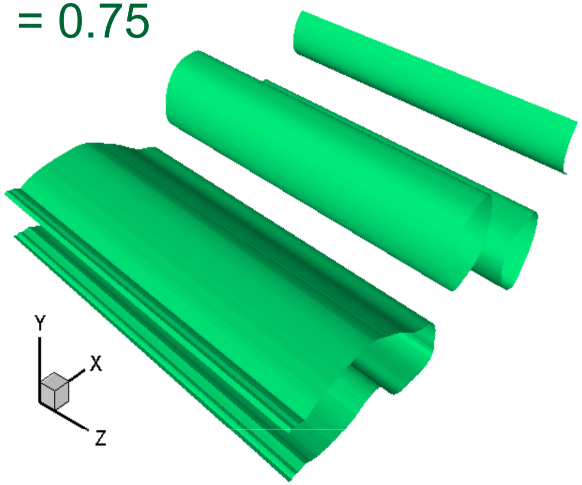
Results - 3D transitional wake (Continued)



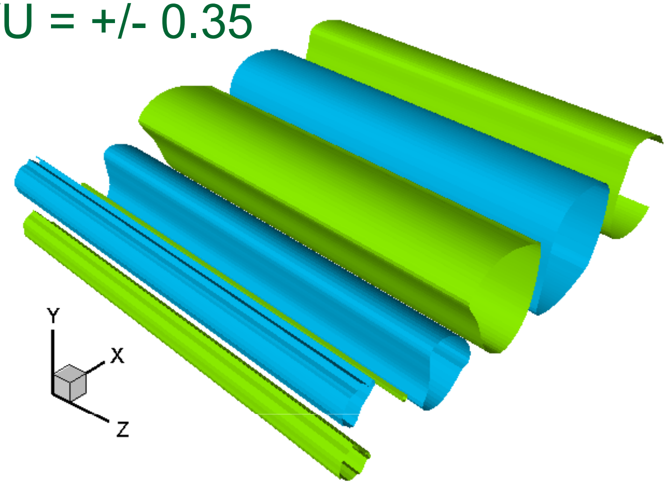
Velocity spectra, DNS, x/D = 1.80, y/D = 0

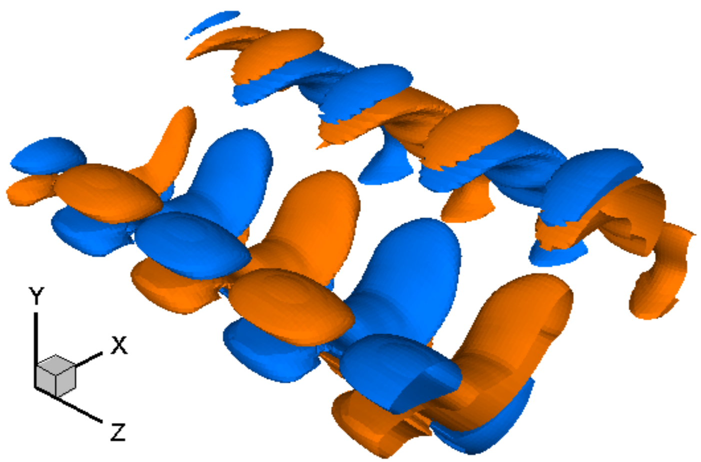
Results – 3D transitional wake (Continued)

u/U = 0.75



v/U = +/- 0.35



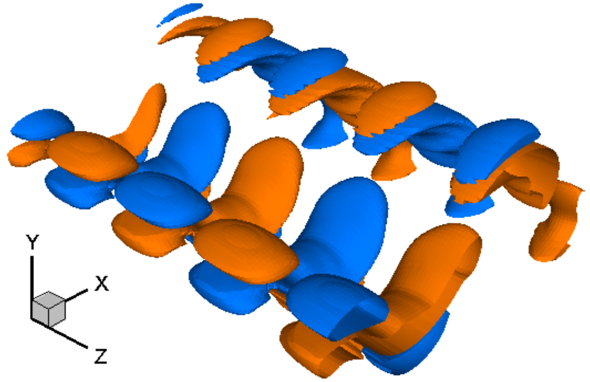


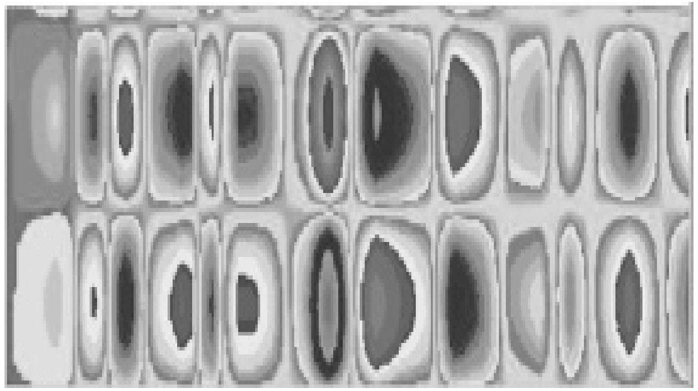
w/U = +/- 0.00006

Early stage of transition

Results - 3D transitional wake (Continued)

