Discrete Kinetic Theory of Gases

Renée Gatignol

Institut Jean le Rond d'Alembert

Université Pierre et Marie Curie & CNRS (UMR 7190)

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Discrete Kinetic Theory of Gases: Outline

1. **Introduction**
2. **Discrete kinetic theory**
3. **Hydrodynamical description for regular discrete models**
4. **Boundary conditions**
5. **Applications**
6. **Conclusion**

1. Introduction

Introduction

Boltzmann equation

$$\mathcal{D} f = \mathcal{J}(f, f)$$

→ **Balance laws**

→ **Entropy inequality**

→ **Constitutive laws**

Dimensionless Boltzmann equation

$$\mathcal{D} f = (1/\varepsilon)\ \mathcal{J}(f, f)$$

$$\varepsilon = \lambda / L$$

→ $\varepsilon << 1$ **Euler, Navier-Stokes, Burnett, … equations**

→ $\varepsilon >> 1$ **Free molecular flows**

→ $\varepsilon \cong 1$ **Transition flows: Wave shock structure, Knudsen layer, …**

→ **Need models**

Introduction: The first works

	Authors	Velocity number	Subject
1957	Carleman	2	H-Theorem
1960	Gross		The velocity discretization is emphasized
1964	Broadwell	6	Shock wave structure
« «	Broadwell	8	Couette and Rayleigh problems
1965	R. G.	6	Shock wave structure
1966	Harris	6	Ternary collisions and H-Theorem
1967	Harris	4	Study of the H-function
1970	R.G.	p	Discrete kinetic theory
1971	Godunov & Sultangazin	6	Kinetic and hydrodynamical descriptions
1972	Hardy & Pomeau	4	Lattice gases

R. G.

1965 - 1972	6 CRAS	Shock structure, H-Theorem, General kinetic equations, Chapman-Enskog expansion, …
1970	Zeitschrift für Flugwissenschaften	« Théorie cinétique des gaz à répartition discrète de vitesses »
1975	Lecture Notes in Physics (Vol. 39)	
1975	Physics Fluids	Discrete kinetic theory
1977	Physics of Fluids	Boundary conditions
1965 - …	4 theses, about 20 papers and 20 proceedings	

H. Cabannes

1975	J. de Mécanique	Shock structure (14 velocities)
1977 - …	On the solutions of the discrete kinetic equations (existence theorems, exact solutions)	
1980	Lecture notes, Berkeley University, « The discrete Boltzmann Equation »	

2. Discrete Kinetic Theory of Gases

Discrete Kinetic Theory of Gases

- In discrete kinetic theory, the main idea is that the velocities of the molecules belong to a given set of vectors
- The Boltzmann equation is replaced by a system of partial differential equations
- This system has an interesting mathematical structure (H. Cabannes, Bellomo, Cercignani, Kawashima, …)
- The discrete models, by their simplicity, help to understand the fundamental problems of rarefied gas dynamics
- The hydrodynamic description of discrete gases is obtained via the Chapman-Enskog expansion

Discrete kinetic theory: Binary collisions

The particles are identical
The particle velocities belong to a given set of vectors:

$$\vec{u}_k\,,\quad k = 1, 2, \ldots, p$$

$N_k(\vec{x}, t)$ denotes the number of particles with velocity $\vec{u}_k$ (i.e. particle « k ») per unit of volume

Macroscopic quantities

$$\begin{cases} n = \sum_k N_k \\ n\,\vec{u} = \sum_k N_k\,\vec{u}_k \\ n\,e = \dfrac{m}{2}\sum_k N_k\,(\vec{u}_k - \vec{u})^2 \end{cases}$$

$$\begin{cases} \vec{\vec{P}} = m\sum_k N_k\,(\vec{u}_k - \vec{u})(\vec{u}_k - \vec{u}) \\ \vec{q} = \dfrac{m}{2}\sum_k N_k\,(\vec{u}_k - \vec{u})^2\,(\vec{u}_k - \vec{u}) \end{cases}$$

Binary collision

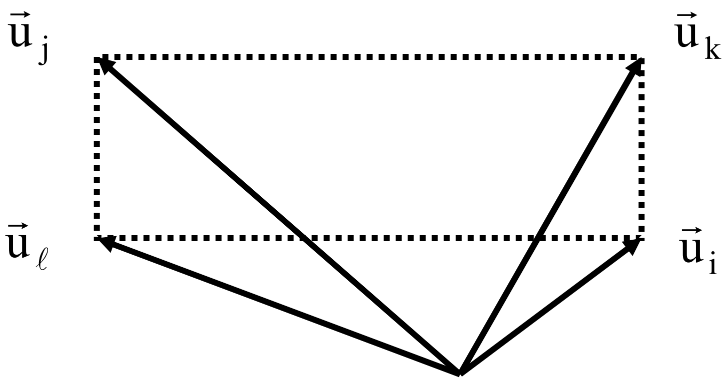
$\vec{u}_i \;,\; \vec{u}_j$

Before the collision

$\Longleftrightarrow$

$\vec{u}_k \;,\; \vec{u}_\ell$

After the collision



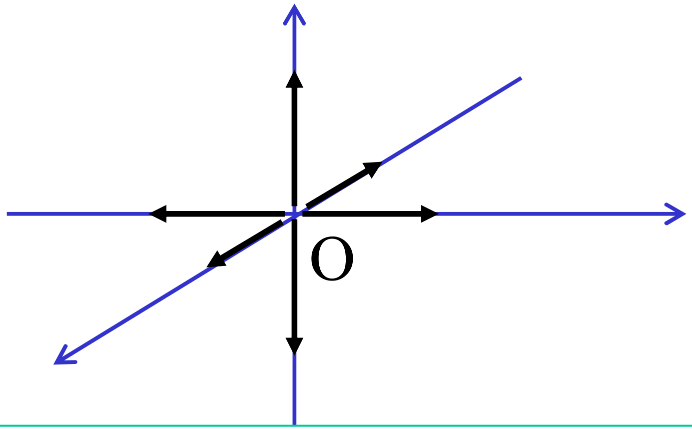
In the collision, the mass, momentum and energy are conserved

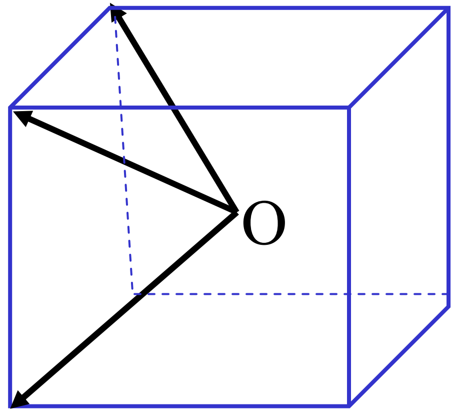
Transition probability $\quad A_{ij}^{k\ell}$

