Dec 05
19

ONERA

Optimal hp Discontinuous Galerkin Method Applications for Computational Aeroacoustics

Christophe Peyret and Philippe Delorme
ONERA/DSNA - 29 ave de la Division Leclerc
F92322 Châtillon cedex France

delorme@onera.fr
peyret@onera.fr

Introduction

Why Develop Methods based on DGM to Compute Euler's linearized equations ?

FEM faces difficulties to solve Linearized Euler's equations
FDM faces difficulties with complex geometries and boudary conditions

DGM Advantages and Disavantages:

Advantages	Disavantages
High Flexiblility Complex Geometries Variational Formulation Adapted to Parallel Computation	Harder to program (OOP required) CPU and RAM Expensive !

Physical Modeling and Mathematical formulation

Linearized Euler's Equations

$$\boldsymbol{\varphi} = \begin{pmatrix} u_1 \\ v_1 \\ a_0\, \rho_1/\rho_0 \end{pmatrix}$$

Symetric Friedrich System

$$\partial_t\, \boldsymbol{\varphi} + \mathcal{A}_i\, \partial_i\, \boldsymbol{\varphi} + \mathcal{B}\, \boldsymbol{\varphi} = \mathbf{0}$$

Matrix $\mathcal{A}_i\, \partial_i$ is symetric

Variational formulation

$$\left\{ \boldsymbol{\varphi}_h \in W^k(\omega_h) \,|\, \forall \boldsymbol{\psi}_h \in W^k(\omega_h)\,;\; \mathcal{L}(\boldsymbol{\varphi}_h, \boldsymbol{\psi}_h) = 0 \right\}$$

$\mathcal{A}_i n_i$ is diagonalizable

$$\mathcal{A}_i n_i = [\mathcal{A}_i n_i]^+ + [\mathcal{A}_i . n_i]^-$$

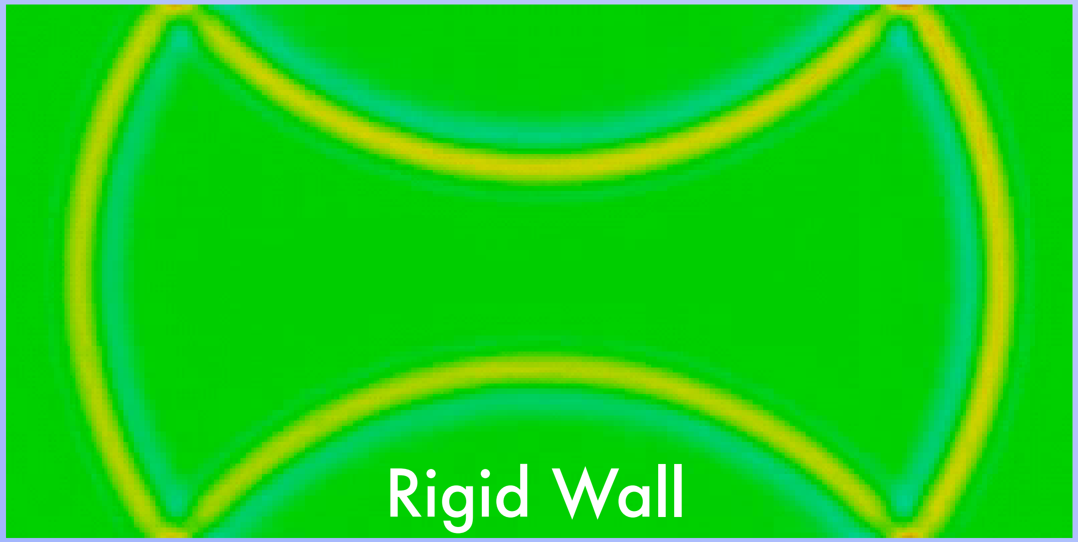
$$\mathcal{L}(\boldsymbol{\varphi}_h, \boldsymbol{\psi}_h) = \int_\Omega \boldsymbol{\psi}_h \,.\, \partial_t\, \boldsymbol{\varphi}_h + \int_\Omega \boldsymbol{\psi}_h \,.\, \mathcal{A}_i\, \partial_i\, \boldsymbol{\varphi}_h + \int_\Omega \boldsymbol{\psi}_h \,.\, \mathcal{B}\boldsymbol{\varphi}_h$$

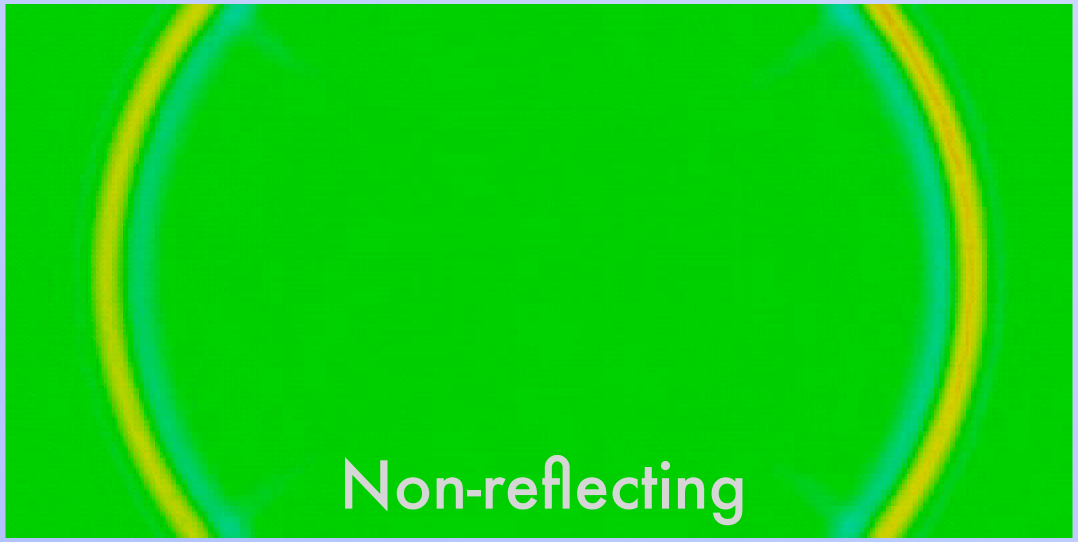
$$+ \oint_{\partial\omega_h | \partial\Omega} \boldsymbol{\psi}_h \,.\, [\mathcal{A}_i n_i]^- (\boldsymbol{\varphi}_h^o - \boldsymbol{\varphi}_h^i) + \oint_{\partial\omega_h \cap \partial\Omega} \boldsymbol{\psi}_h \,.\, (\mathcal{M}\boldsymbol{\varphi}_h - \boldsymbol{g}) - \int_\Omega \boldsymbol{\psi}_h \,.\, \boldsymbol{g}$$