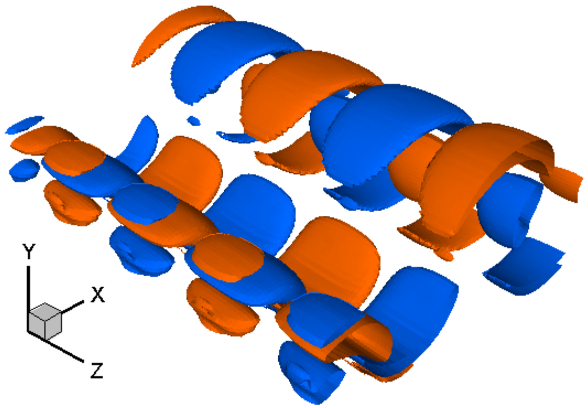
w/U = +/- 0.00006

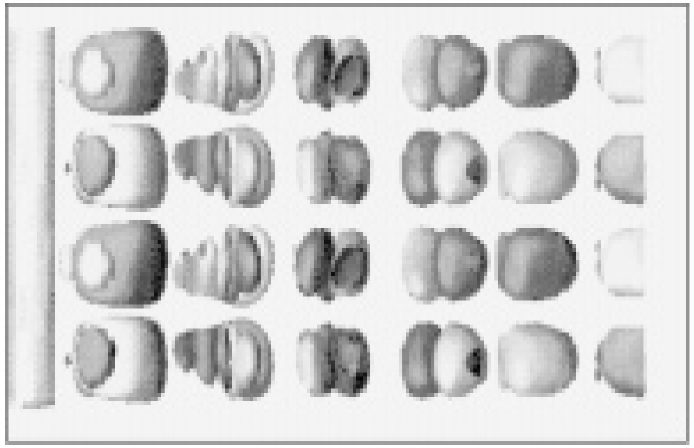




DNS, Braza,
FTC, 1999, **63**,315-341

$\omega_x/\omega_{x,max}$ = +/- 0.1

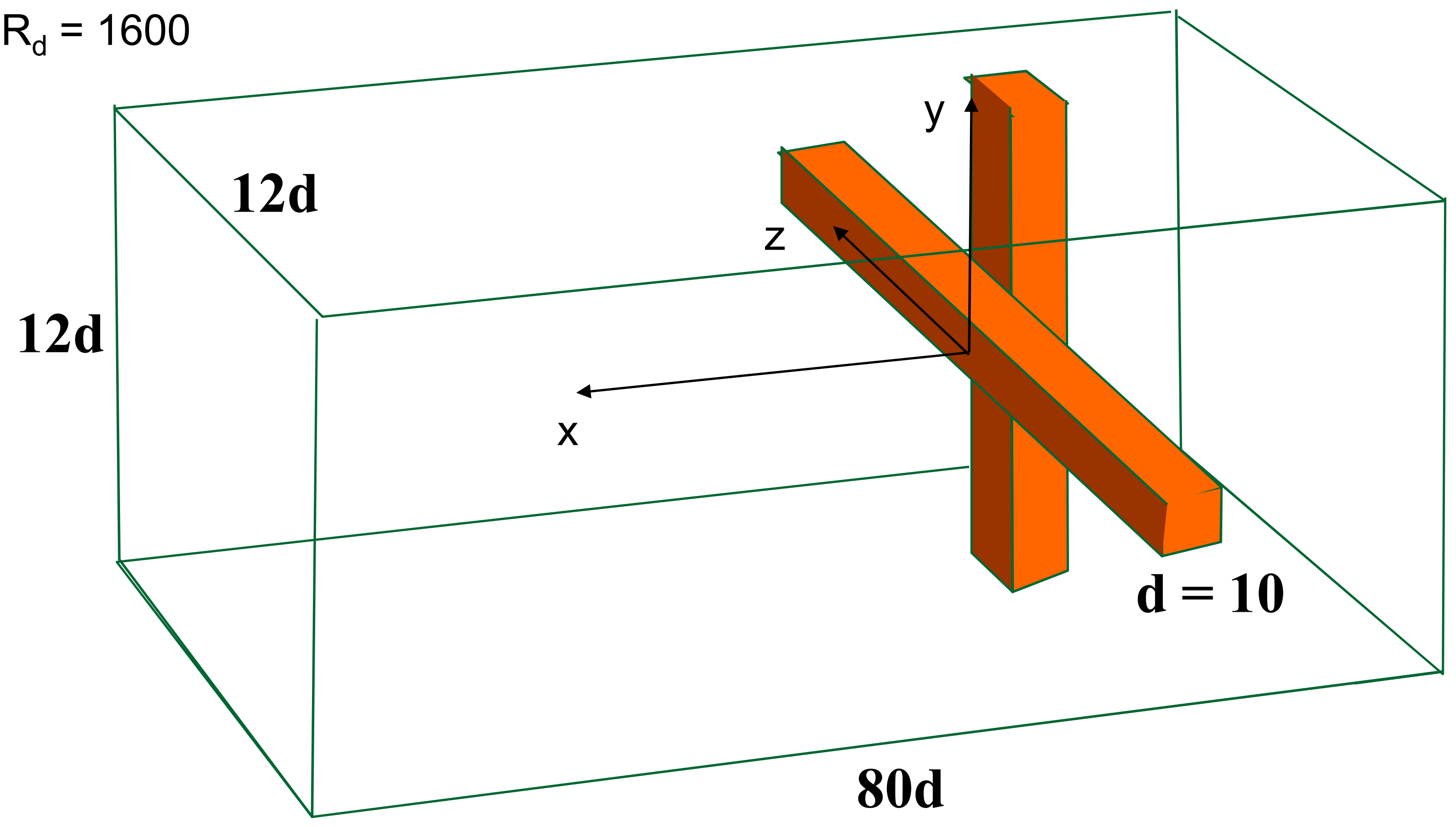




Lattice Boltzmann simulation of a crossbar wake

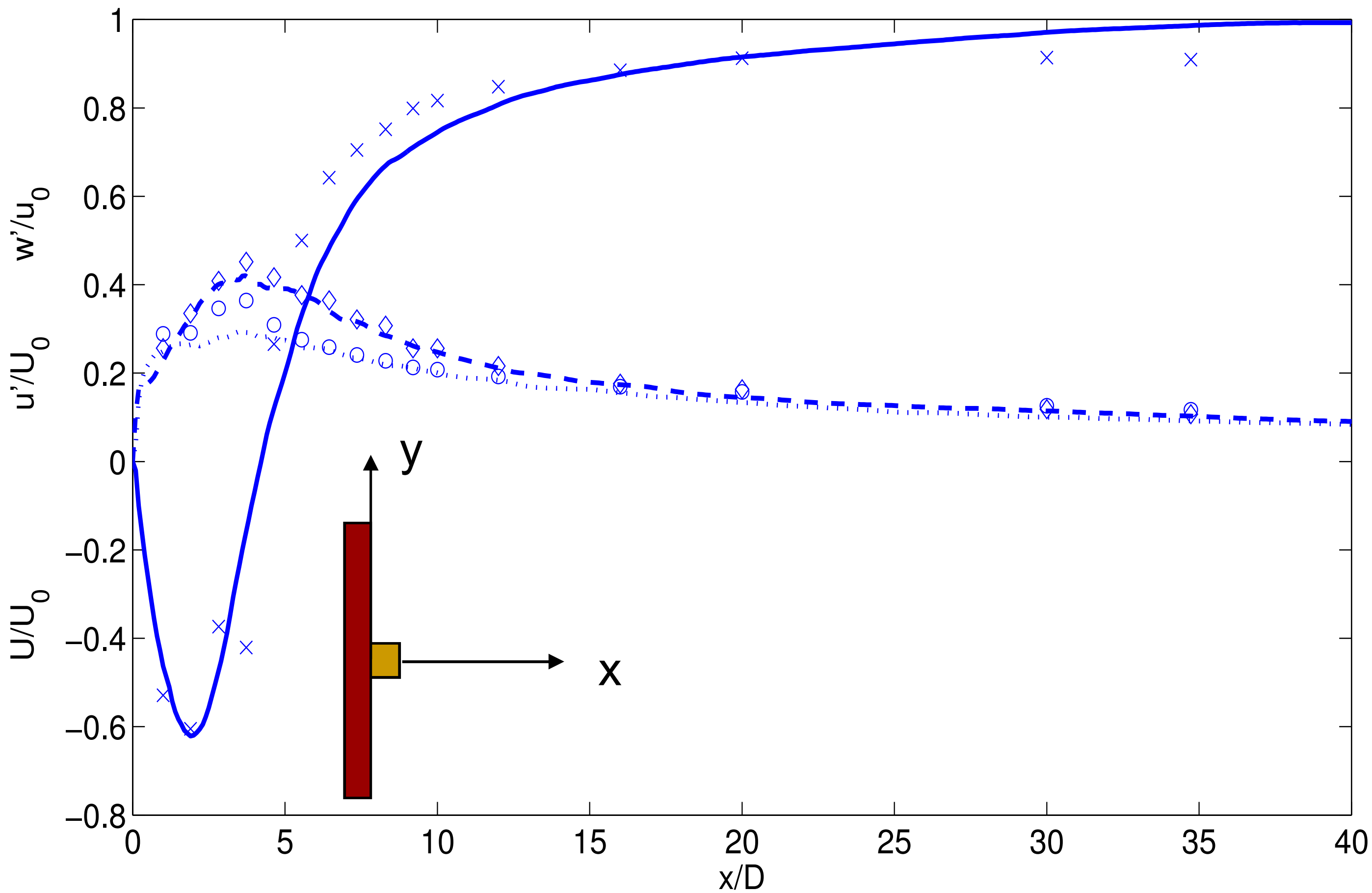
Crossbar and Computational domain

$R_d = 1600$



Results - Crossbar wake

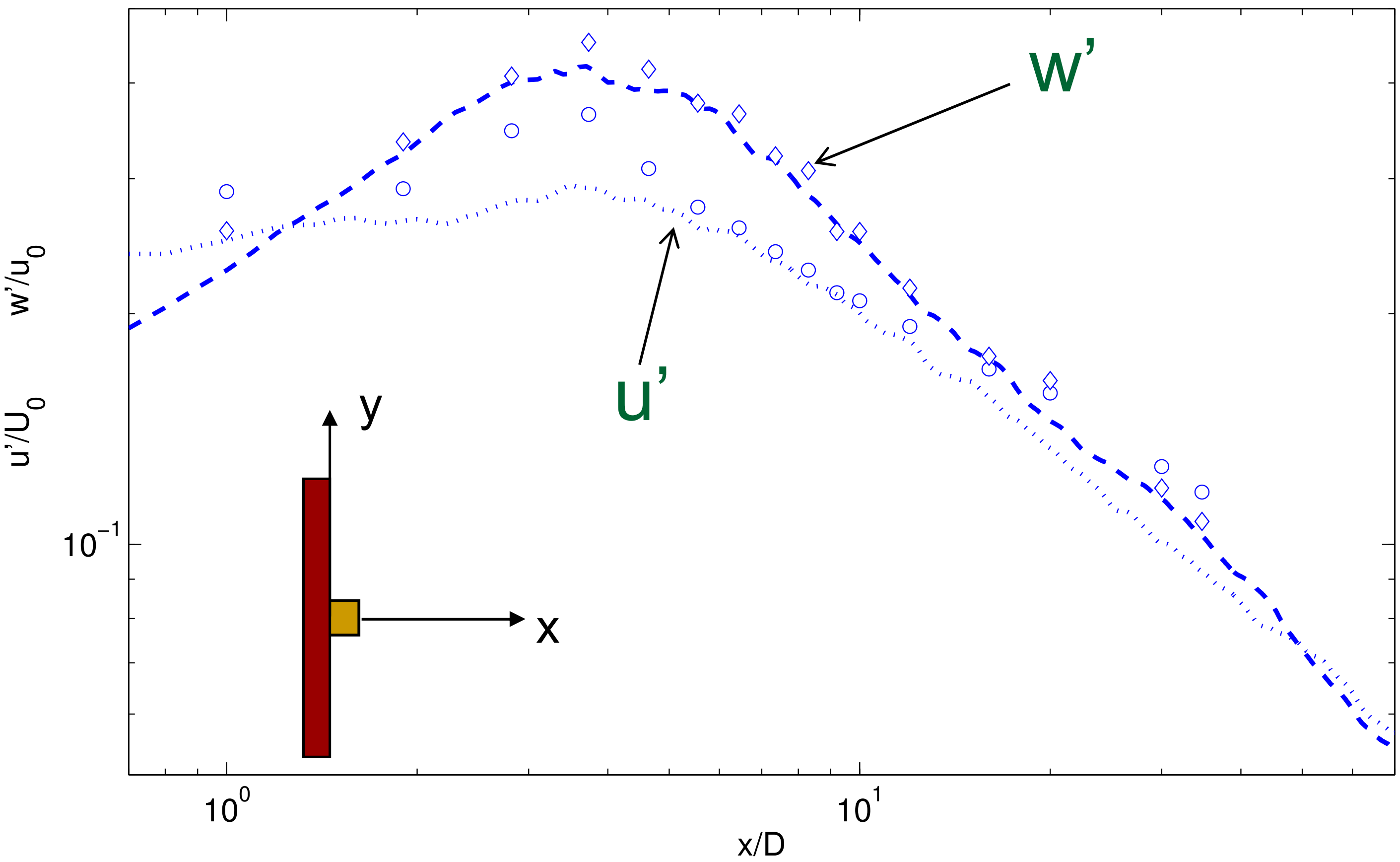