Microreversibility property $\quad A_{ij}^{k\ell} = A_{k\ell}^{ij}$

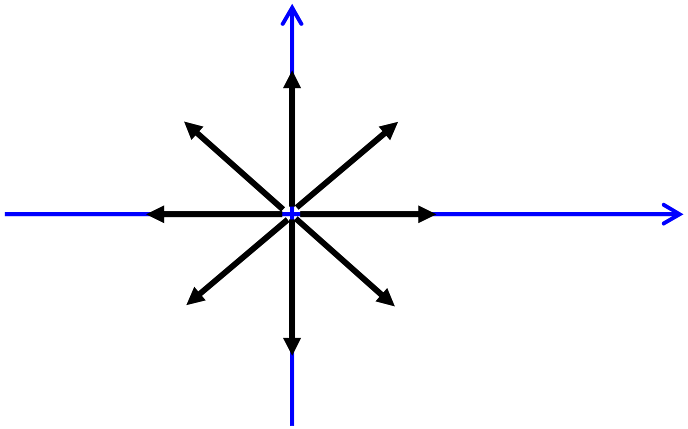
Discrete kinetic theory: Examples

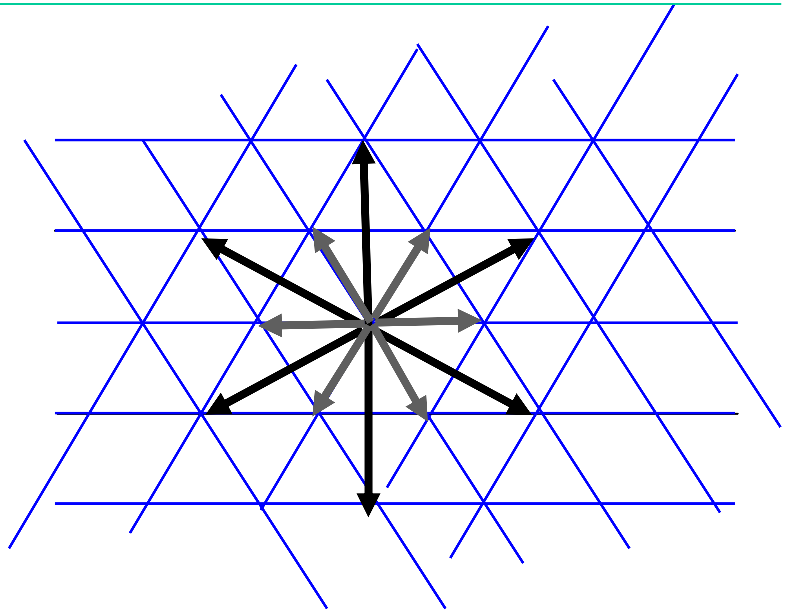
Spatial models with 6 velocities or with 8 velocities (Broadwell, 1964)





Coplanar models





Kinetic equations (binary collisions)

$$\begin{cases} \dfrac{\partial}{\partial t} N_k + \vec{u}_k \cdot \vec{\nabla} N_k = G_k - L_k \quad , \quad k = 1, 2, \ldots, p \\ \dfrac{\partial}{\partial t} N_k + \vec{u}_k \cdot \vec{\nabla} N_k = \dfrac{1}{2} \sum_{i j \ell} \left(A_{ij}^{k\ell} N_i N_j - A_{k\ell}^{ij} N_k N_\ell \right), \quad k = 1, 2, \ldots, p \end{cases}$$

Notations

$$\mathbf{N} = \left(N_1, N_2, \ldots, N_p \right)$$

$$< \mathbf{U}, \mathbf{V} > = \sum_k U_k V_k$$

$\mathcal{F}(\mathbf{U}, \mathbf{V})$ Linear mapping of $R^p \times R^p$ into R^p

Kinetic equations

$$\frac{\partial}{\partial t} \mathbf{N} + \mathcal{A} \mathbf{N} = \mathcal{F}(\mathbf{N}, \mathbf{N})$$

Symmetry property

$$< \phi , \mathcal{F}(\mathbf{U},\mathbf{V}) > = -\frac{1}{8}\sum_{ijk\ell} A_{k\ell}^{ij}\left(\varphi_k + \varphi_\ell - \varphi_i - \varphi_j\right)\left(U_i V_j + U_j V_i\right)$$

$$\phi = \left(\varphi_1, \varphi_2, \ldots, \varphi_p\right) \in R^p$$

Summational invariants: $\phi \in R^p$ such as

$$A_{ij}^{k\ell}\left(\varphi_k + \varphi_\ell - \varphi_i - \varphi_j\right) = 0 \qquad \forall\, i, j, k, \ell$$

→ Linear subspace $\mathbf{F}$ ($\mathbf{F} \subset R^p$ dimension of $\mathbf{F} = q$)

- Base in $\mathbf{F}$: $\mathbf{v}^1, \mathbf{v}^2, \ldots \mathbf{v}^q$
- Base in R^p : $\mathbf{v}^1, \mathbf{v}^2, \ldots \mathbf{v}^q, \mathbf{w}^{q+1}, \ldots \mathbf{w}^p$

Kinetic densities

$$\mathbf{N} = \sum_{\alpha=1}^{\alpha=q} \mathbf{a}_\alpha \mathbf{v}^\alpha + \sum_{\beta=q+1}^{\beta=p} \mathbf{b}_\beta \mathbf{w}^\beta$$

Macrocospic variable ($\mathbf{a}_\alpha$)   Microscopic variable ($\mathbf{b}_\beta$)

$$\frac{\partial}{\partial t}\mathbf{N} + \mathcal{A}\mathbf{N} = \mathcal{F}(\mathbf{N},\mathbf{N})$$

Equations for the $\mathbf{a}_\alpha$ and the $\mathbf{b}_\beta$ variables

$$\begin{cases} \dfrac{\partial \mathbf{a}_\alpha}{\partial t} + \langle \mathcal{A}\mathbf{N}\,,\,\mathbf{V}^\alpha \rangle = 0 \;, \qquad \alpha = 1, 2, \ldots, q \qquad \text{conservation laws} \\[2ex] \dfrac{\partial \mathbf{b}_\beta}{\partial t} + \langle \mathcal{A}\mathbf{N}\,,\,\mathbf{W}^\beta \rangle = \langle \mathcal{F}(\mathbf{N},\mathbf{N})\,,\,\mathbf{W}^\beta \rangle \;, \qquad \beta = q+1, q+2, \ldots, p \end{cases}$$

H – Theorem: H is decreasing with $H = \langle \mathbf{N}\,,\,\ln \mathcal{F}(\mathbf{N},\mathbf{N}) \rangle$

Maxwellian state: $\ln \mathbf{N} \in \mathbf{F} \Leftrightarrow \mathcal{F}(\mathbf{N},\mathbf{N}) = 0 \Leftrightarrow \ln \mathbf{N} = \sum_{\alpha=1}^{\alpha=q} \mathbf{c}_\alpha \mathbf{V}^\alpha$