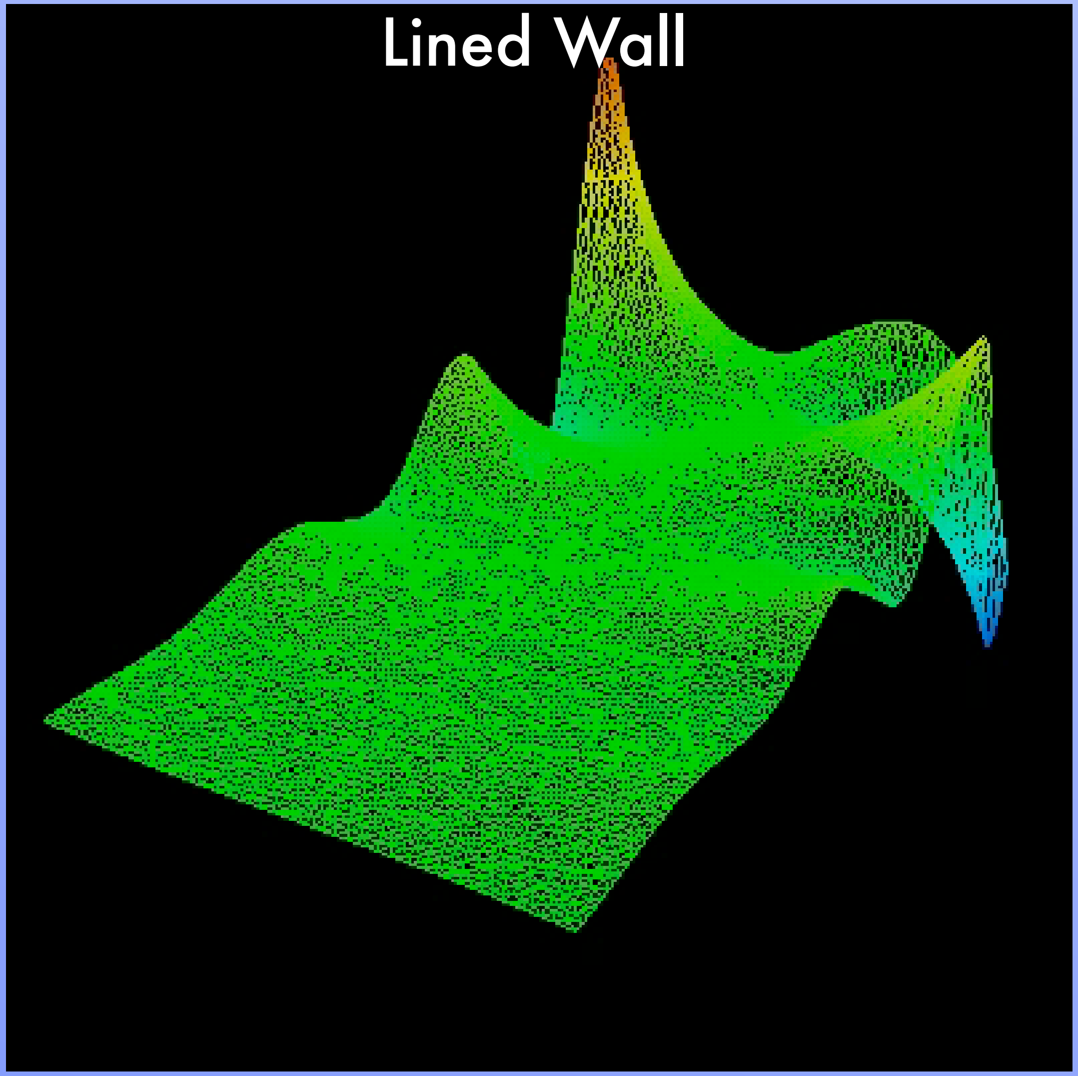
Fully Upwind Scheme

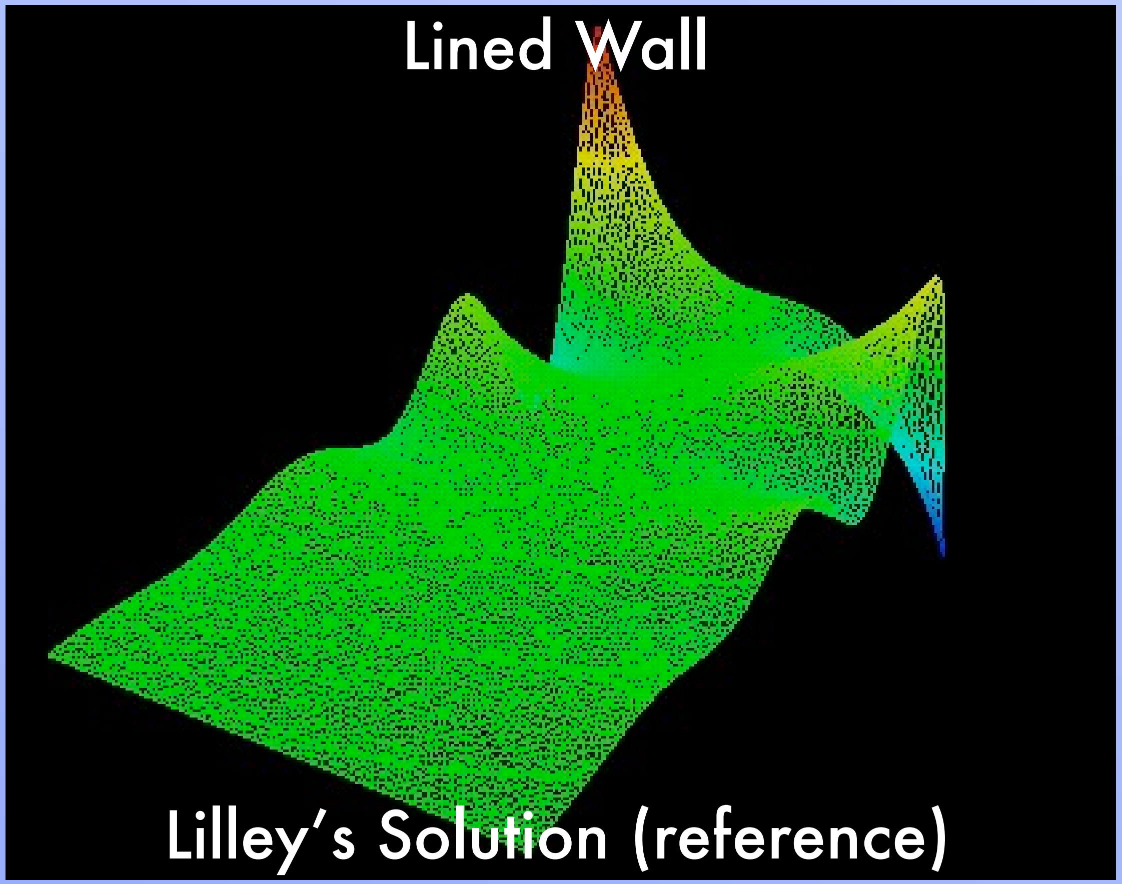
ONERA

Boundary Conditions

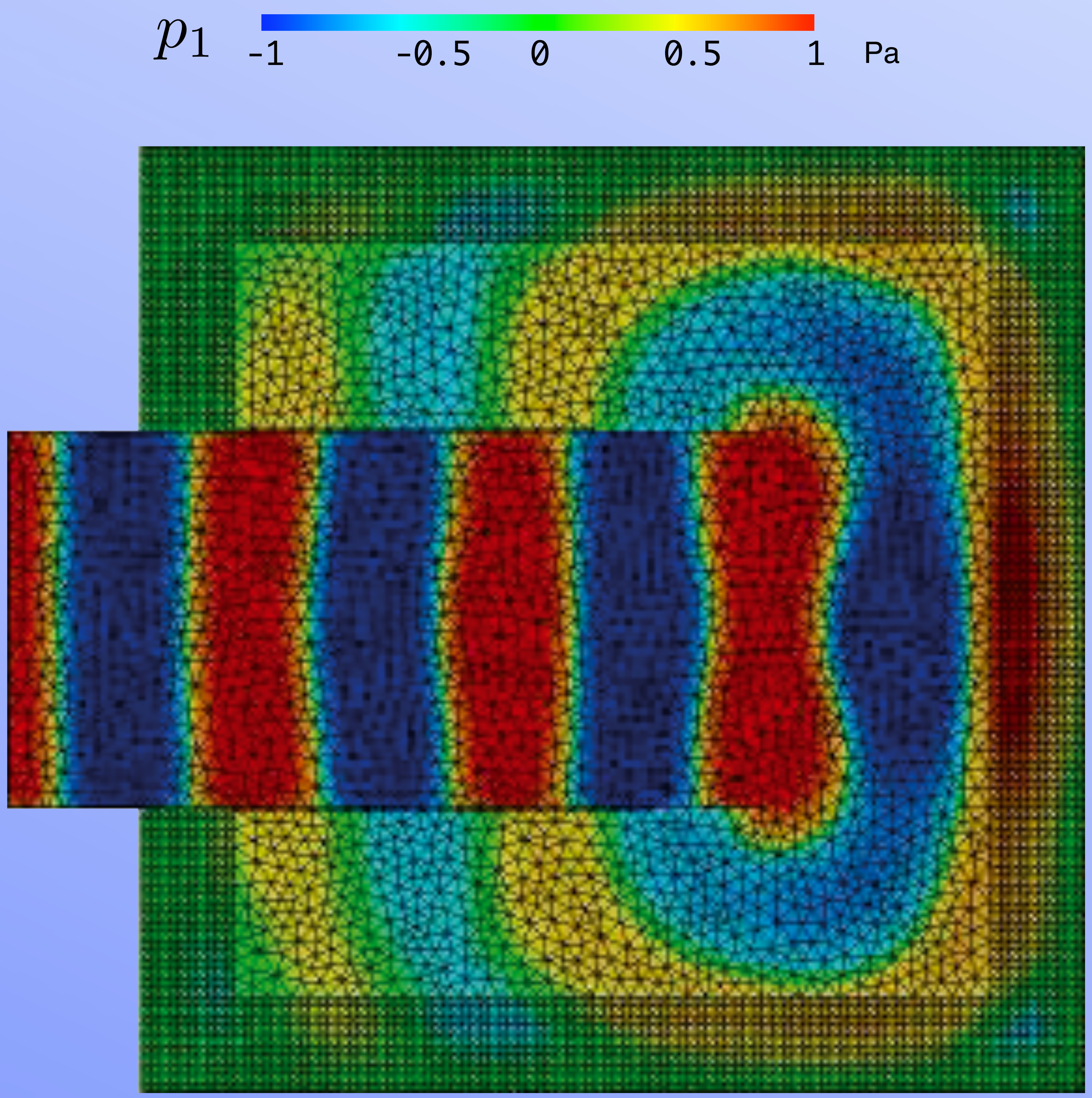








Perfectly Matched Layers