Comparison between LBM and LDV



U, u', and w' along the centerline, Symb.: LVD, lines: LBM

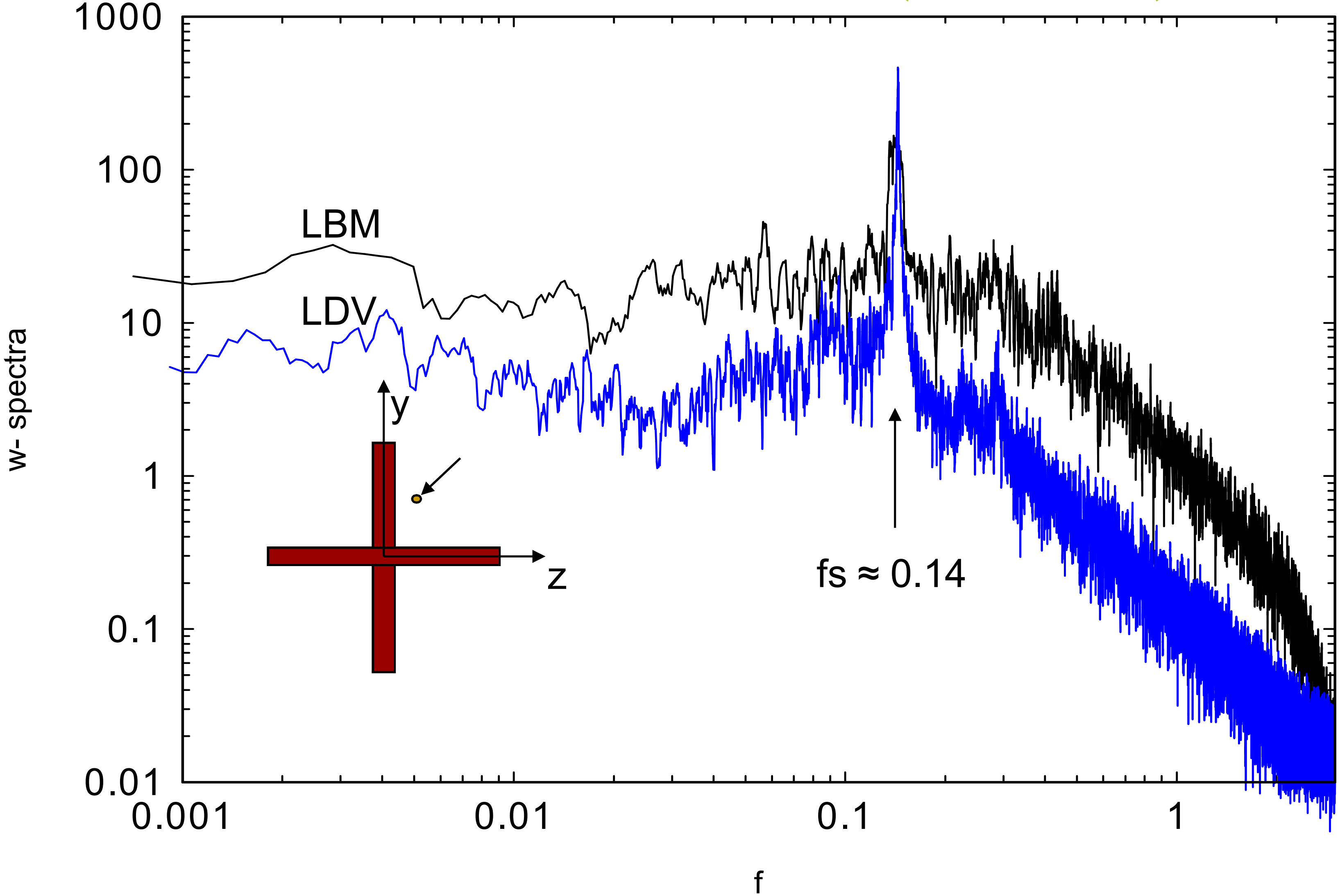
Results - Crossbar wake (Continued)

Comparison between LBM and LDV



u' and w' along the centerline, Symb.: LVD, lines :LBM

Results - Crossbar wake (Continued)

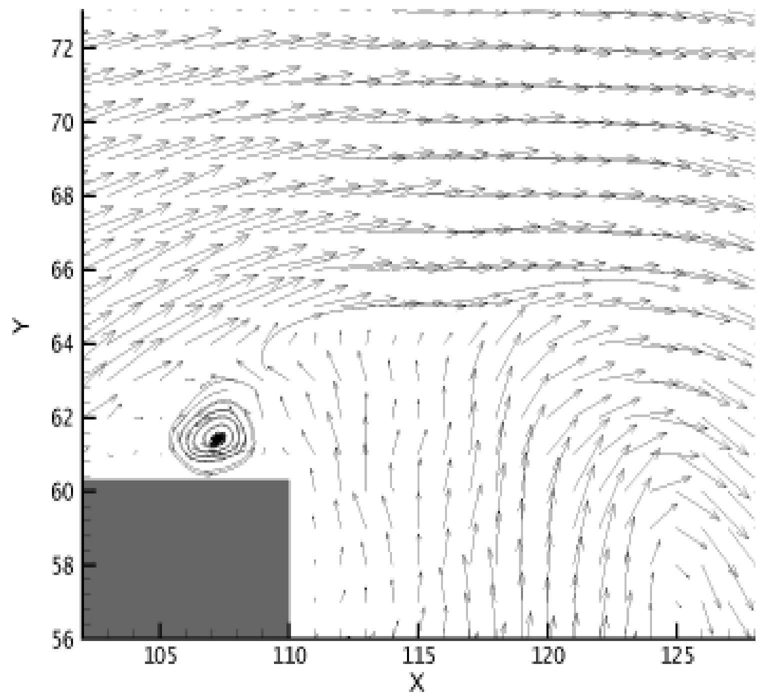


w-spectrum at y/d = 3, z/d = 1.5 and x/d = 5

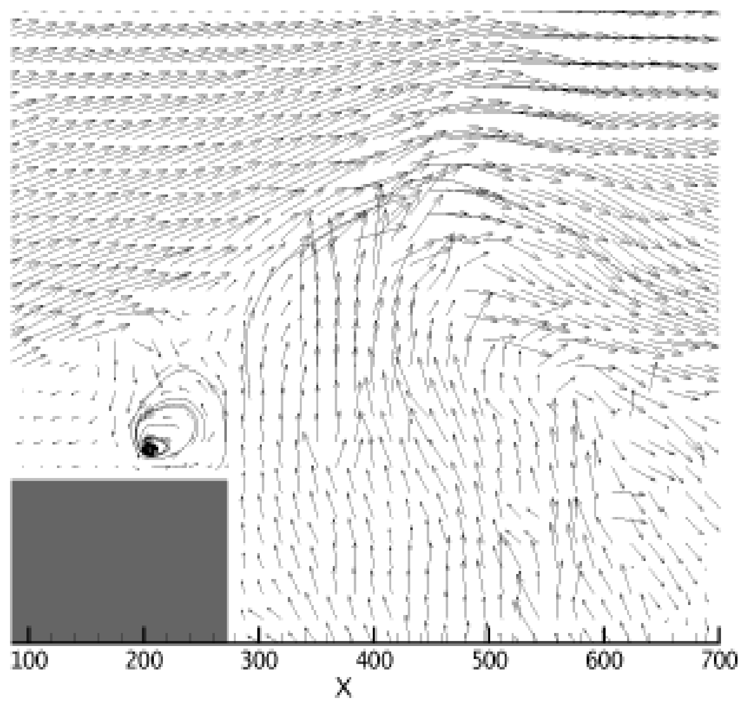
Results - Crossbar wake (Continued)