Euler equations associated with the model: Equations for the variables $\mathbf{a}_\alpha$ or equivalently for the $\mathbf{c}_\alpha$ variables

$$\sum_{\delta=1}^{\delta=q} \frac{\partial^2 \mathcal{L}(\mathbf{c}_1,\mathbf{c}_2,\ldots,\mathbf{c}_q)}{\partial \mathbf{c}_\alpha \partial \mathbf{c}_\delta}\frac{\partial \mathbf{c}_\delta}{\partial t} + \sum_{\delta=1}^{\delta=q} \frac{\partial^2 \mathcal{M}(\mathbf{c}_1,\mathbf{c}_2,\ldots,\mathbf{c}_q)}{\partial \mathbf{c}_\alpha \partial \mathbf{c}_\delta}\frac{\partial \mathbf{c}_\delta}{\partial x} = 0 \;, \qquad \alpha = 1, 2, \ldots, q$$

Two problems are present in discrete kinetic theory:

1. **The existence of macrocospic variables other than mass, momentum and energy**

2. **The anisotropic character generally related to the discrete models**

In order to reduce and possibly to eliminate them, multiple collisions are introduced and some symmetry properties on the models are adopted

The multiple collisions

A r – collision is a collision between r particles:

$\vec{u}_{i_1}, \vec{u}_{i_2}, \ldots, \vec{u}_{i_r}$ Before the collision $I_r = (i_1, i_2, \ldots, i_r)$	⟷	$\vec{u}_{j_1}, \vec{u}_{j_2}, \ldots, \vec{u}_{j_r}$ After the collision $J_r = (j_1, j_2, \ldots, j_r)$

Transition probability: $A_{I_r}^{J_r}$

$\delta(k, I_r, J_r)$ is the algebraic number of particles « k » created in the r – collision $I_r \rightarrow J_r$

$\sum_{I_r \, J_r} \delta(k, I_r, J_r) \, A_{I_r}^{J_r} \, N_{i_1} N_{i_2} \ldots N_{i_r}$ is the algebraic number of particles « k » created in all the r – collisions (per unit time)

Kinetic equations with r – collisions (r = 2,3,…,R)

$$\frac{\partial}{\partial t} N_k + \vec{u}_k \cdot \vec{\nabla} N_k = \frac{1}{2} \sum_{r=2,3,\dots,R} \sum_{I_r\, J_r} \delta(k, I_r, J_r) A_{I_r}^{J_r} N_{i_1} N_{i_2} \dots N_{i_r}$$

$k = 1, 2, \dots, p$

$$\frac{\partial}{\partial t}\mathbf{N} + \mathcal{A}\mathbf{N} = \mathcal{C}(\mathbf{N})$$

Summational invariants $\phi = (\varphi_1, \varphi_2, \dots, \varphi_p) \in R^p$

$$A_{I_r}^{J_r} \sum_k \delta(k, I_r, J_r)\, \varphi_k = 0 \qquad \forall\, I_r, J_r, r$$

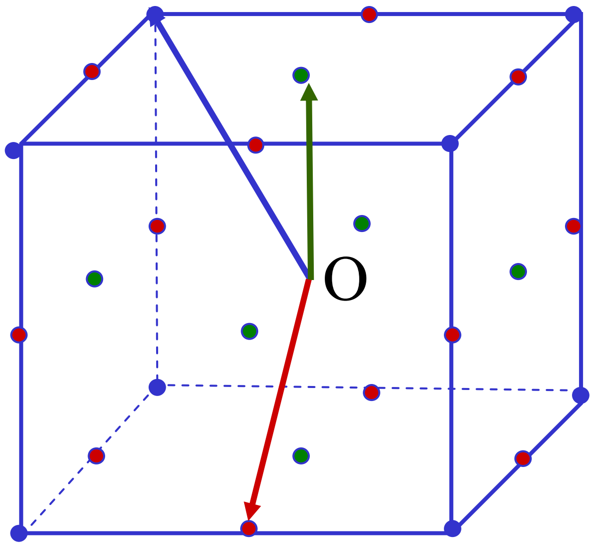
→ Linear subspace $\mathbf{F}$ $(\mathbf{F} \subset R^p)$

Two remarks

- By taking into account multiple collisions, the dimension of $\mathbf{F}$ is decreasing
- By taking into account all the r – collisions, it is possible to find the dimension of $\mathbf{F}$, without explicitly determining all the collisions between the particles (Ph. Chauvat)

Examples: Dimension of **F** is 4 or 5

Spatial models related to the cube



Velocity number

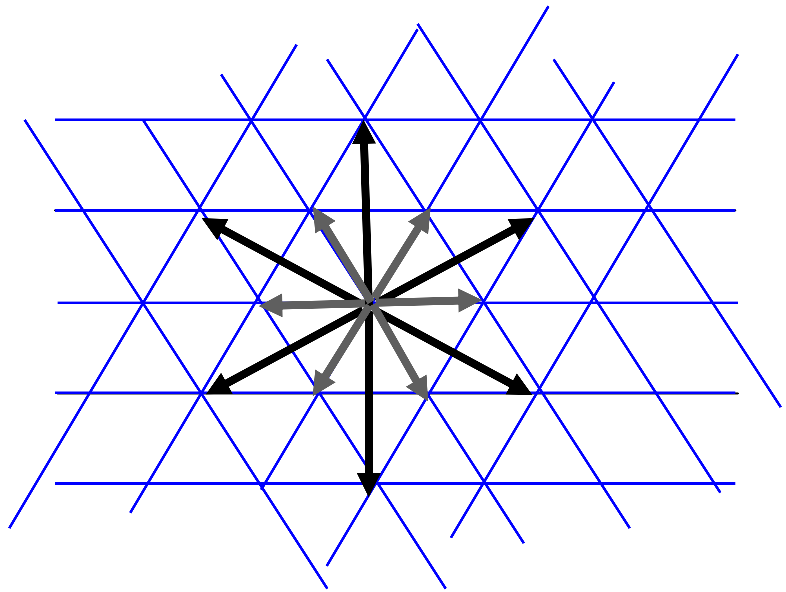
6

8

14

26

Coplanar models related to the hexagonal lattice



Generalizations

$$\vec{u}_k = a_k \vec{I} + b_k \vec{J} + c_k \vec{K} \,,$$

$$(a_k, b_k, c_k) \in Z^3$$

$$\dim \mathbf{F} = 5$$

Chapman – Enskog expansion

$$\frac{\partial}{\partial t}\mathbf{N} + \mathcal{A}\mathbf{N} = \frac{1}{\varepsilon}\mathcal{C}(\mathbf{N},\mathbf{N})$$

$\varepsilon << 1$ (ε Knudsen number)

But: To obtain balance laws for the variables $\mathbf{a}_\alpha$

$$\left\{\begin{array}{l} \dfrac{\partial \mathbf{a}_\alpha}{\partial t} + <\mathcal{A}\mathbf{N}, \mathbf{V}^\alpha> = 0 \;, \qquad \alpha = 1, 2, \ldots, q \\ \text{with } \mathbf{N}=\mathbf{N}(\mathbf{a}_\alpha,\mathbf{b}_\beta) \text{, the variables } \mathbf{b}_\beta \text{ depending on the } \mathbf{a}_\alpha \end{array}\right.$$