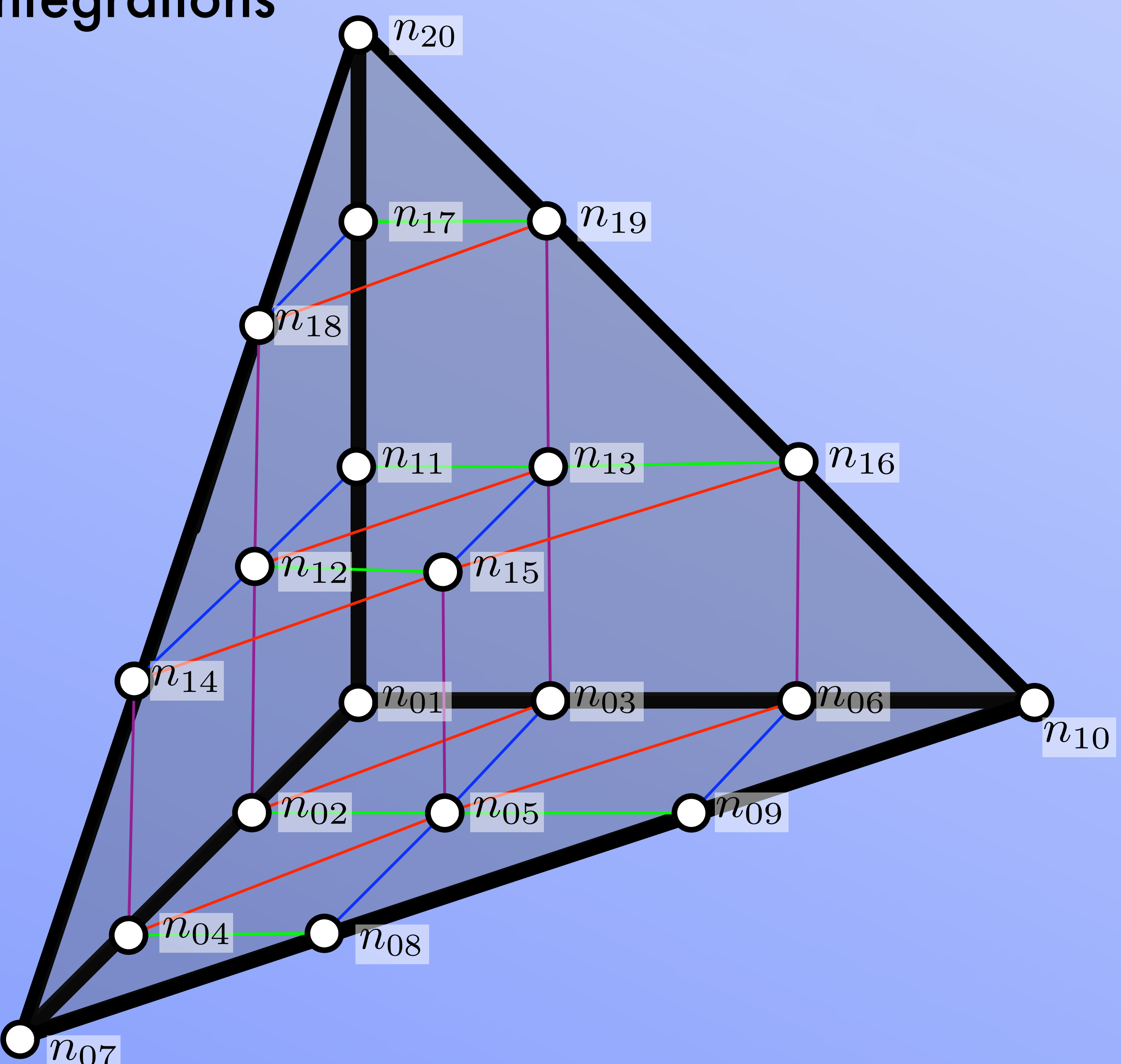


Part One
Optimal *hp* DGM principles

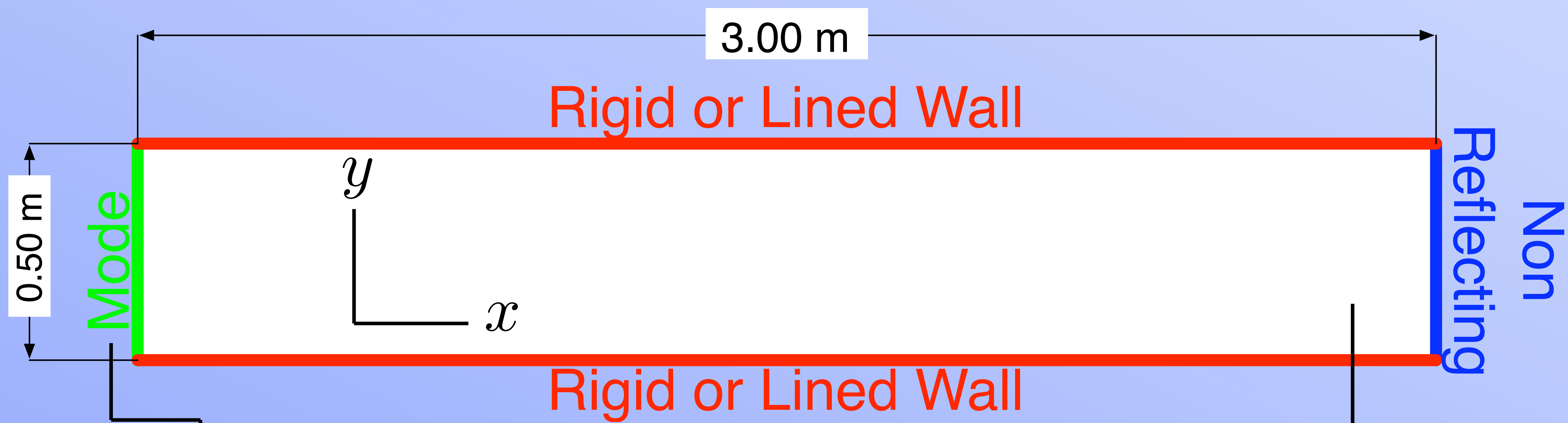
"Optimal" Functional Basis

- Easy to Build (and Program)
- Higher Order
- Quadratures for Num Integrations
- Ortogonal Basis

