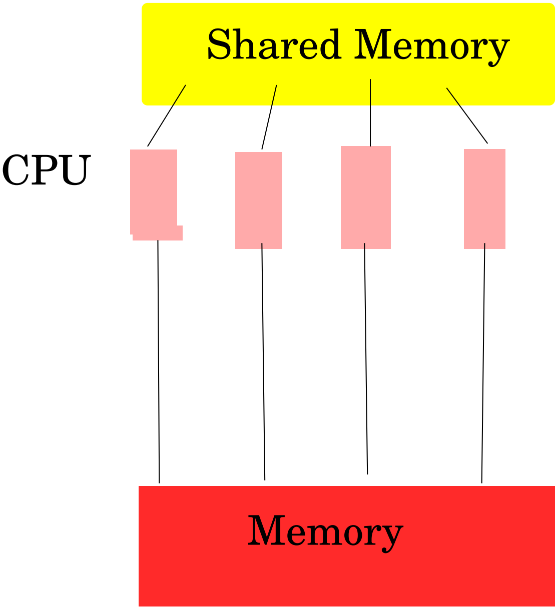
GPU
the "poor man's" computer for Lattice Boltzmann Simulations

Plan
Description of GPU
Programming GPU
Some booby traps...
Examples of simulations
Remarks about “accuracy” of LBS models.

GPU

The generation of images (mostly games...) from limited information requires a lot of simple computations. Instead of a complicated CPU + Large cache, the same silicon "real estate" is used for a large number of simplified CPU-units and no cache.


Shared Memory
CPU
Memory

Typical Properties
Memory 1 Gbyte (up to 4 Gbytes)
organized in banks : it takes roughly 400 cycles to "recover" after a read !!
CPU 128 (256 or somewhat more)
usually work with 32 bits words (more recent cards have 64 bits arithmetic, apparently one such processor for every 8 CPU)
Shared Memory 16384 bytes !!

Programming

NVIDIA has developped an extension of gcc (nvcc)

one adds at the beginning

__host__ $\rightsquigarrow$ operates on host

__global__ $\rightsquigarrow$ operates on GPU

Prepare a c-program for the host, test it !

Transfer all necessary data in the GPU

Replace for GPU do-loops by thread indices that are generated by the GPU

ex. for($j = 0; j < Nx; j++$) for CPU

j=threadIdx.x; thread_Index in the GPU

After computing in GPU, transfer back to CPU the desired results.

Main difficulty
Master the system of threads.
They are organized in a hierarchical way :
Block of threads that are organized in a grid of such blocks. One has control over the size of blocks and of the size of grid, but the order in which threads are executed is not reproducible, so don't try to be too smart to link various steps in a computation.
The access to the main memory must avoid operation like
f[(j+1)%N]=f[j]
Use the shared__memory as much as possible, however it is very small (back to the old days....).

Typical performance
D3Q19 in full “d’Humières” MRT algorithm,
on 9800 GT approximately 6 nsec/LUPS limited to 196^3
on Tesla C870 approx 4 nsec/LUPS 256^3
on Tesla C1060 approx 2 nsec/LUPS a little more

Some Results for simple 2-D Models
D2Q13
Velocities
0, 1, 0,-1, 0, 2, 1,-1,-2,-2,-1, 1, 2
0, 0, 1, 0,-1, 1, 2, 2, 1,-1,-2,-2,-1
Euler

$$\begin{vmatrix} \partial_t & \partial_r & 0 \\ \frac{\alpha+44}{26}\partial_r & \partial_t & 0 \\ 0 & 0 & \partial_t \end{vmatrix} \tag{1}$$

Navier

$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & -S\frac{1021+156c_1-22\alpha}{572}\partial_r^2 & 0 \\ 0 & 0 & -S\frac{51+4c_1}{44}\partial_r^2 \end{vmatrix} \tag{2}$$

Correction to shear viscosity from equivalent equations at order 4 in space derivatives.