Chapman – Enskog expansion

$$\mathbf{N} = \mathbf{N}^{(0)} + \varepsilon\mathbf{N}^{(1)} + \varepsilon^2\mathbf{N}^{(2)} + \cdots$$

$$\mathcal{J}(\mathbf{N}^{(1)}) = \frac{\partial}{\partial t}\mathbf{N}^{(0)} + \mathcal{A}\,\mathbf{N}^{(0)}$$

$\mathcal{J}$: Linearized collision operator

$$\left\{\begin{array}{l} \mathcal{C}(\mathbf{N}^{(0)},\mathbf{N}^{(0)}) = 0 \\ \mathbf{N}^{(0)} \text{ Maxwellian densities} \\ \text{Euler equations for } \mathbf{a}_\alpha \end{array}\right.$$

$$\left\{\begin{array}{l} \mathbf{N}^{(0)} + \varepsilon\mathbf{N}^{(1)} \quad \text{Navier – Stokes equations for } \mathbf{a}_\alpha \end{array}\right.$$

3. Hydrodynamical description for regular discrete models

Regular discrete models

The successful simulations undertaken with the lattice gas method introduced by Frisch, Hasslacher and Pomeau, … have provided a new light on the discrete models of gas

" Quasi – isotropic " models (Chauvat, Coulouvrat, R.G.)

$$\mathcal{U}_\ell = \left\{ \vec{u}_k^{\,\ell} \;,\; \left| \vec{u}_k^{\,\ell} \right| = c_\ell \;,\; k = 1, 2, \ldots, p_\ell \right\}$$

$$\mathcal{U} = \left\{ \vec{u}_k \;,\; k = 1, 2, \ldots, p \right\} = \cup_{\ell=1}^{\ell=L} \mathcal{U}_\ell$$

Mean properties

- $\mathcal{G}$: Isometry group in R^D
- $g\,(\mathcal{U}) = \mathcal{U} \quad \forall\, g \in \mathcal{G}$
- $\dim\ F = D + 2$ (The multiple collisions are introduced)

Examples: Coplanar models related to the hexagonal lattice,
Spatial models related to the cubic lattice

Hydrodynamical description of the gas

Maxwellian state $$N_k^{(0)} = \exp\left(\alpha + \vec{\beta}\cdot\vec{u}_k + \gamma\left(\vec{u}_k^2 - a^2\right)\right)$$

$$n = \sum_k N_k^{(0)} , \qquad n\,\vec{u} = \sum_k N_k^{(0)}\,\vec{u}_k , \qquad n\,e = \frac{1}{2}\sum_k N_k^{(0)}\,\vec{u}_k^2$$

$$\alpha , \vec{\beta} , \gamma \Leftrightarrow n , \vec{u} , e \qquad \text{Bijection}$$

Homogeneous Maxwellian state $$N_k^{(0)} = \frac{n}{p} \quad \left(\vec{u} = 0 , e = \frac{1}{2} a^2\right)$$

Quasi - homogeneous Maxwellian state $$\Delta e / a^2 << 1 , \quad |\vec{u}| / a << 1$$

$$N_k^{(0)} = \frac{n}{p}\left\{1 + \frac{D}{a^2}\vec{u}.\vec{u}_k + \frac{2\left(\vec{u}_k^2 - a^2\right)}{a_2^4 - a^4}\left(e - \frac{a^2}{2}\right) + \frac{D^2}{2a^4}\left(\vec{u}_k\,\vec{u}_k - \frac{\vec{u}_k^2}{D}\,\vec{\vec{I}}\right) : \vec{u}\,\vec{u}\right.$$
$$\left. + \frac{2D^2}{a^2}\left(\frac{\vec{u}_k^2 - a^2}{a_2^4 - a^4} - \frac{1}{a^2}\right)\left(e - \frac{a^2}{2}\right)(\vec{u}.\vec{u}_k)\right\}$$

Notations: $$a_r = \left(\frac{1}{p}\sum_k |\vec{u}_k|^{2r}\right)^{1/2r} \qquad a_1 = \left(\frac{1}{p}\sum_k |\vec{u}_k|^2\right)^{1/2} \equiv a$$

Euler equations associated with the model

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0 \\ \dfrac{\partial(\rho\vec{u})}{\partial t} + \eta\,\vec{\nabla}\cdot(\rho\vec{u}\,\vec{u}) + \vec{\nabla}\varpi = (\eta-1)\vec{\nabla}\left(\rho\dfrac{\vec{u}^2}{D}\right) + \vec{\nabla}\cdot\left(\rho\vec{\vec{M}}_{an}\right) \\ \dfrac{\partial}{\partial t}\left(\rho e + \rho\dfrac{\vec{u}^2}{2}\right) + \eta\,\vec{\nabla}\cdot\left(\left(\rho e + \rho\dfrac{\vec{u}^2}{2} + \varpi\right)\vec{u}\right) = \varphi\,\vec{\nabla}\cdot\left(\rho\left(e - \rho\dfrac{a^2}{2}\right)\vec{u}\right) \end{cases}$$

$$\eta = \frac{a_2^4}{a^4}\,\frac{D}{D+2}$$

$$\varphi = \frac{2a_2^8 - a^4 a_2^4 - a^2 a_3^6}{a^4\left(a_2^4 - a^4\right)}$$

η and φ depend on the discrete models

Continuous fluids: $\eta = 1$ $\varphi = 0$

Remarks: We can provide equivalent forms of the Euler equations; for example with a pressure tensor not necessarily spherical or a vector heat flux not necessarily zero

Navier -Stokes equations associated with the model

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\,\vec{u}) = 0 \\[2ex] \dfrac{\partial(\rho\,\vec{u})}{\partial t} + \eta\,\vec{\nabla}\cdot(\rho\,\vec{u}\,\vec{u}) + \vec{\nabla}\varpi = (\eta-1)\,\vec{\nabla}\left(\rho\dfrac{\vec{u}^{\,2}}{D}\right) + \vec{\nabla}\cdot\left(\rho\,\vec{\vec{M}}_{an}\right) \\[1ex] \qquad + \vec{\nabla}\cdot\left(2\mu\,\vec{\vec{D}} + \lambda\,\mathrm{Tr}(\vec{\vec{D}})\,\vec{\vec{I}}_D\right) + \vec{\nabla}\cdot\vec{\vec{B}}_{an} \\[2ex] \dfrac{\partial}{\partial t}\left(\rho e + \rho\dfrac{\vec{u}^{\,2}}{2}\right) + \eta\,\vec{\nabla}\cdot\left(\left(\rho e + \rho\dfrac{\vec{u}^{\,2}}{2} + \varpi\right)\vec{u}\right) = \varphi\,\vec{\nabla}\cdot\left(\rho\left(e - \rho\dfrac{a^2}{2}\right)\vec{u}\right) \\[1ex] \qquad + \vec{\nabla}\cdot\left(\kappa\,\vec{\nabla} e\right) \end{cases}$$

$$\mu \cong \frac{\rho}{(D+2)\,a^2}\,\frac{1}{L}\sum_{\ell=1,2,\dots,L} \beta_\ell\, a_\ell^4\ , \quad 2\mu + D\lambda = 0\ , \quad \kappa \cong \frac{\rho}{D\,a^2}\,\frac{1}{L}\sum_{\ell=1,2,\dots,L} \delta_\ell\, a_\ell^4$$