High Order Lagrangian Elements



Infinite 2D Duct with constant cross section



$$\lambda = 0.10\ \mathrm{m}$$
$$f = 3.4\ \mathrm{kHz}$$
$$n = 0$$

30 periods for 3m length

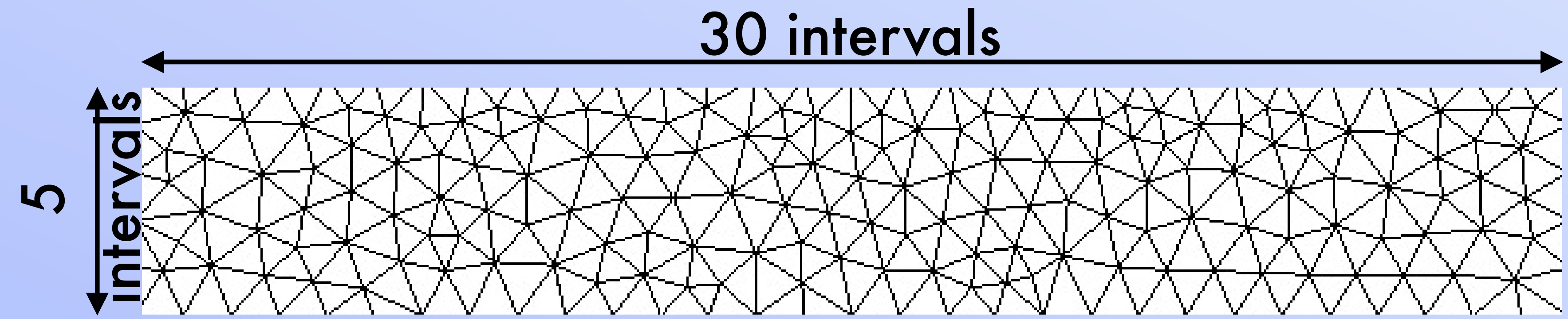
Approximated Non Reflecting condition works perfectly

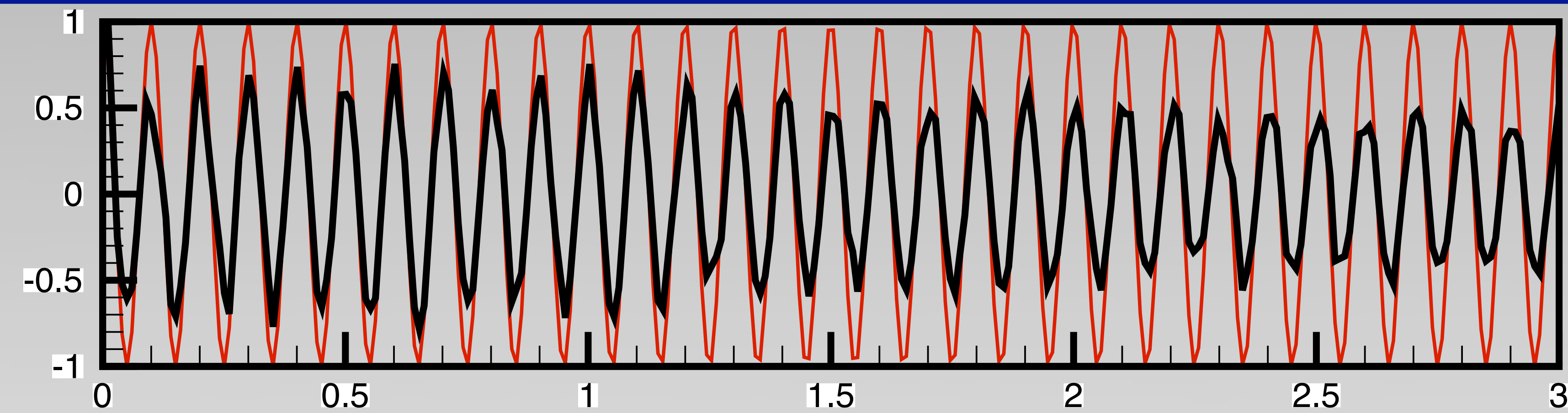
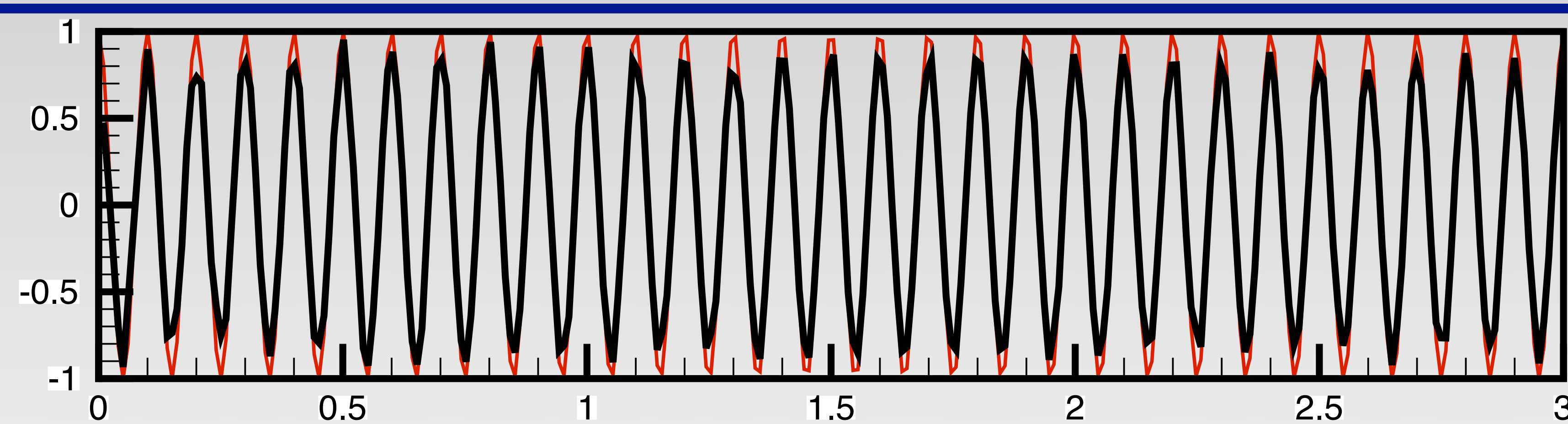
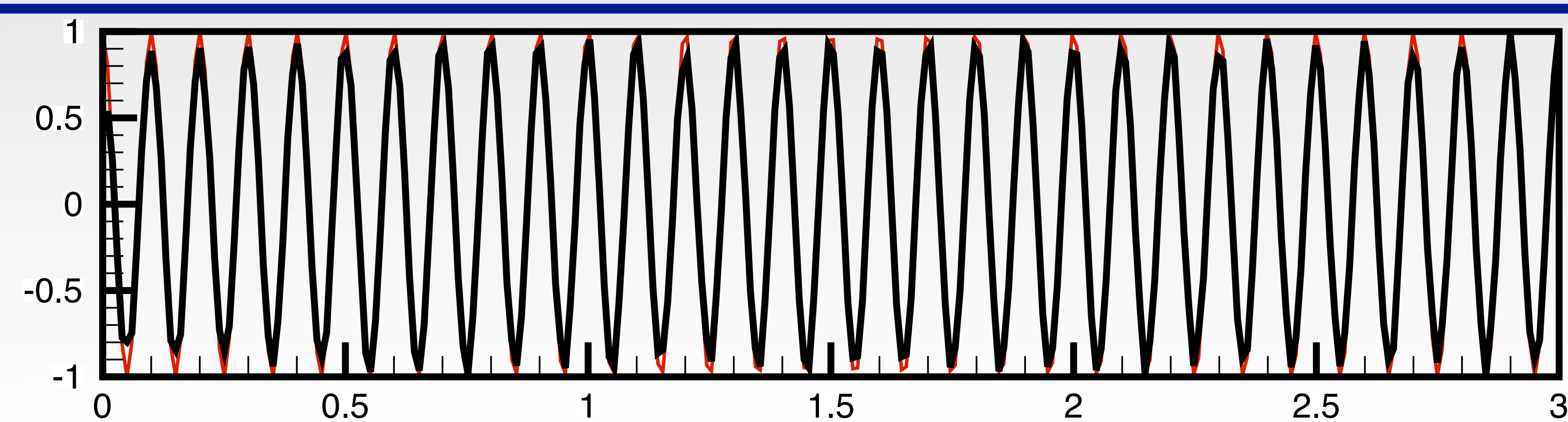
Air

$$a_0 = 340\ \mathrm{m.s}^{-1}$$
$$\rho_0 = 1.23\ \mathrm{kg.m}^{-3}$$
$$p_0 = 101\,325\ \mathrm{Pa}$$