For wavevector : $k\{l, m\}$

$$
\begin{aligned}
&-\tfrac{3}{4}\alpha(36S^2-7)ml^5\\
&+\tfrac{1}{66}(6S^2-1)(16c_1+61)l^4\\
&+\tfrac{3}{4}\alpha(36S^2-7)m(1-m^2)l^3\\
&-\tfrac{1}{66}(6S^2-1)(16c_1+61)l^2\\
&+\tfrac{1}{23232}(557+7680c_1S^2+12264S^2-48c_1^2+384c_1^2S^2-872c_1)
\end{aligned}
$$

D2Q16

Velocities

2, 1,-1,-2,-2,-1, 1, 2, 3, 1,-1,-3,-3,-1, 1, 3

1, 2, 2, 1,-1,-2,-2,-1, 1, 3, 3, 1,-1,-3,-3,-1

Euler

$$\begin{vmatrix} \partial_t & \partial_r & 0 \\ \frac{5(\alpha+3)}{4}\partial_r & \partial_t & 0 \\ 0 & 0 & \partial_t \end{vmatrix} \tag{3}$$

Navier

$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & -S\frac{5(2+c_1-\alpha)}{4}\partial_r^2 & 0 \\ 0 & 0 & -S\frac{5(5+c_1)}{12}\partial_r^2 \end{vmatrix} \tag{4}$$

Correction to shear viscosity from equivalent equations at order 4 in space derivatives.

For wavevector : $k\{l, m\}$

$$-\frac{1}{54}(-1+6S^2)(91+133c_1+96c_3)l^4$$
$$+\frac{1}{54}(-1+6S^2)(91+133c_1+96c_3)l^2$$
$$+\frac{1}{1728}(1189-2304S^2c_3-2592c_1S^2$$
$$+600S^2c_1^2+682c_1+384c_3-75c_1^2-3384S^2)$$

D2Q25

Velocities

0, 1, 0,-1, 0, 1,-1,-1, 1, 2, 1,-1,-2,-2,-1, 1, 2, 3, 1,-1,-3,-3,-1, 1, 3

0, 0, 1, 0,-1, 1, 1,-1,-1, 1, 2, 2, 1,-1,-2,-2,-1, 1, 3, 3, 1,-1,-3,-3,-1

Euler

$$\begin{vmatrix} \partial_t & \partial_r & 0 \\ \frac{\alpha+132}{50}\partial_r & \partial_t & 0 \\ 0 & 0 & \partial_t \end{vmatrix} \tag{5}$$

Navier

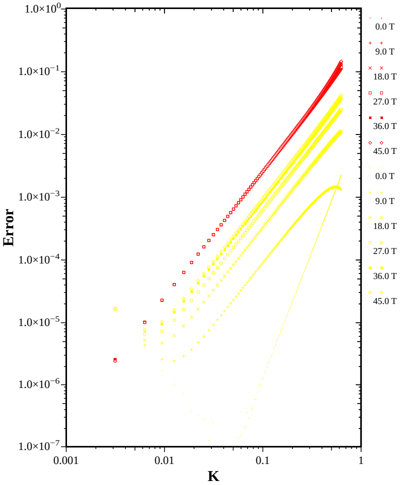
$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & -S\frac{3471+75c_1-22\alpha}{1100}\partial_r^2 & 0 \\ 0 & 0 & -S\frac{85+c_1}{44}\partial_r^2 \end{vmatrix} \tag{6}$$

Correction to shear viscosity from equivalent equations at order 4 in space derivatives.

For wavevector : $k\{l, m\}$

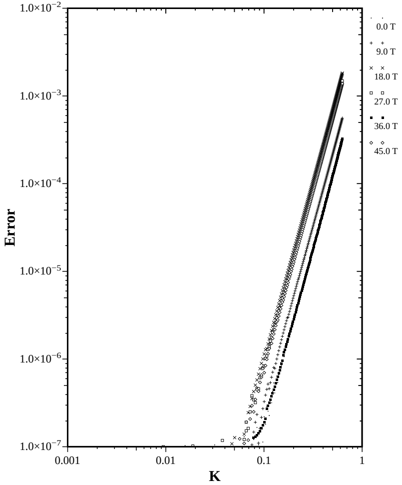
$$
\frac{1}{34914}(6S^2-1)(2755c_1+44c_2+85169)l^4
-\frac{1}{34914}(6S^2-1)(2755c_1+44c_2+85169)l^2
+\frac{1}{23232}(4373+1968c_1S^2+24c_1^2S^2-3c_1^2+17112S^2-158c_1)
$$