Comparison between LBM and PIV

LBM



PIV

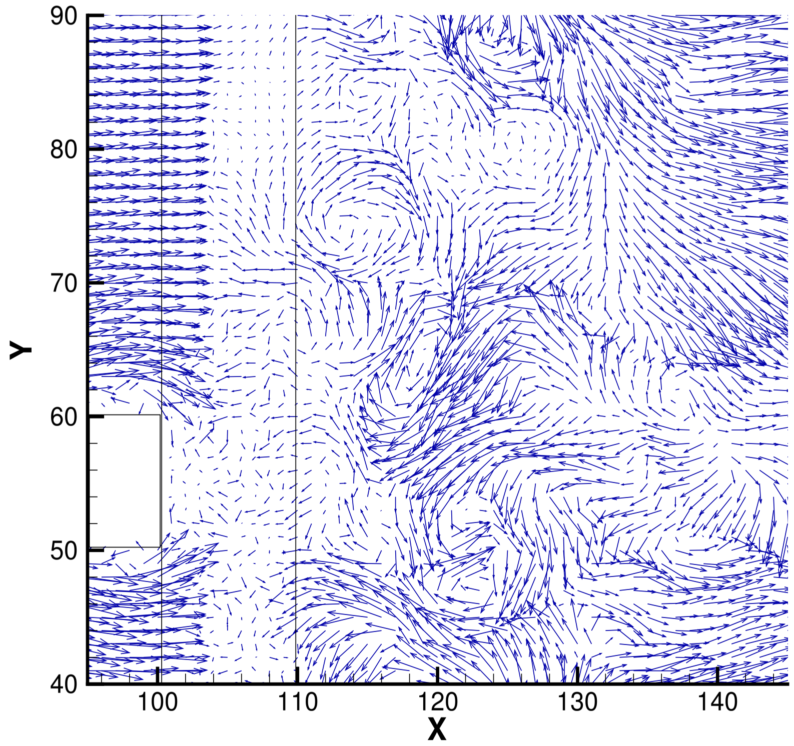


Velocity field at $z = 3D$

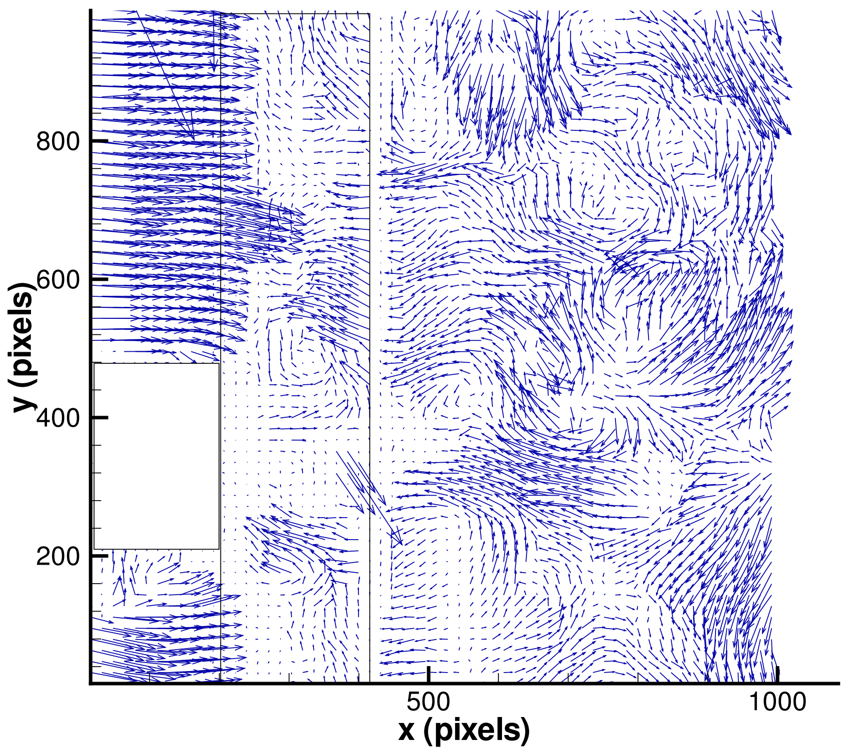
Results – Crossbar wake (Continued)

Comparison between LBM and PIV

LBM

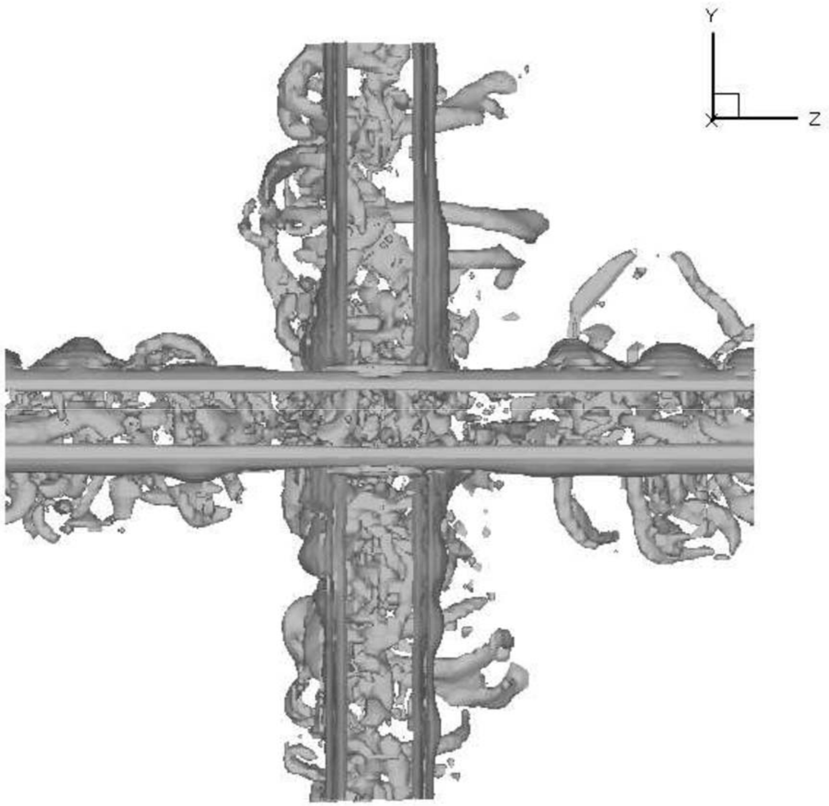


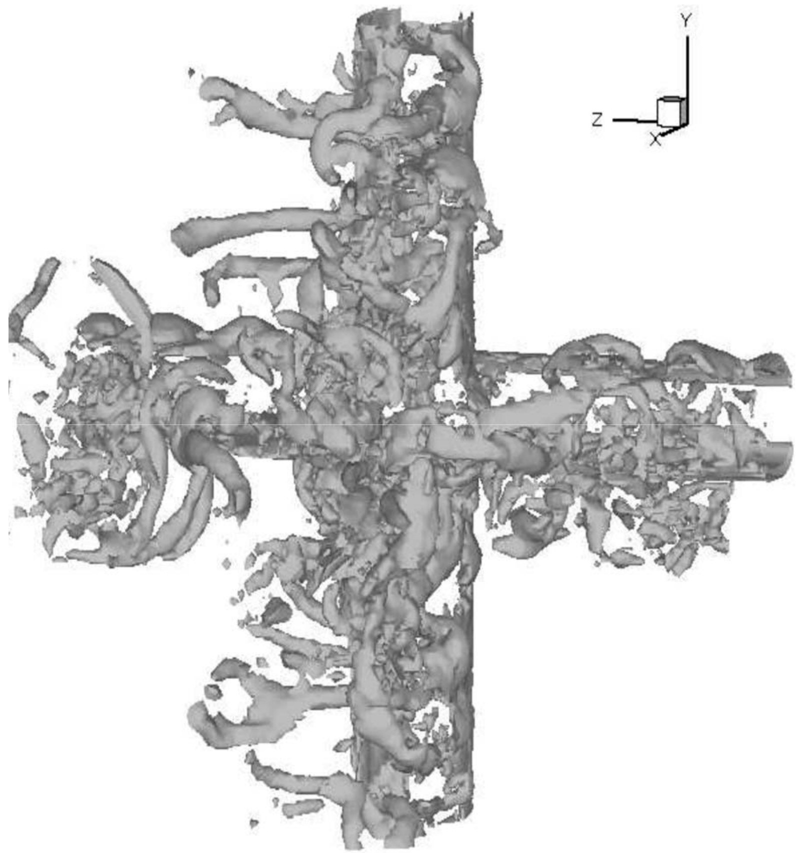
PIV



Velocity field at $z = 0.3D$

Results - Crossbar wake (Continued)