Remarks: $\sum_k \vec{u}_k\vec{u}_k\vec{u}_k\vec{u}_k$ isotropic $\longrightarrow$ $\vec{\vec{M}}_{an} = 0 \qquad \vec{\vec{B}}_{an} = 0$

Numerical results for the transport coefficients

Model	$\bar{\mu}$	$\bar{\kappa}$	*Pr*
I	0.15	0.24	0.62
II	0.24	0.33	0.73
III	0.22	0.24	0.92
IV	0.25	0.41	0.61
Chauvat (1989)*	0.14	0.50	0.28
Chahine (1967)**	0.14	0.56	0.25

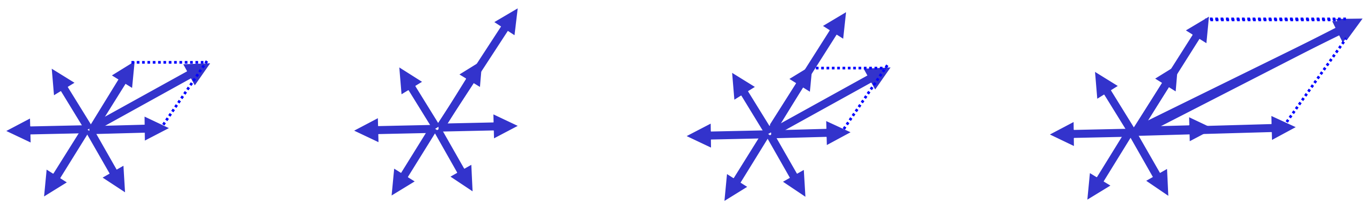
$$\mu = \frac{ma}{S\sqrt{2}}\bar{\mu}$$

$$\kappa = \frac{ma}{S\sqrt{2}}\bar{\kappa}$$

$$Pr = \frac{\mu}{\kappa}$$

* model I ($\eta = 0.67$, $\varphi = 0$

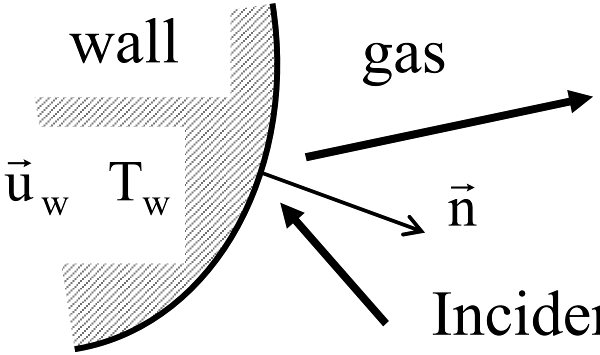
** coplanar continuous model



Models: I (12) II (12) III (18) IV (18)

4. Boundary Conditions

Boundary conditions on an impermeable wall (R.G., 1975)



Emerging particles $\vec{u}_r$ $(r \in R)$

Incident particles $\vec{u}_i$ $(i \in I)$

$$\left|(\vec{u}_r - \vec{u}_w)\cdot\vec{n}\right| N_r = \sum_{i \in I} B_{ir} \left|(\vec{u}_i - \vec{u}_w)\cdot\vec{n}\right| N_i \qquad \forall\, r \in R$$

B_{ir} : probability for a particle of velocity $\vec{u}_i$ impinging the wall, to be reflected with the velocity $\vec{u}_r$

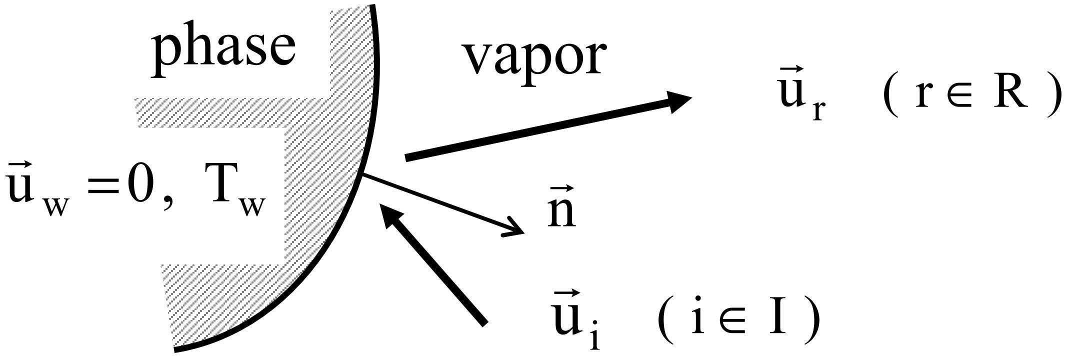
H – Theorem in a vessel

Particular case of the diffuse reflexion: $N_r = \lambda N_{rw}$

The densities N_{rw} are Maxwellian densities associated with the macroscopic variables of the wall $n_w = 1, \vec{u}_w, T_w, \ldots$

λ (as n) is unknown; λ is known when the problem is solved

Boundary conditions on an interface



Gas in Maxwellian equilibrium with the condensed phase

$$N_{kw} = \exp\left(\alpha + \vec{\beta}\cdot\vec{u}_k + \gamma\left|\vec{u}_k\right|^2\right)$$

$$\sum_k N_{kw} = 1 , \qquad \sum_k N_{kw}\,\vec{u}_k = 0 , \qquad \frac{1}{2}\sum_k N_k\,(\vec{u}_k - \vec{u})^2 = \frac{3}{2}\,\frac{k\,T_w}{m}$$

Boundary conditions for the vapor $\quad N_r = n_{sat}\, N_{rw} \;, \quad \forall r \in R$

n_{sat} : saturation density of the vapor at the temperature T_w

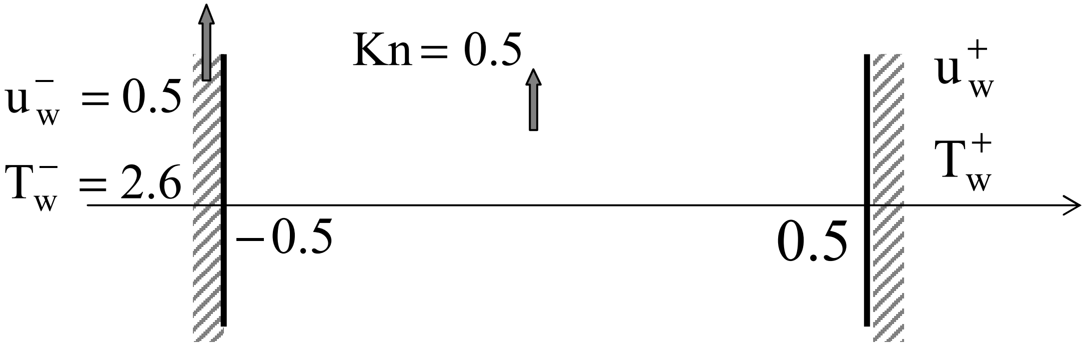
Remark: This boundary condition is valid only when the models are symmetrical about the normal $\vec{n}$. This condition is similar to that of the continuous kinetic theory