Error



Error
1.0×10^0
1.0×10^−1
1.0×10^−2
1.0×10^−3
1.0×10^−4
1.0×10^−5
1.0×10^−6
1.0×10^−7
0.001
0.01
0.1
1
K
0.0 T
9.0 T
18.0 T
27.0 T
36.0 T
45.0 T
0.0 T
9.0 T
18.0 T
27.0 T
36.0 T
45.0 T

D2Q25



1.0×10⁻²
1.0×10⁻³
1.0×10⁻⁴
1.0×10⁻⁵
1.0×10⁻⁶
1.0×10⁻⁷
Error
0.001
0.01
0.1
1
K
0.0 T
9.0 T
18.0 T
27.0 T
36.0 T
45.0 T

D2Q17

Velocities

0, 1, 0,-1, 0, 1,-1,-1, 1, 2, 0,-2, 0, 2,-2,-2, 2

0, 0, 1, 0,-1, 1, 1,-1,-1, 0, 2, 0,-2, 2, 2,-2,-2

Euler

$$\begin{vmatrix} \partial_t & \partial_r & 0 \\ \frac{\alpha+60}{34}\partial_r & \partial_t & 0 \\ 0 & 0 & \partial_t \end{vmatrix} \tag{7}$$

Navier

$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & -S\frac{169+17c_1-2\alpha}{68}\partial_r^2 & 0 \\ 0 & 0 & -S\frac{17+c_1}{12}\partial_r^2 \end{vmatrix} \tag{8}$$

Correction to shear viscosity from equivalent equations at order 4 in space derivatives.

For wavevector : $k\{l, m\}$

$$-\frac{S}{18}(6S^2-1)(7c_1+12c_2+57)l^4$$
$$+\frac{S}{18}(6S^2-1)(7c_1+12c_2+57)l^2$$
$$+\frac{S}{2880}(2035-5c_1^2+174c_1+384c_2$$
$$+S^2(-9320+40c_1^2-704c_1^2-2304c_2))$$

D2Q17 with 4 conservations

Velocities

0, 1, 0,-1, 0, 1,-1,-1, 1, 2, 0,-2, 0, 2,-2,-2, 2

0, 0, 1, 0,-1, 1, 1,-1,-1, 0, 2, 0,-2, 2, 2,-2,-2

Euler

$$\begin{vmatrix} \partial_t & \partial_r & 0 & 0 \\ \frac{30}{17}\partial_r & \partial_t & \frac{1}{34}\partial_r & 0 \\ 0 & \frac{109+17c_1}{3}\partial_r & \partial_t & 0 \\ 0 & 0 & 0 & \partial_t \end{vmatrix} \tag{9}$$

Navier

$$\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & -S\frac{17+c_1}{12}\partial_r^2 & 0 & 0 \\ T\frac{1090c_1+\beta_\rho}{109} & 0 & -T\frac{1054-109c_1-102\beta_e}{654}\partial_r^2 & 0 \\ 0 & 0 & 0 & -S\frac{17+c_1}{12}\partial_r^2 \end{vmatrix} \tag{10}$$

Viscosity
$\nu = \frac{17+c_1}{12}(\frac{1}{s_{jx}} - \frac{1}{2})$
Diffusivity
$\kappa = 3\frac{620+60\beta_e-\beta_\rho}{109(17+c_1)}(\frac{1}{s_{xx}} - \frac{1}{2})$
Sound
$\frac{\sqrt{102+6c_1}}{6}$
$\gamma = \frac{1}{2}(\nu + \kappa) + \frac{1054-109c_1+102\beta_e}{1308}(\frac{1}{s_{xx}} - \frac{1}{2})$

PLOT