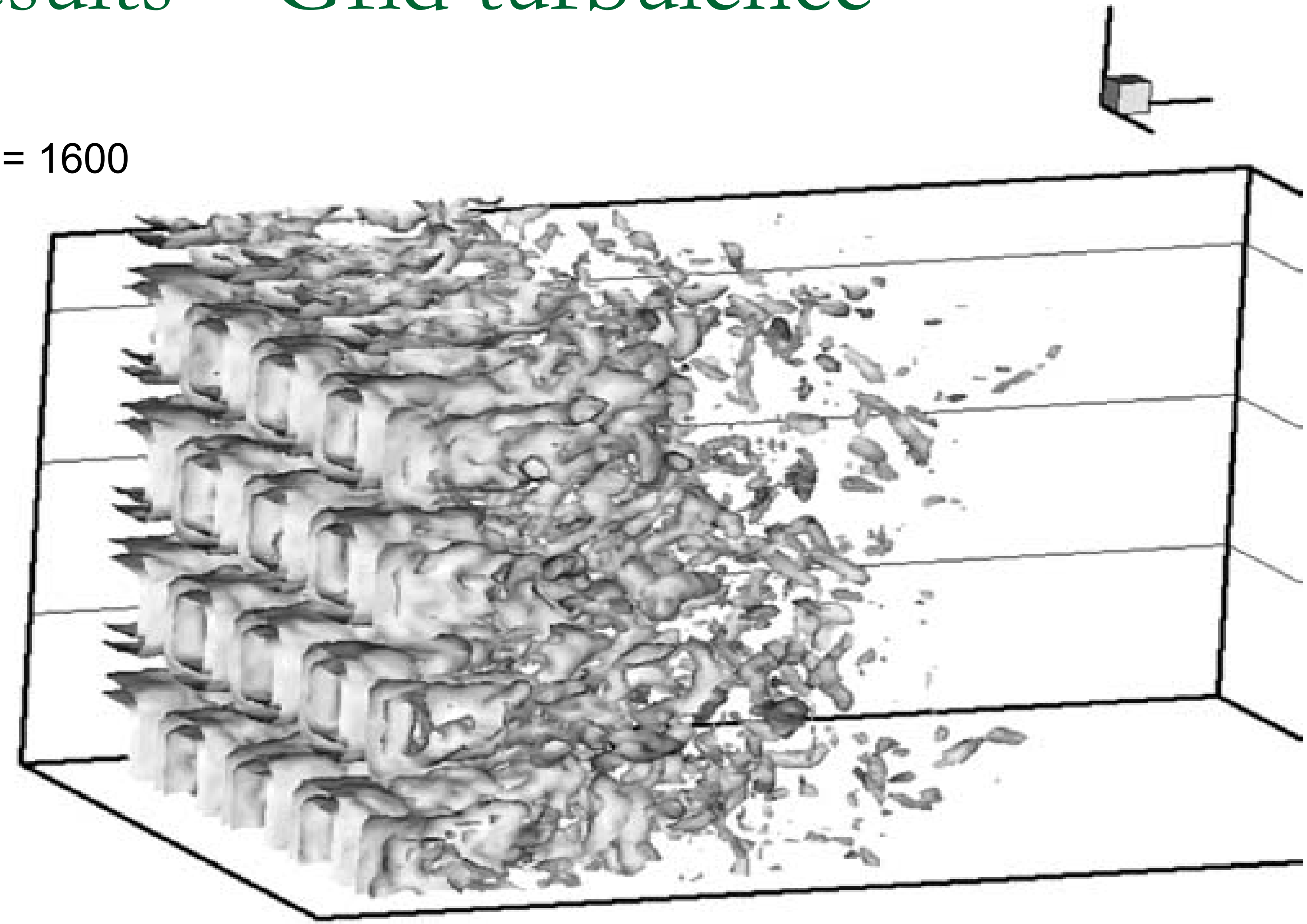




Instantaneous contour of $\omega / \omega_{max} = 0.3$. Left: front view, right: back view.

Results – Grid turbulence

$R_d = 1600$



Iso-surfaces of ω^2 *behind the grid.*

Results – Grid turbulence (Continued)

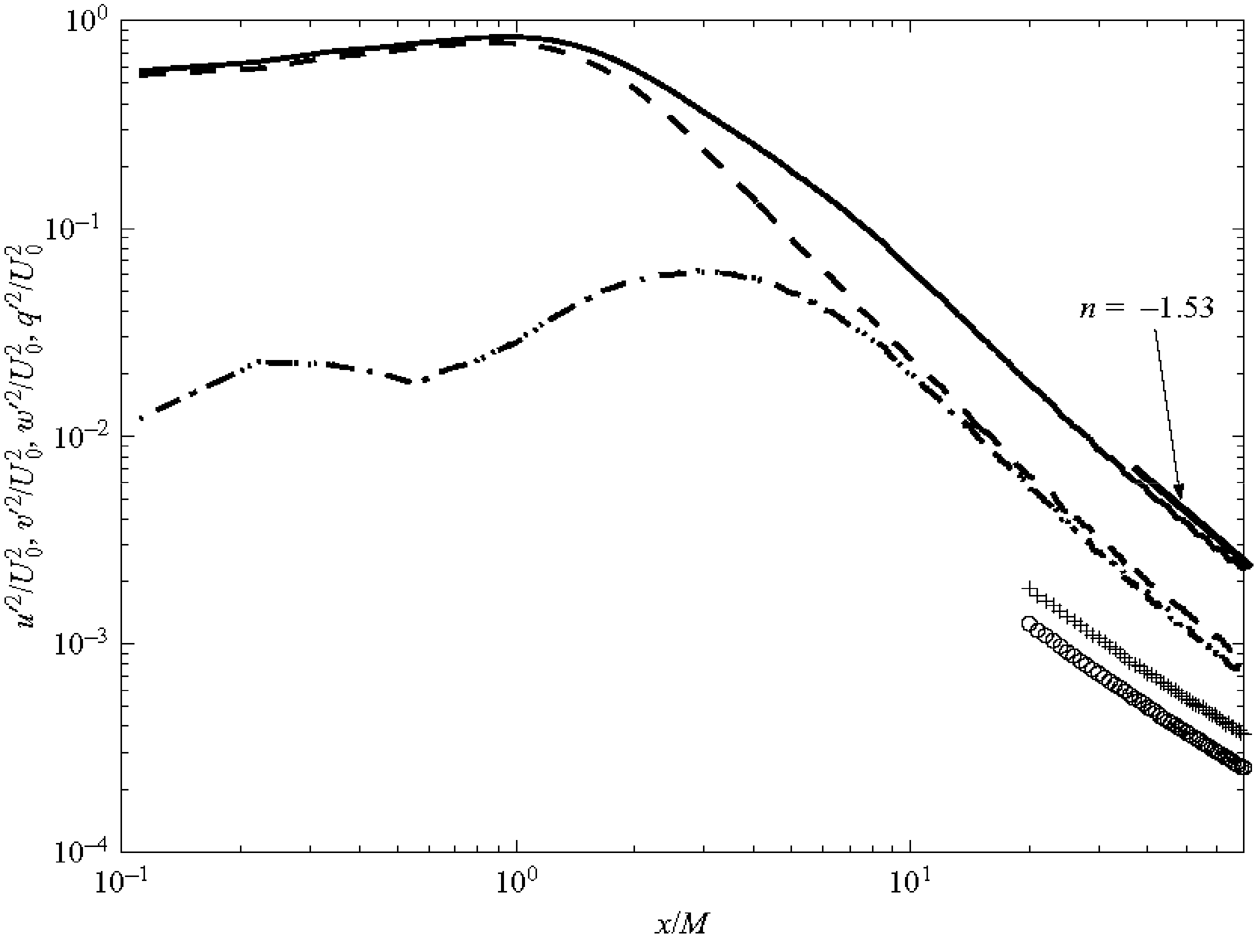


FIGURE 8. Decay of the turbulent kinetic energy (solid line) and its components (- - -, u'^2; —·—, v'^2; · · ·, w'^2) downstream of the grid. Symbols: experiments (Lavoie *et al.* 2005), ×, u'^2; ○, v'^2.