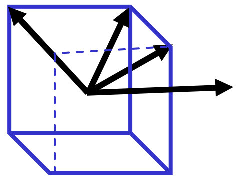
5. Applications

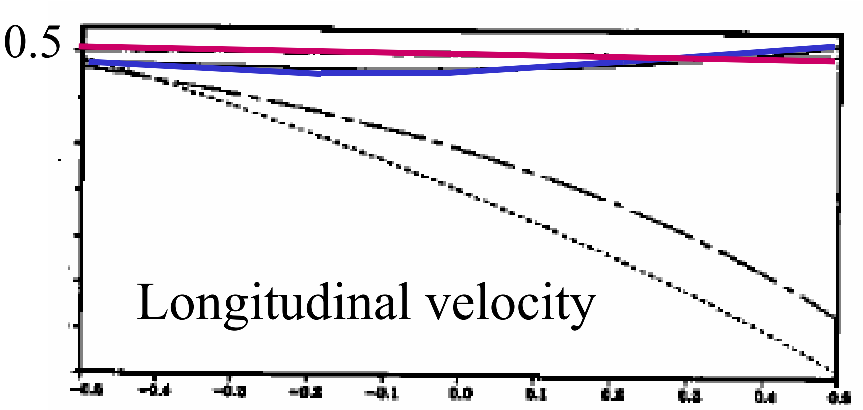
Applications

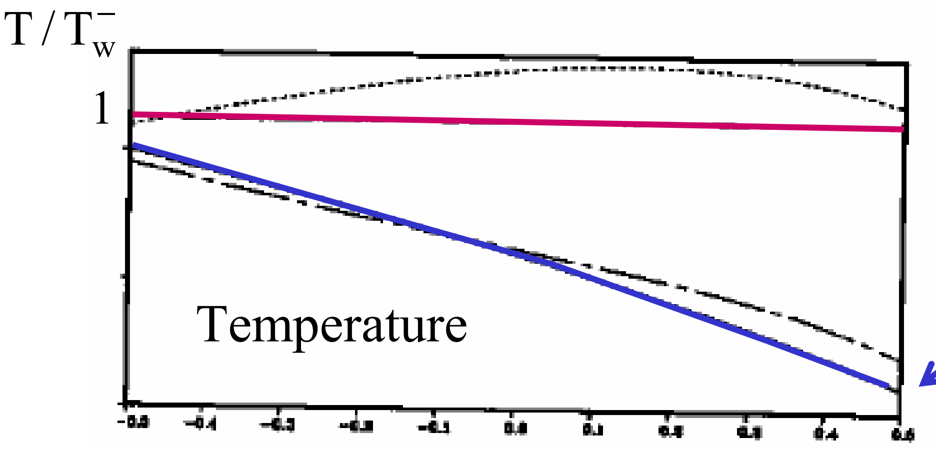
- **Shock wave structure**
- **Unsteady and steady Couette flows (Knudsen layer, initial layer, …)**
- **Flow and heat transfer between two parallel plates**
- **Evaporation / condensation between two interfacex (temperature inversion)**
- **Evaporation or condensation on a liquid interface**
- **Flow in a microchannel**
- **…**

Flow and heat transfer between two parallel plates (d'Almeida)









