Consistent two-relaxation-times LBE model for porous flow and transport

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With the symmetry argument in hand, the minimal two-relaxation-times (TRT) model is introduced for flow and transport in porous media. The model is then extended to the highly non-linear anisotropic advection-diffusion problems, such as the Richard's equation for variably saturated Darcy's flow. This and several other examples of complex flows indicate the suitability of the model for high values of the transport coefficients, which also accelerate the convergence toward a steady state. The study is then focused on the consistency of the bulk solutions and boundary schemes for an arbitrary choice of the transport coefficients, starting from the Stokes and Brinkman equations.

Our study undoubtedly shows that the LBE schemes need at least one free collision relaxation rate to avoid a non-linear dependency of the truncated errors on the selected transport coefficients. Then the TRT operator is the minimal collision configuration which allows to fully control the numerical *steady* solutions by the non-dimensional physical numbers. The analysis is based on the directional finite-difference recurrence equations of the TRT scheme, recently introduced. Then the *optimal* OTRT sub-class is derived. It guarantees *the same stable velocity for any Peclet number*, with or without numerical diffusion, even for the negative equilibrium distributions. An adequate choice of the TRT free parameter enables the OTRT to gain in efficiency against forward time central finite-difference schemes and in efficiency/consistency/accuracy against the most popular LBGK model. However, unsuitable choices may result in unstable and/or inaccurate schemes.

References

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