Results – Grid turbulence (Continued)

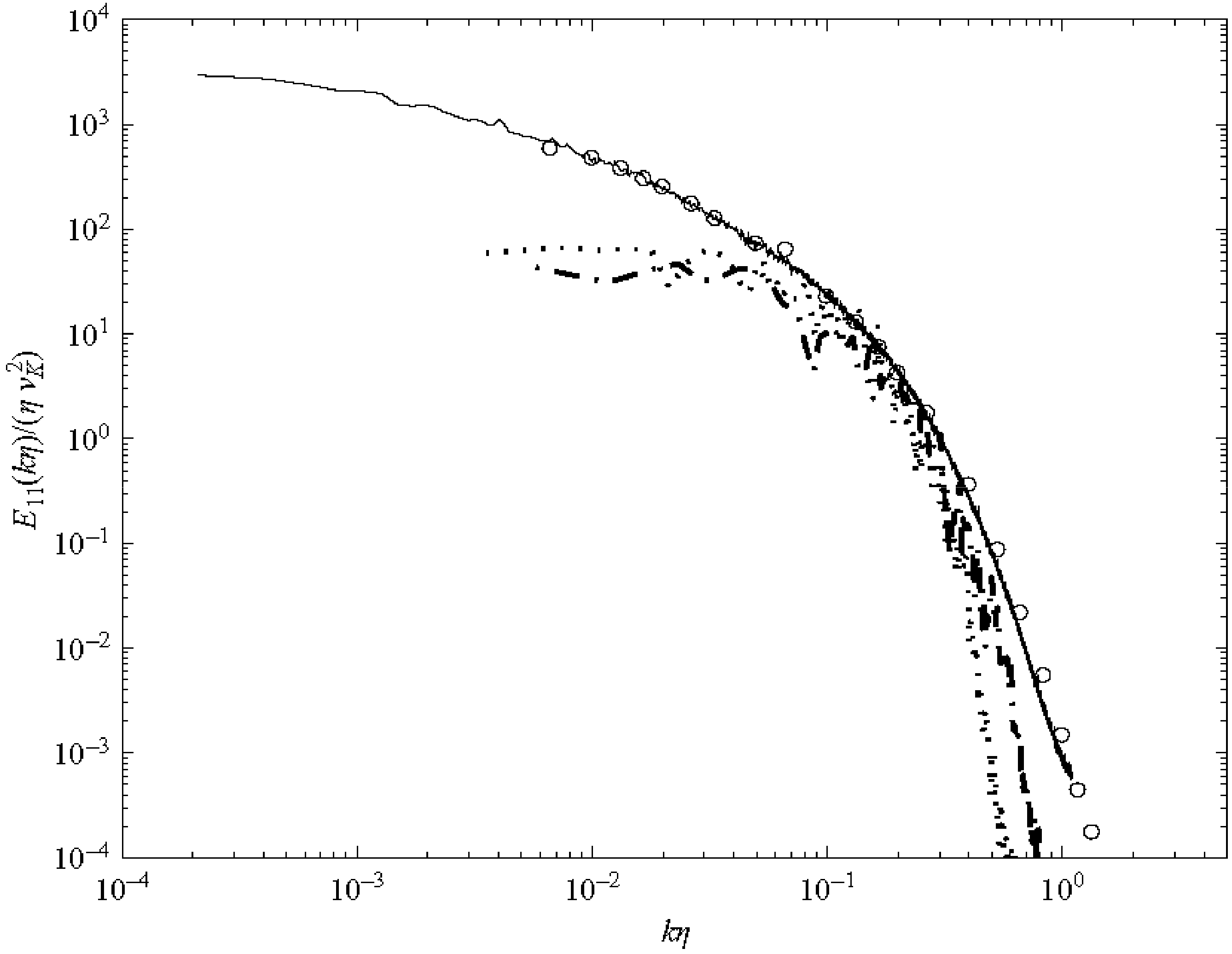
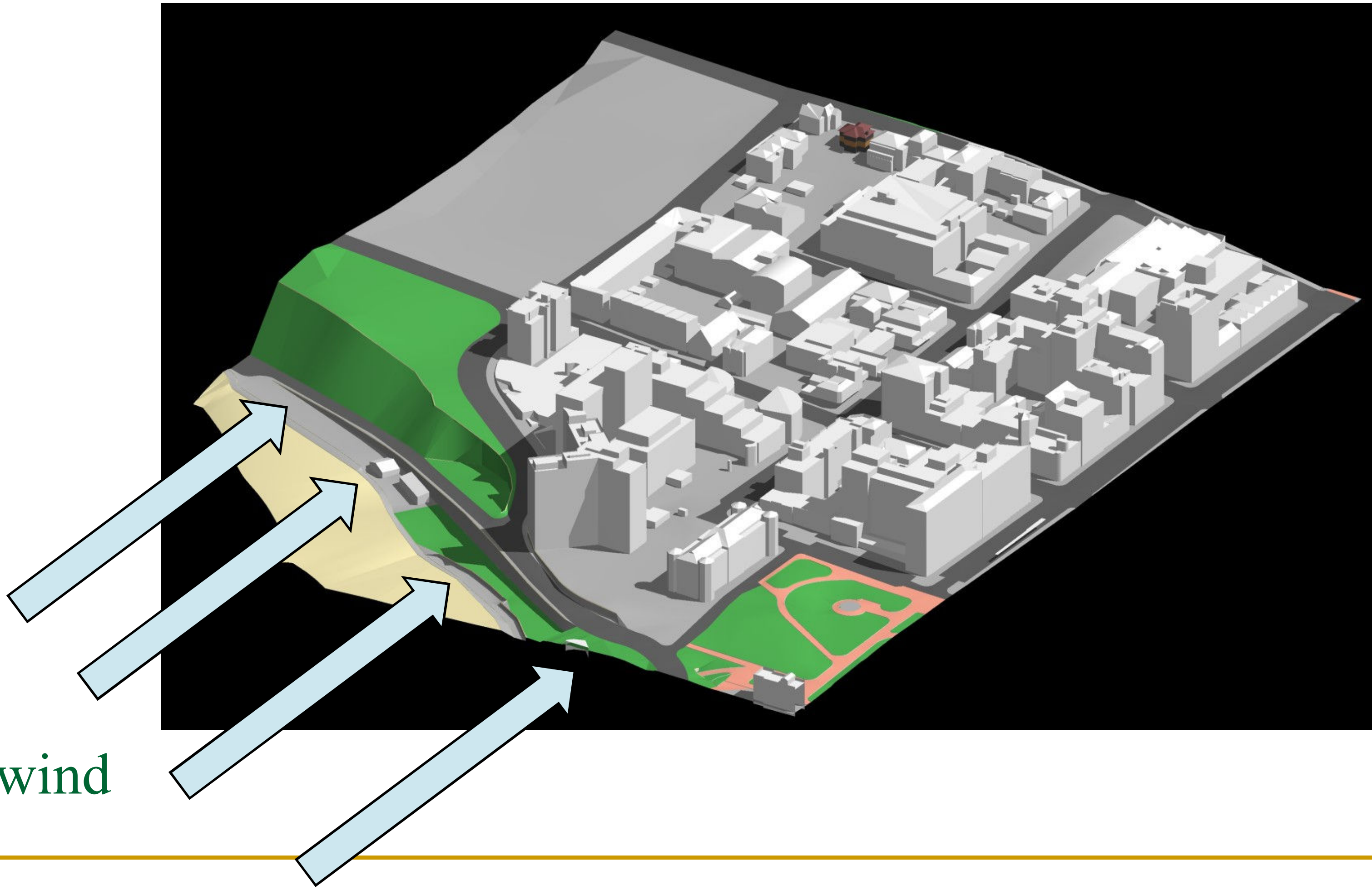


FIGURE 17. Longitudinal one dimensional spectra $E_{11}(k)$ in Kolmogorov units at $x/M = 20$ $(\cdots)$ and 60 $(-\cdot-)$. —, experiment of Lavoie *et al.* (2005), symbols: experiment of Comte-Bellot & Corrsin (1971).

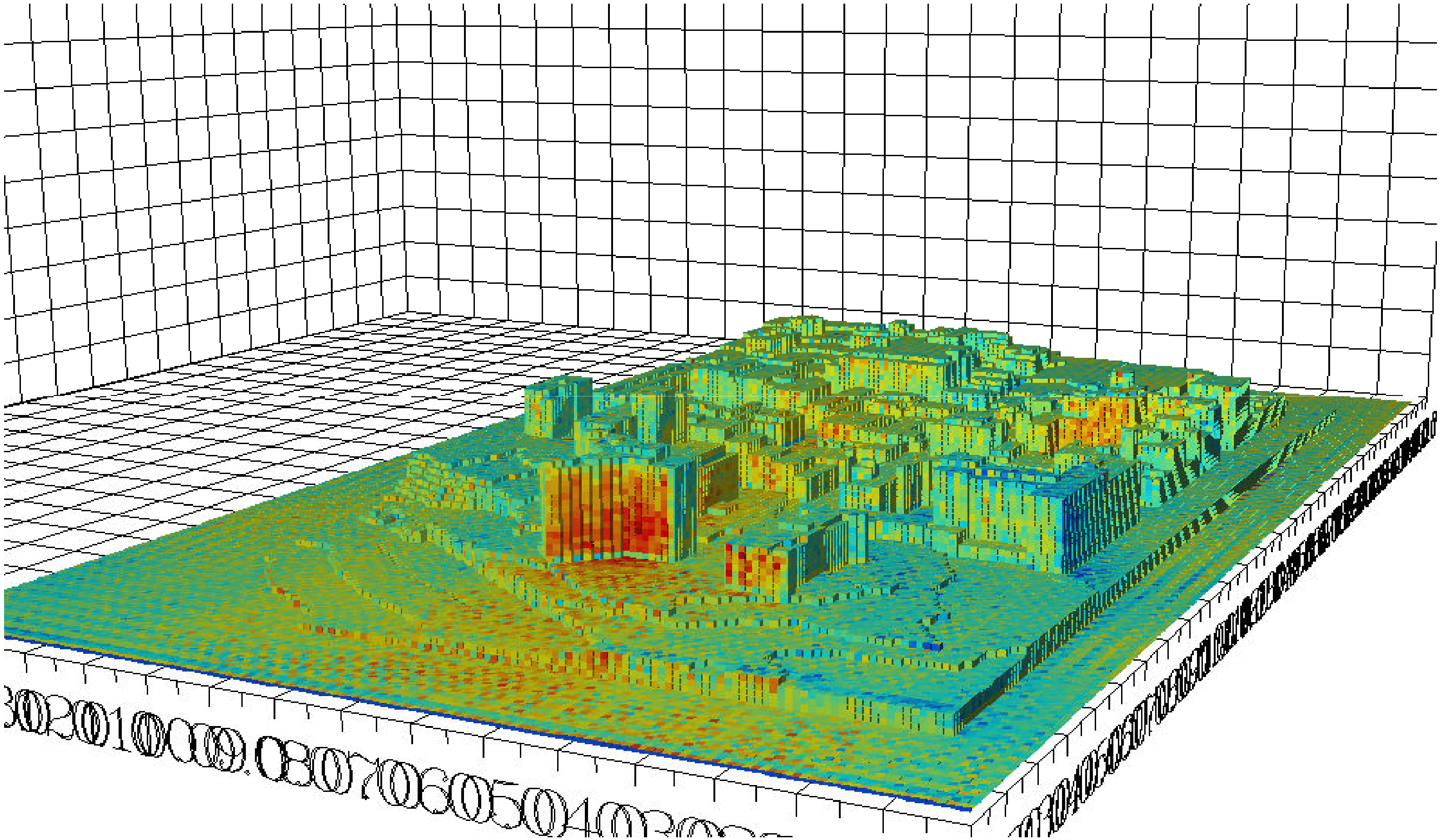
Lattice Boltzmann simulation of urban flows