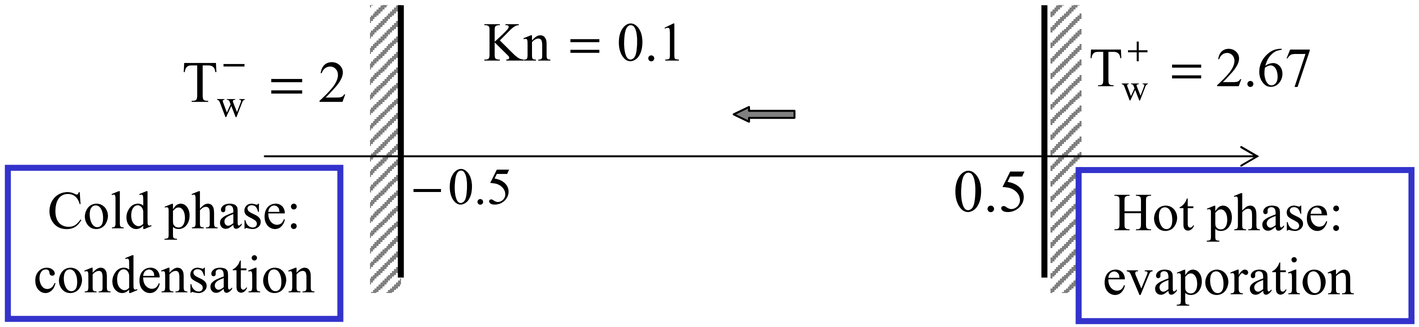
Red line: test case

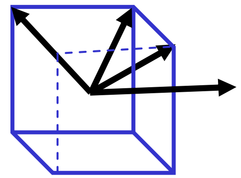
$T_w^+ = 2.6$, $u_w^+ = 0.5$

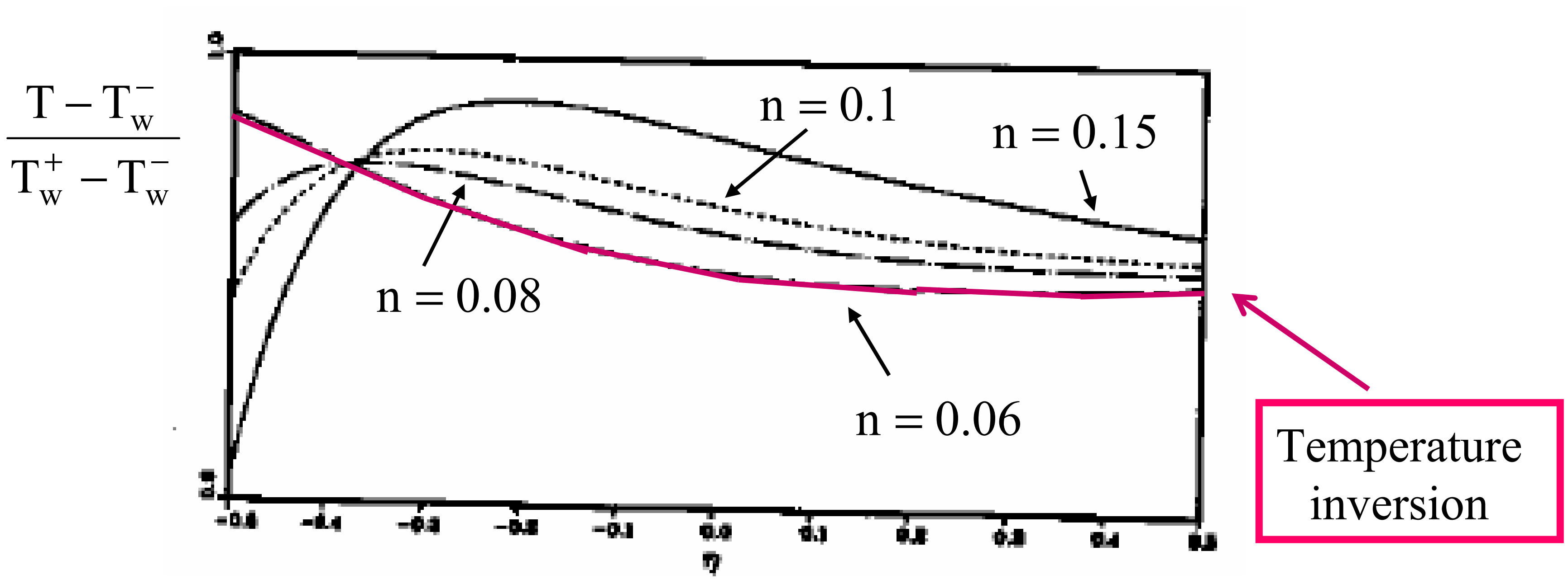
Blue line: Thermophoresis phenomenon

$T_w^+ = 2$, $u_w^+ = 0.2$

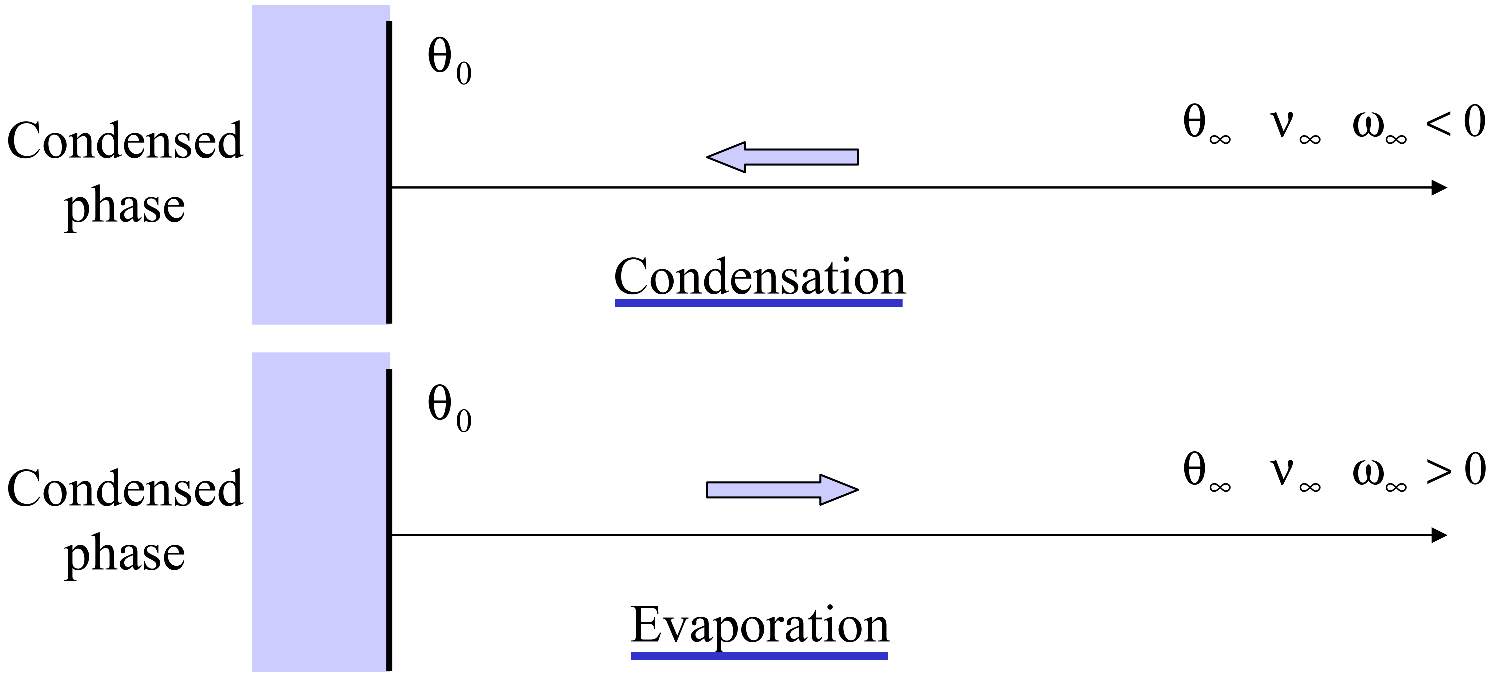
Evaporation / condensation between two interfaces (d'Almeida)







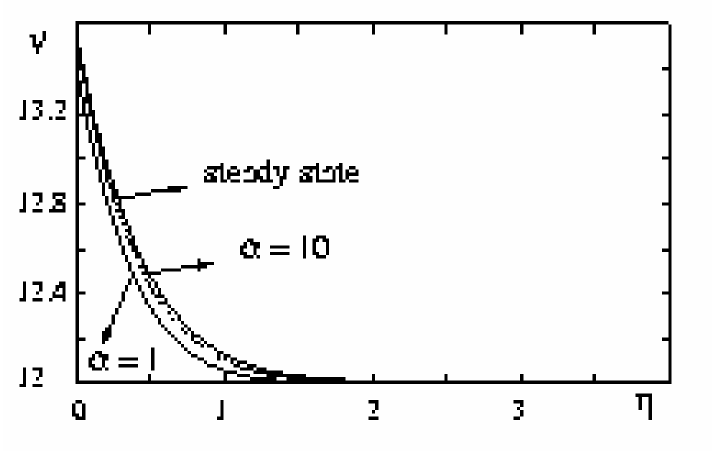
Evaporation or condensation on a liquid interface (Nicodin)



These problems depend on 3 parameters : $\dfrac{\theta_\infty}{\theta_0}$, ν_∞ , $M_\infty = \dfrac{|\omega_\infty|}{\sqrt{\theta_\infty}} > 0$