Analytical Solution for Rigid Wall No Flow $\quad p(x,y) = \sin(2\pi f t - \frac{2\pi}{\lambda} x)$

$h \approx \lambda$



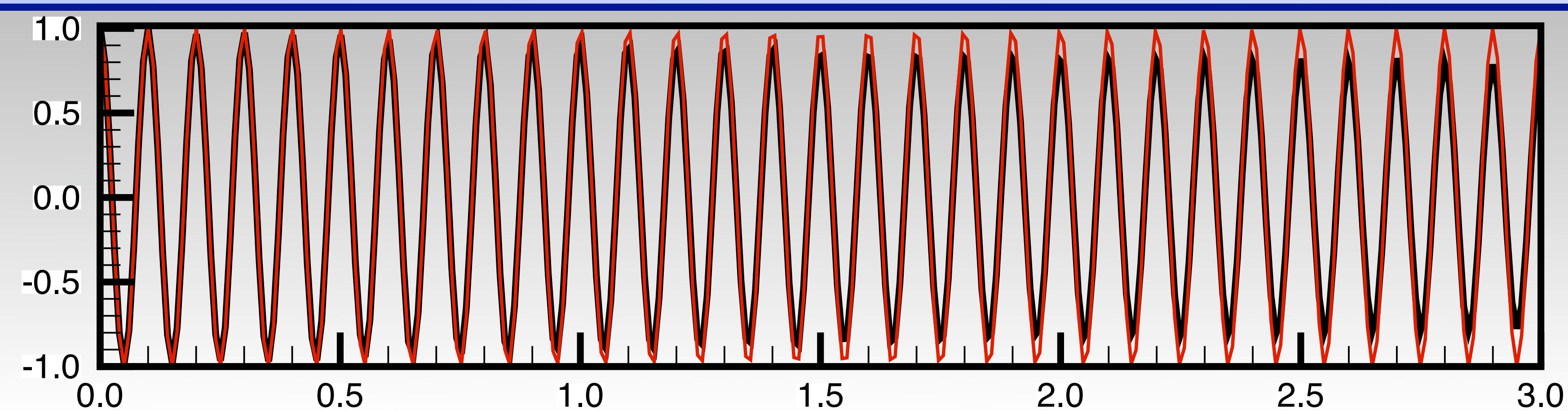
	CPU (s)	deg	
P4	10.55	5430	
P5	28.43	7602	
P6	68.61	10136	

(CPU are obtained on a Dual Apple G5 2 GHz)

ONERA

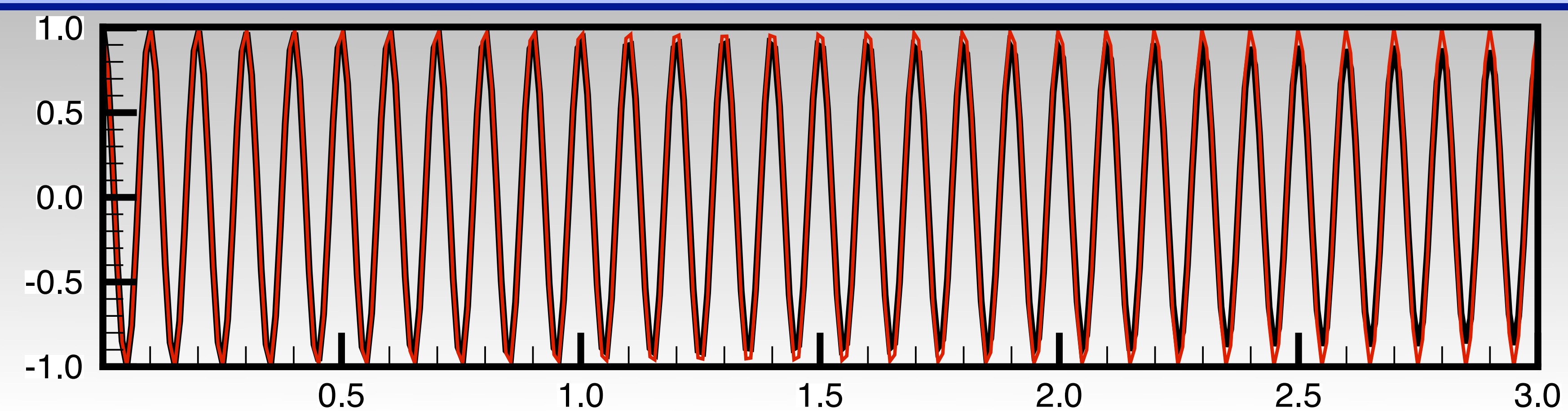
$h \approx \lambda/10$ 300 x 50 intervals

	CPU (s)	deg
P1	1047.85	110658



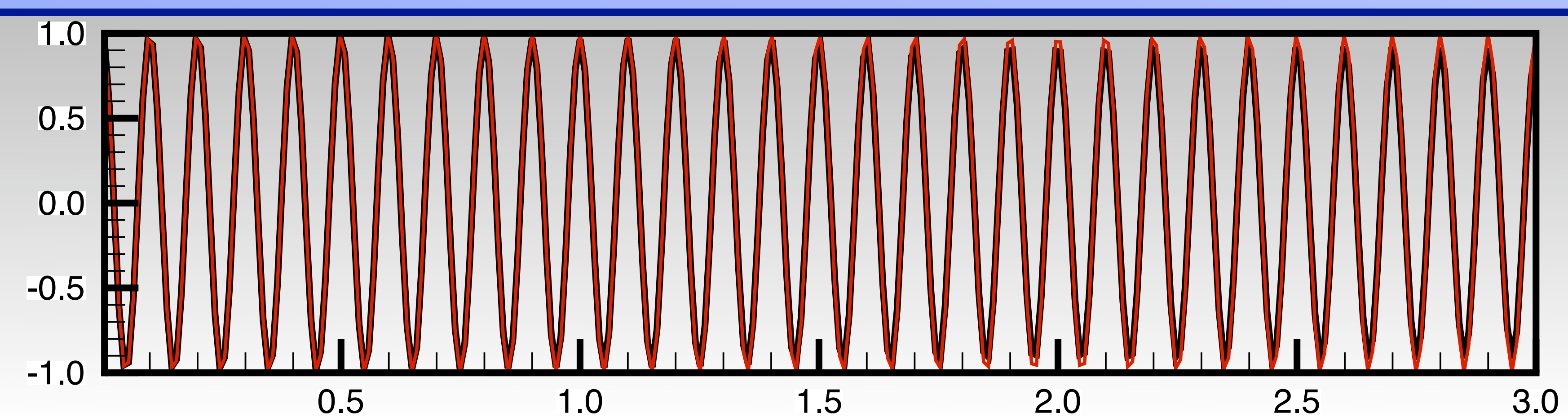
$h \approx \lambda/12$ 360 x 60 intervals