North-East side of Newcastle



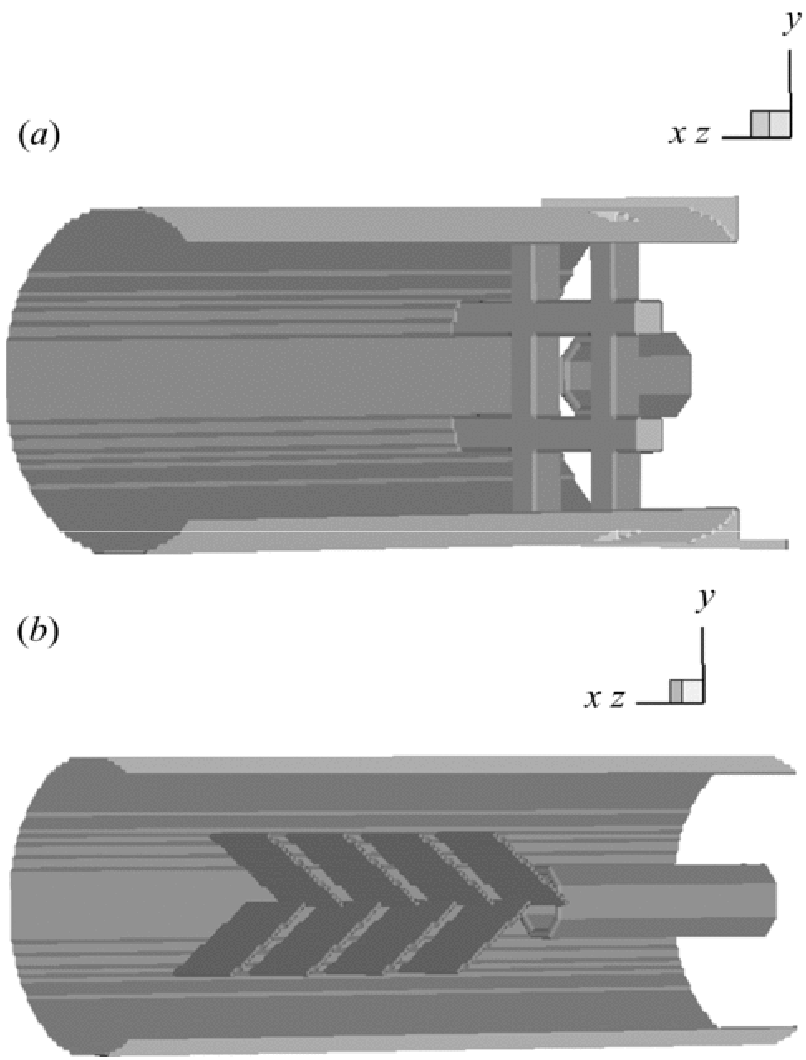
Ocean wind

Results - Urban flow (Continued)



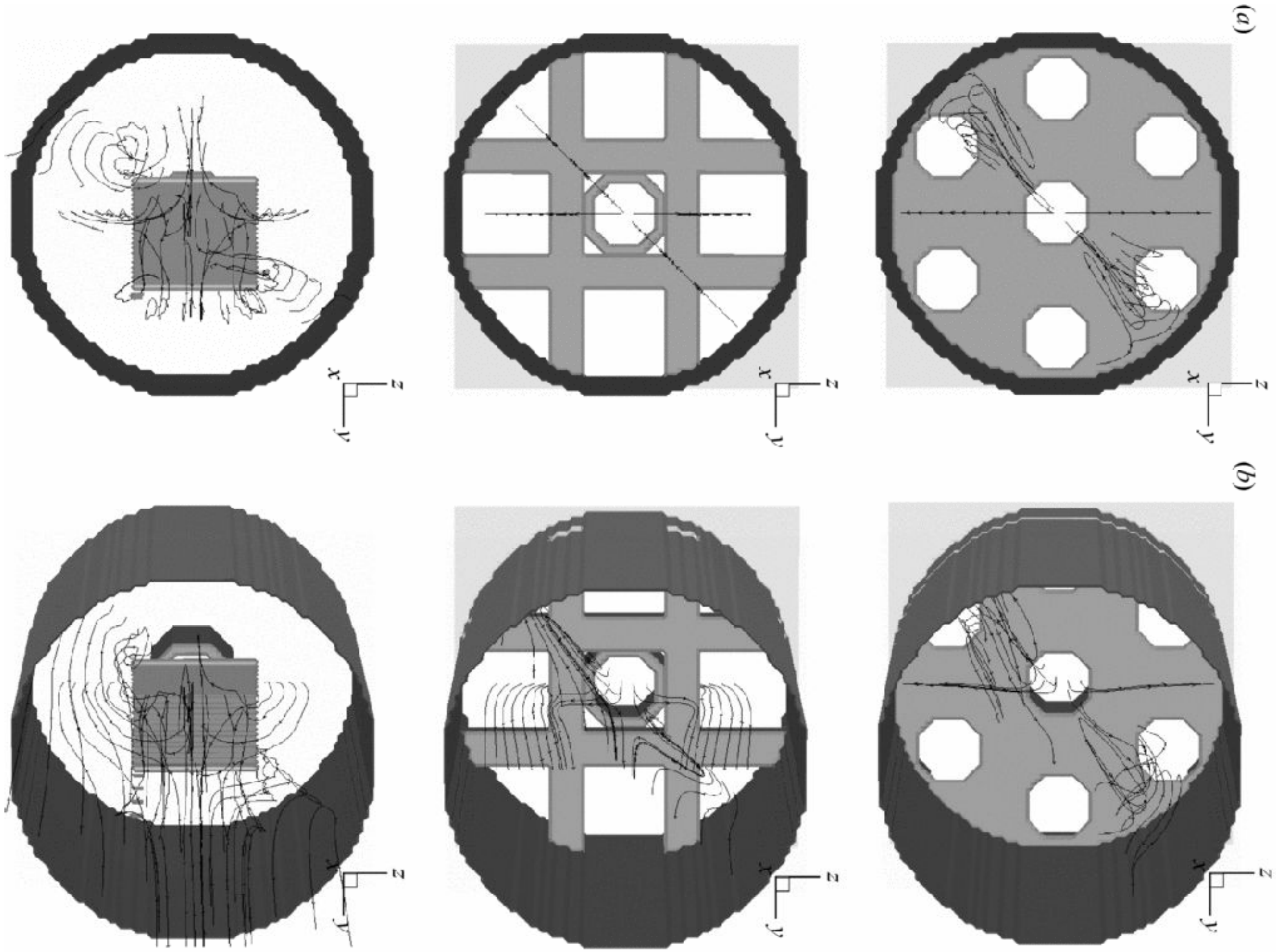
Instantaneous surface pressure

Results - Micromixer



Passive micromixer (coaxial)