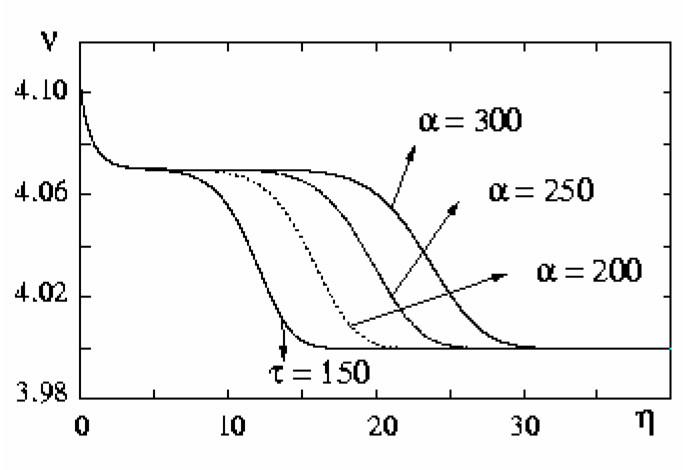
The results obtained with a very simple model (16 velocities only) are in very good agreement with those of Sone, Aoki and their collaborators with continuous theory

Condensation problem (Nicodin)



$$\frac{\theta_\infty}{\theta_0} = 1\,,\quad \nu_\infty = 12\,,\; M_\infty = 1.414$$

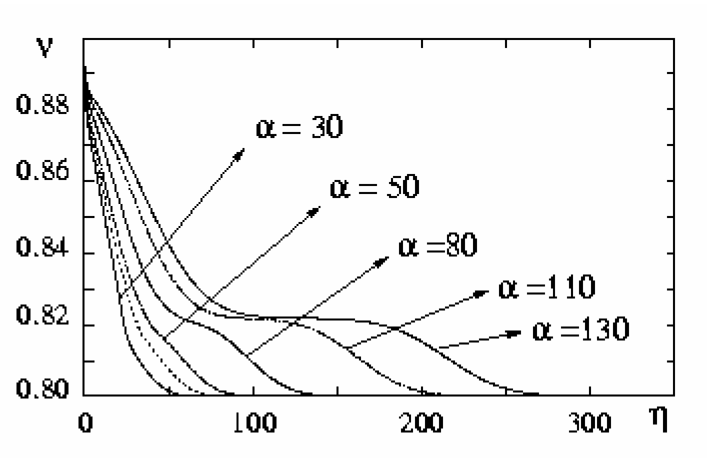
The vapor is compressed on the condensed phase ($\eta = 0$). The steady state is obtained for about 30 mean free times



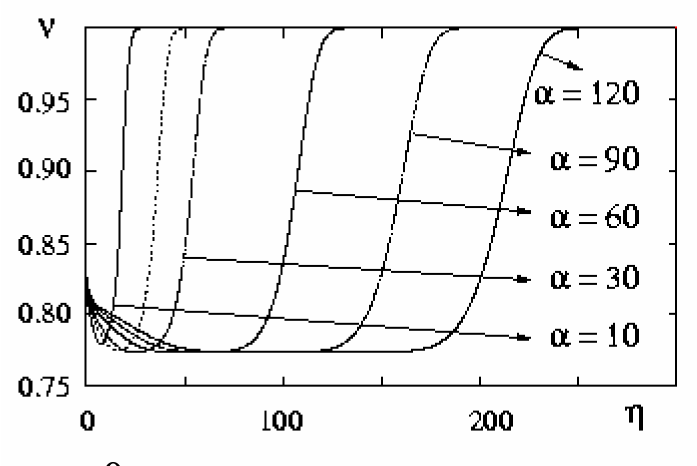
$$\frac{\theta_\infty}{\theta_0} = 1,\quad \nu_\infty = 4\,,\; M_\infty = 0.707$$

There is a Knudsen layer near the condensed phase ($\eta = 0$). A compression wave (shock wave) propagates to infinity

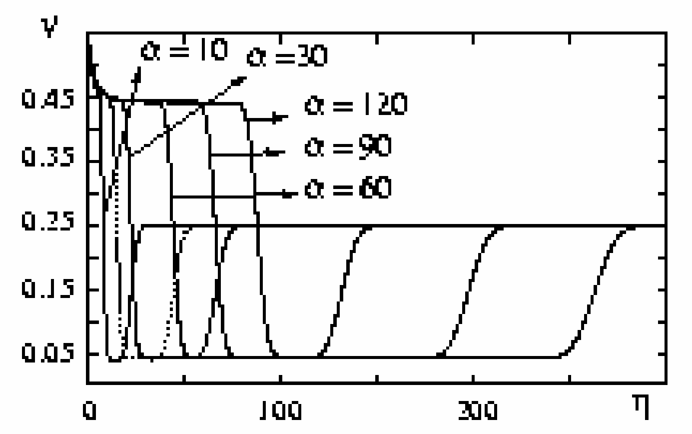
Evaporation problem (Nicodin)



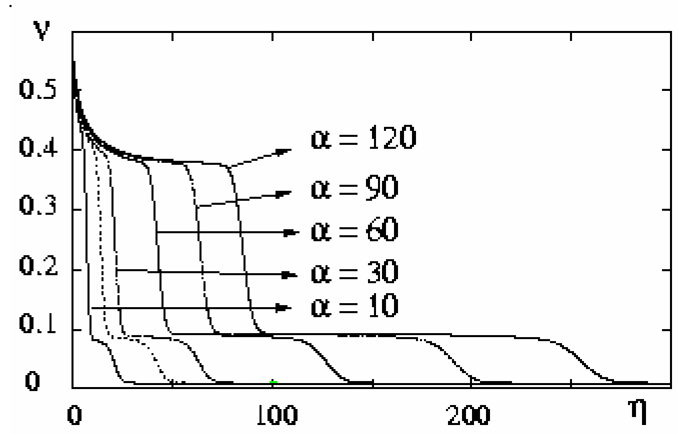
$$\frac{\theta_\infty}{\theta_0} = 1\,,\quad \nu_\infty = 0.8\,,\ M_\infty = 0.058$$



$$\frac{\theta_\infty}{\theta_0} = 1\,,\quad \nu_\infty = 1\,,\ M_\infty = 0.316$$



$$\frac{\theta_\infty}{\theta_0} = 1\,,\quad \nu_\infty = 0.01\,,\ M_\infty = 1$$



$$\frac{\theta_\infty}{\theta_0} = 1\,,\quad \nu_\infty = 0.25\,,\ M_\infty = 1.73$$

6. Conclusion

Conclusion

Many generalizations

- **Gas mixtures (Cercignani, Cornille, …)**
- **Chemical reactions (Pandolfi, …)**
- **Numerical approaches (Leguillon, Teman, Golstein, …)**
- **Semi discrete Boltzmann equation (Cabannes, Toscani, …)**
- **…**

Many mathematical papers

- **Existence theorems, exact solutions, asymptotic analysis, …**
- **Cabannes, Bardos, Beale, Bellomo, Bobylev, Bony, Cercignani, Cornille, Godunov, Golse, Hamdache, Illner, Kawashimha, Levermore, Nishida, Platkowski, Sultangazin, Tartar, Vedenyapin, …**

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Yassine Benbouali (DEA, 2003)
………

Thank you for your attention