	CPU (s)	deg
P1	2288.29	159660



$h \approx \lambda/14$ 420 x70 intervals

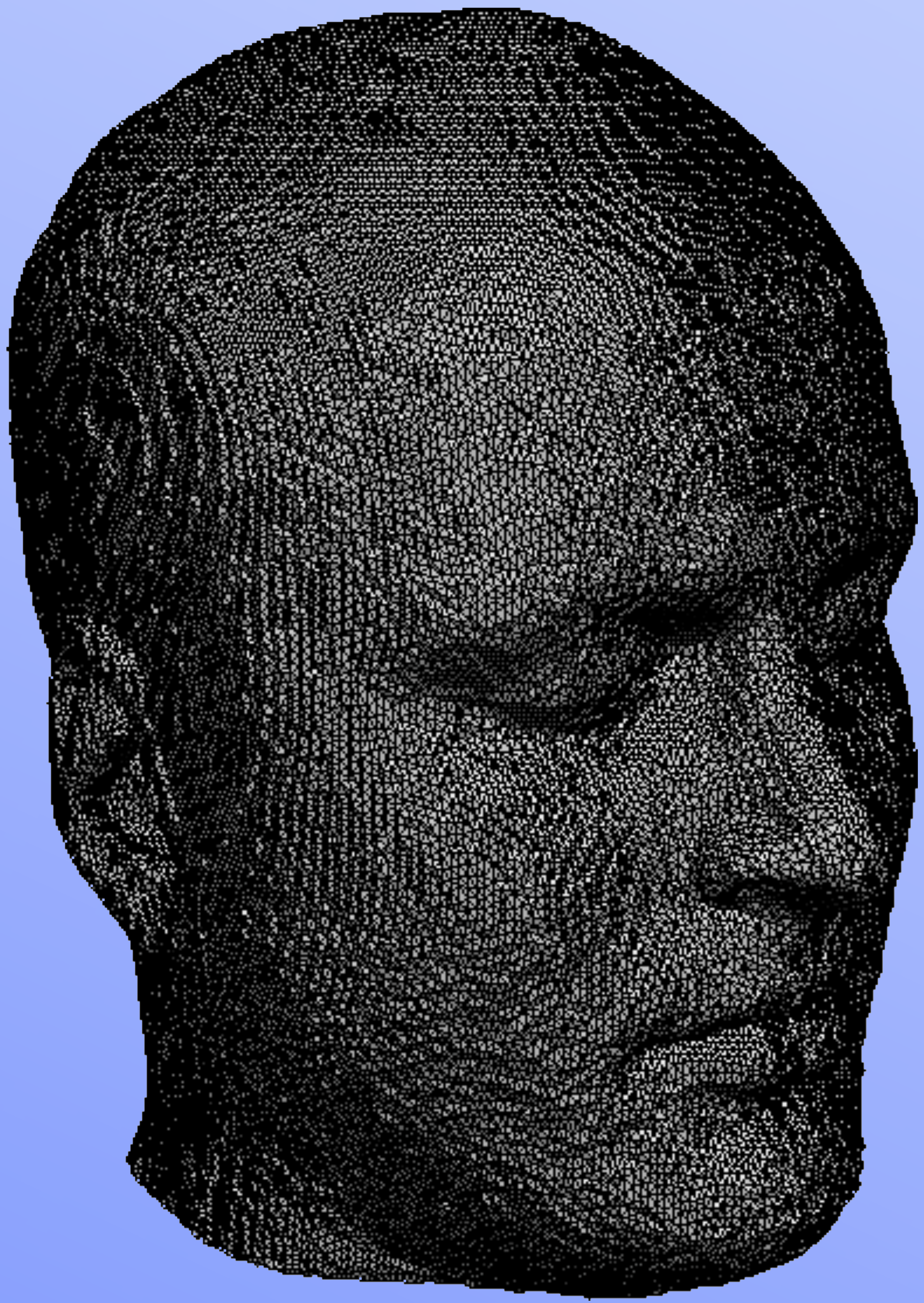
	CPU (s)	deg
P1	4709.81	216684



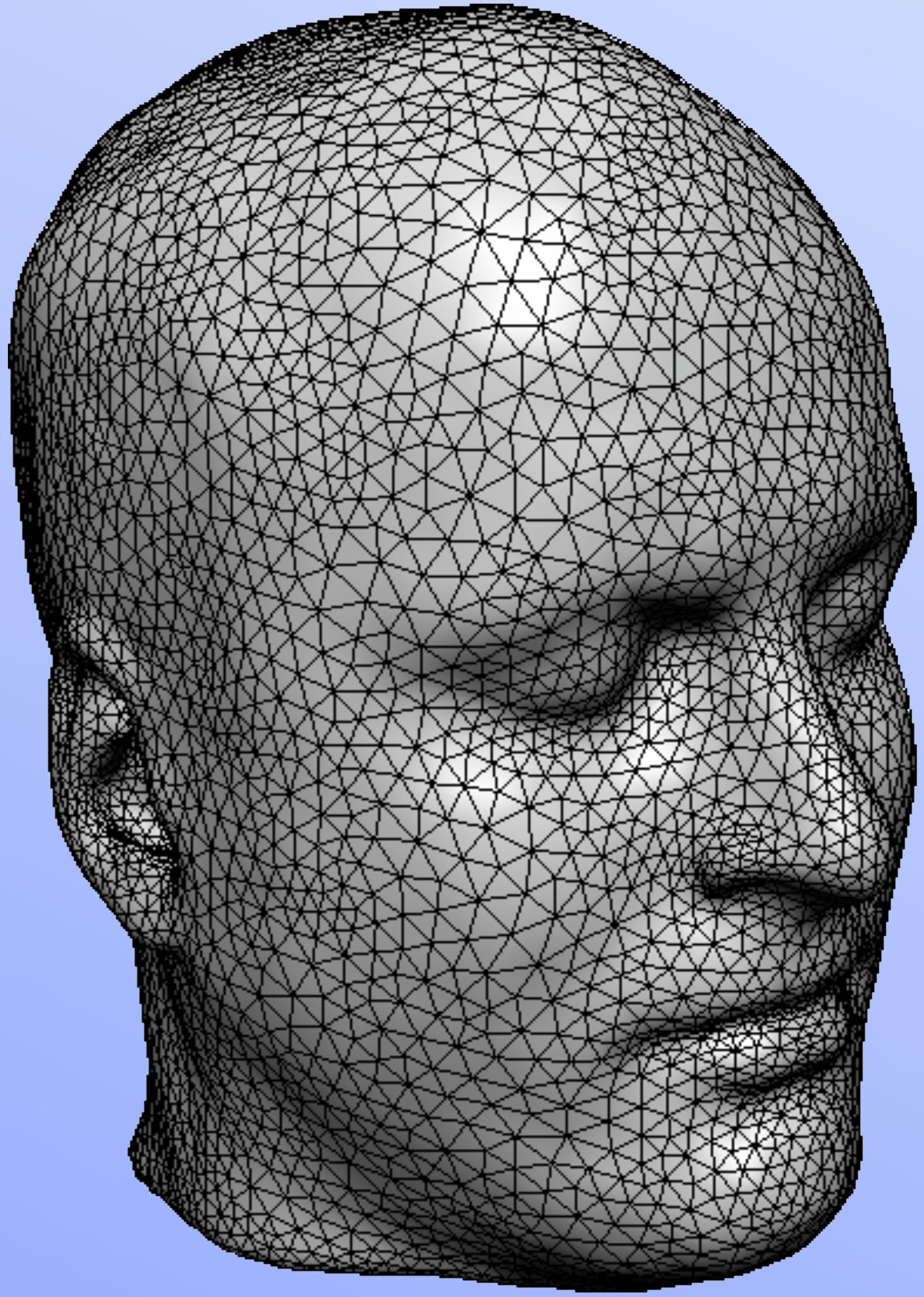
Mesh Refinement / Element Order

element	$n(P_i)$	$m(P_i)$	$\alpha(P_i)$	$h_{min}(\leq 5\%)$	$h_{min}(\leq 10\%)$
P0	1	0		λ/20	λ/40
P1	3	1	1.0	λ/14	λ/12
P2	6	4	0.5	λ/4	λ/4
P3	10	9	0.37	λ/3	λ/2
P4	15	16	0.31	λ/2	λ/3
P5	21	25	0.28		λ
P6	28	36	0.26	λ	