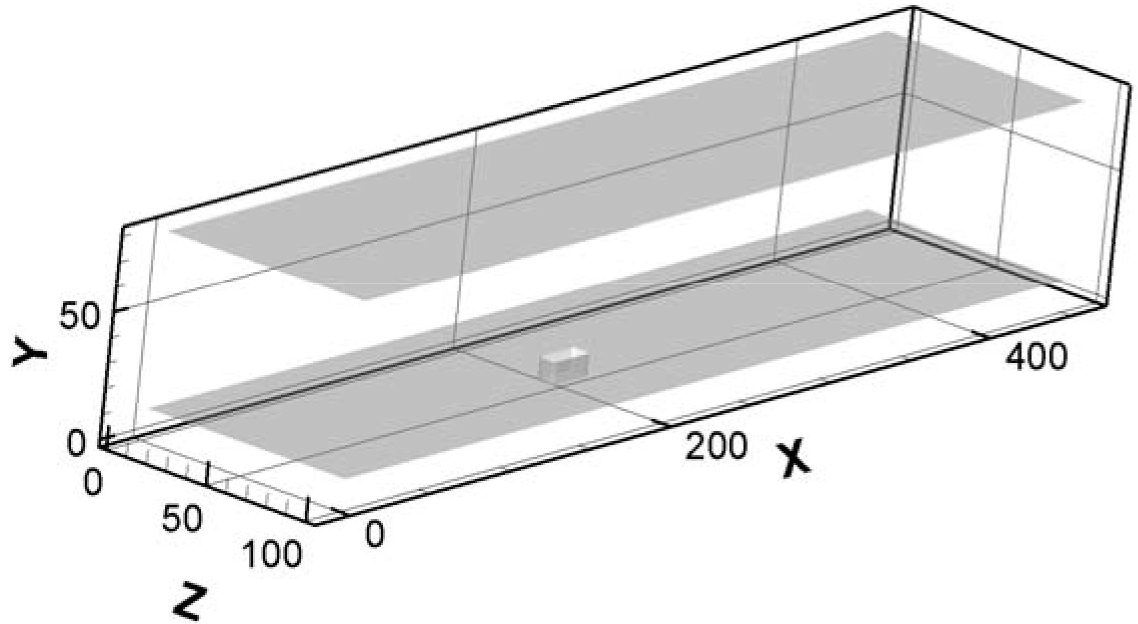
Results - Micromixer (Continued)



Passive micromixer (coaxial)

Results - Micromixer (Continued)

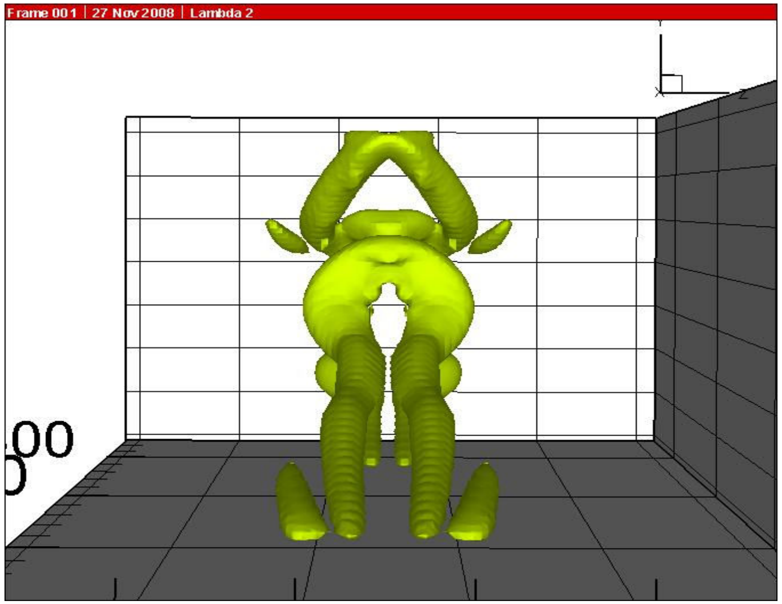
Control by Pulsed jet

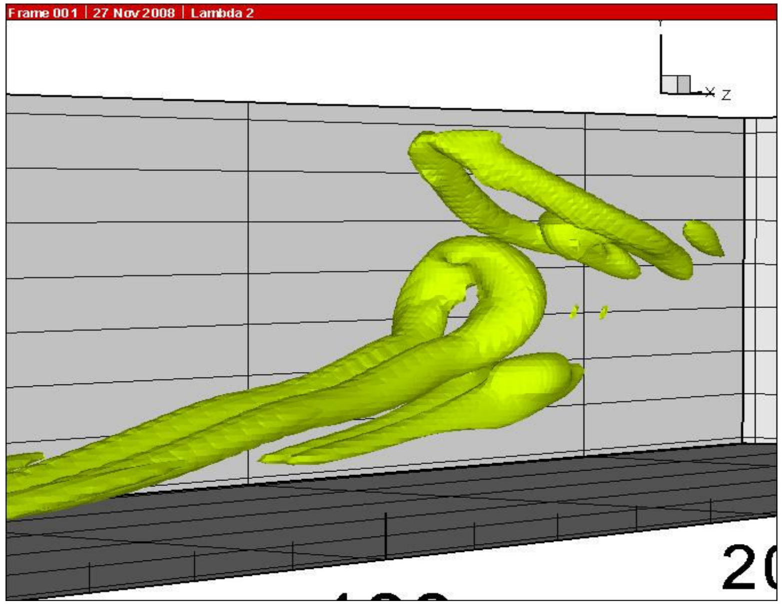


Active micromixer (channel flow with pulsed jets)

Results - Micromixer (Continued)

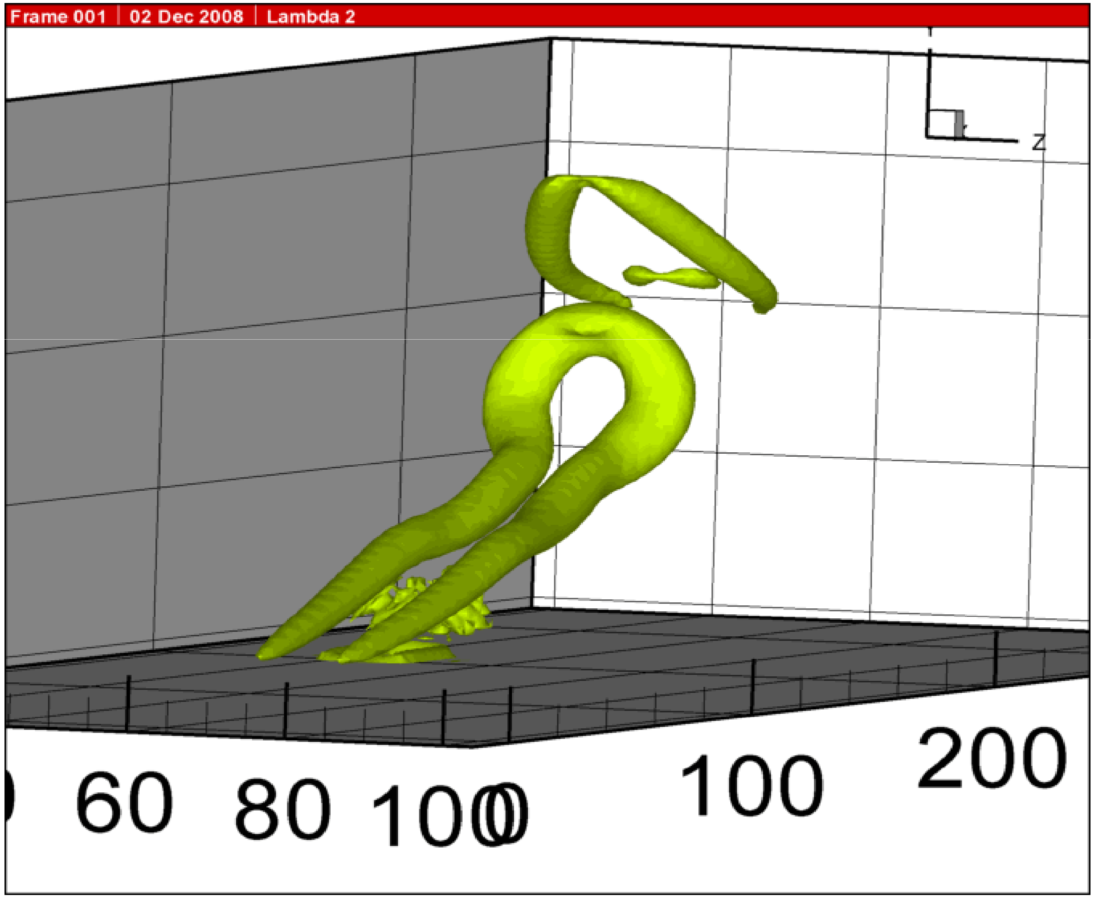
One pulsed jet





Results - Micromixer (Continued)

Two pulsed jets



Conclusions

The presented LBM simulation showed:

1 - LBM is a reliable alternative DNS method to the classical procedure based on the resolution of NS-equations.
(at least for incompressible flows)

2 - LBM can be used for research and development.

BUT !

Need to convince more researchers (e.g. turbulence community) to use LBM (How?)