Remeshing Tool

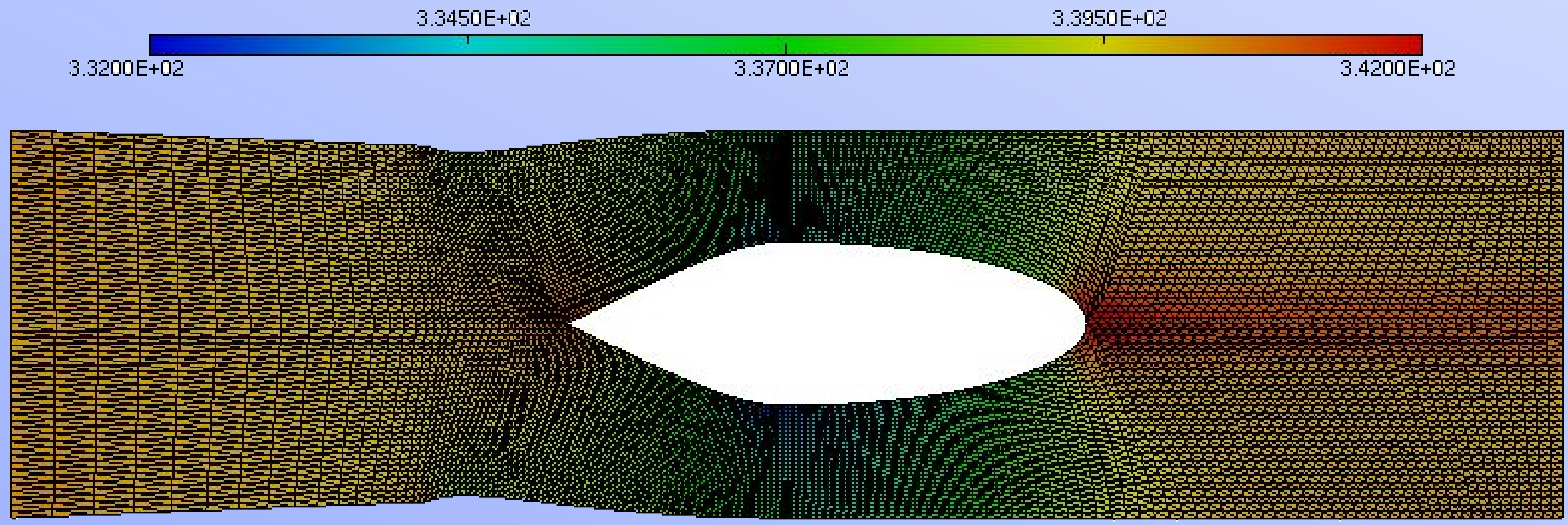


Original mesh
np: 67 108   nt: 134 212

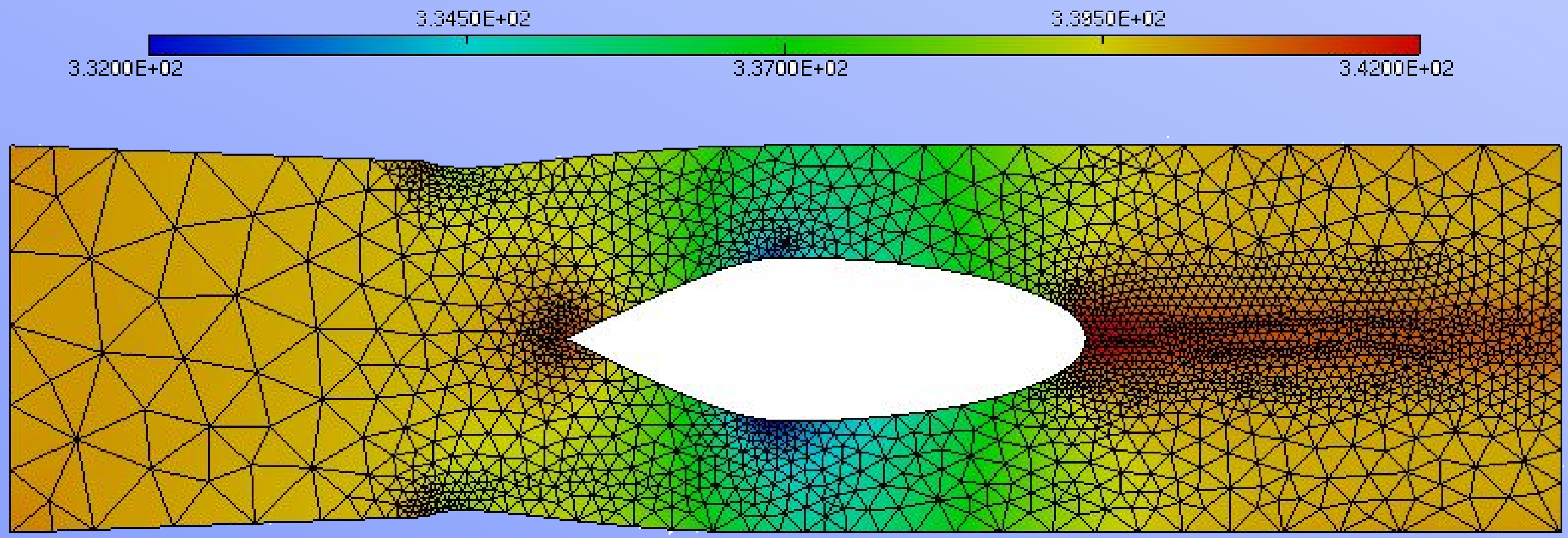


Adapted mesh
np: 9 796   nt: 19 588

Optimal hp DGM CAA Meshes



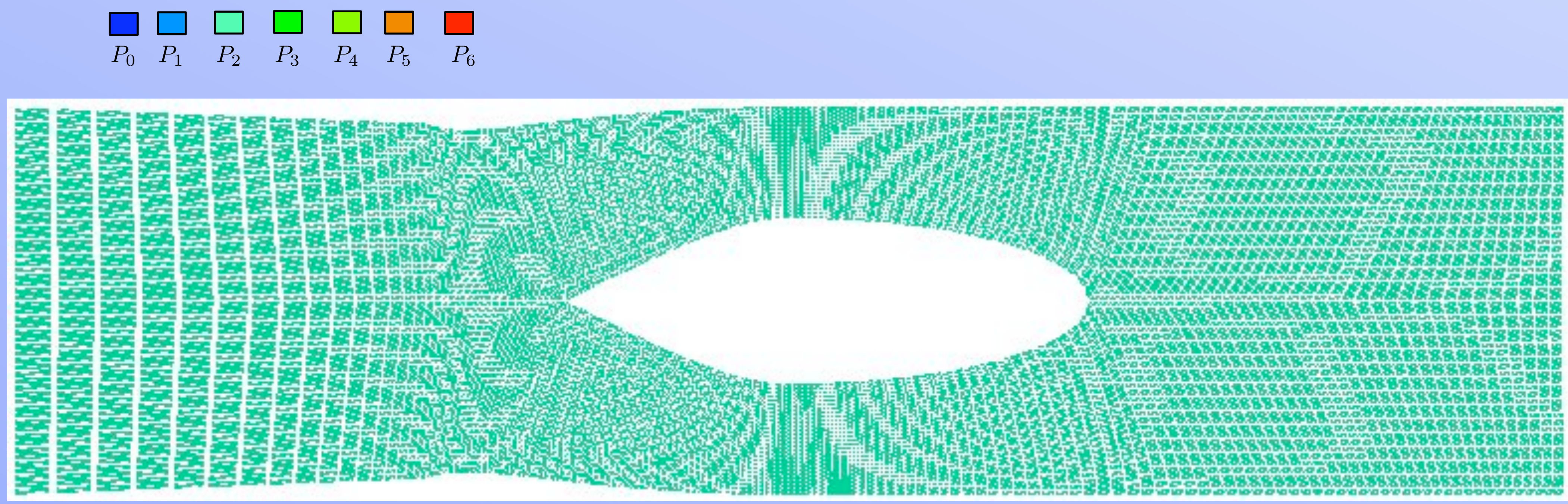
CFD Mesh: 17024 triangles

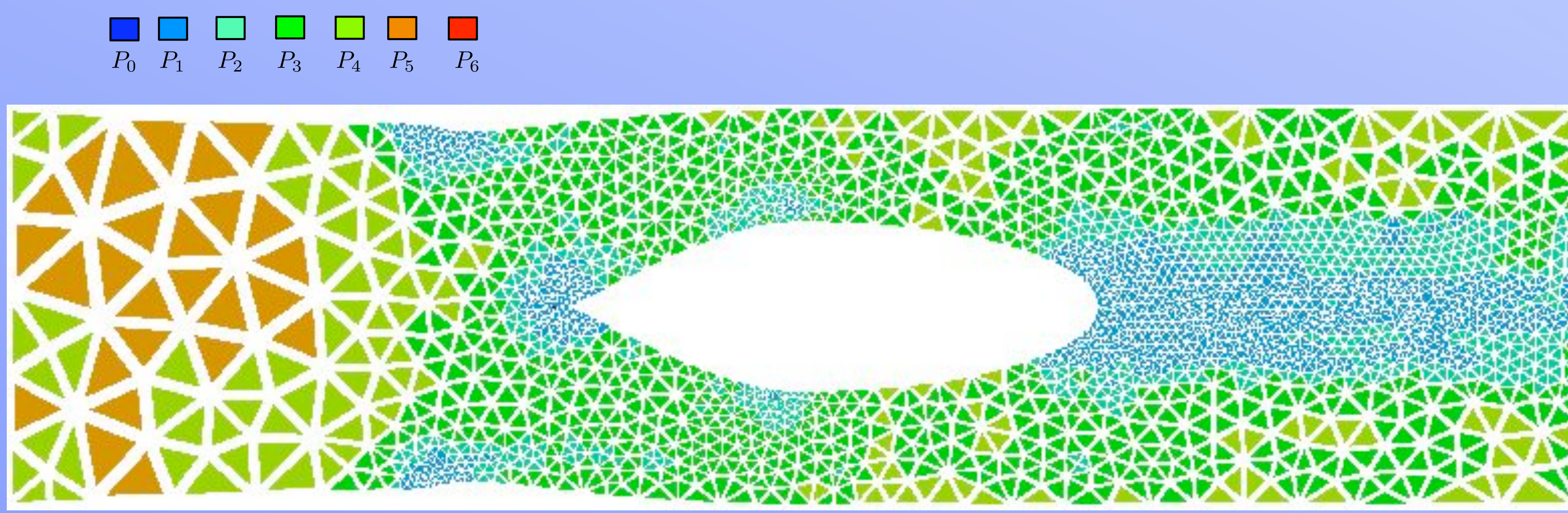


Optimal Mesh: 3729 triangles

Optimal hp DGM CAA Orders

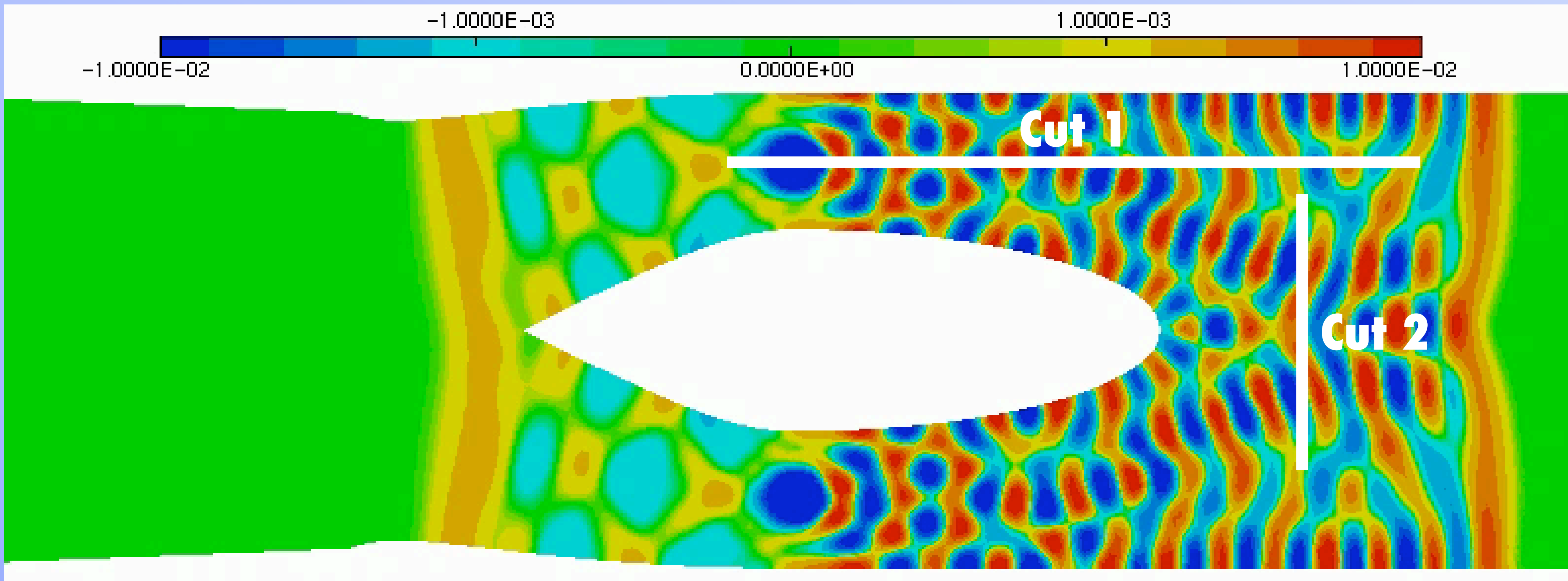
$f = 2\,\mathrm{kHz}$



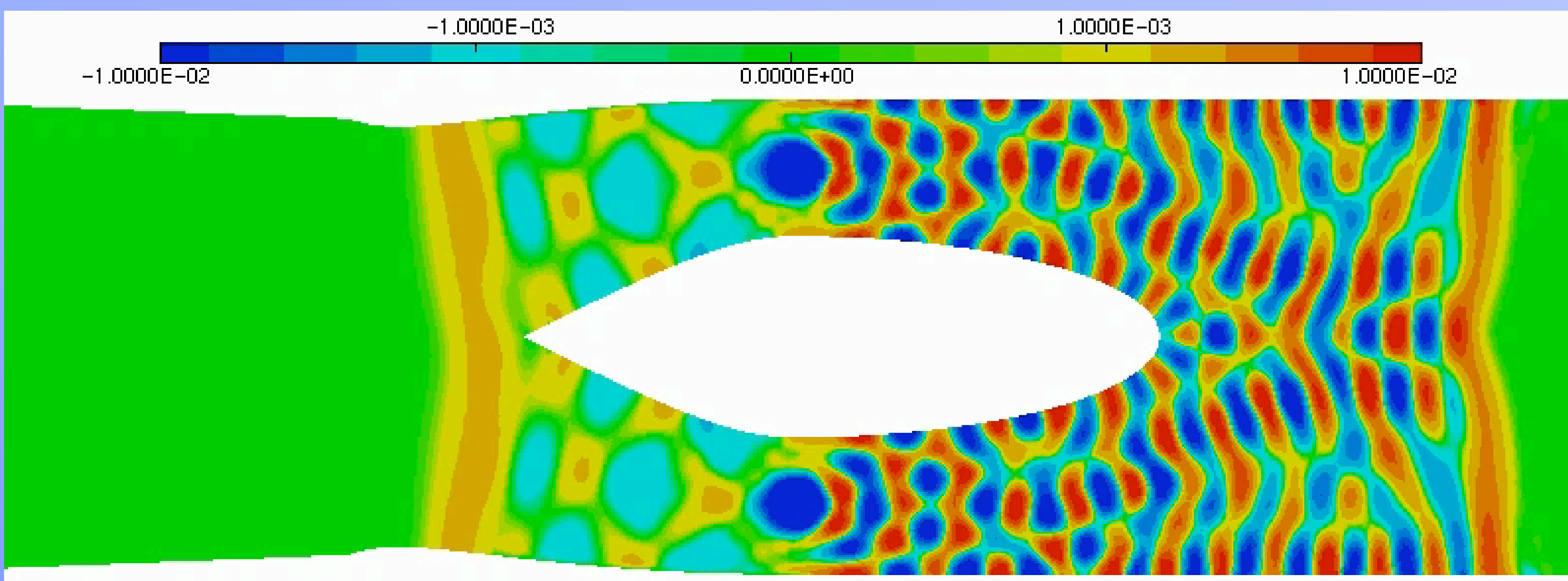


Optimal hp DGM CAA Results

$f = 2\,\mathrm{kHz}$



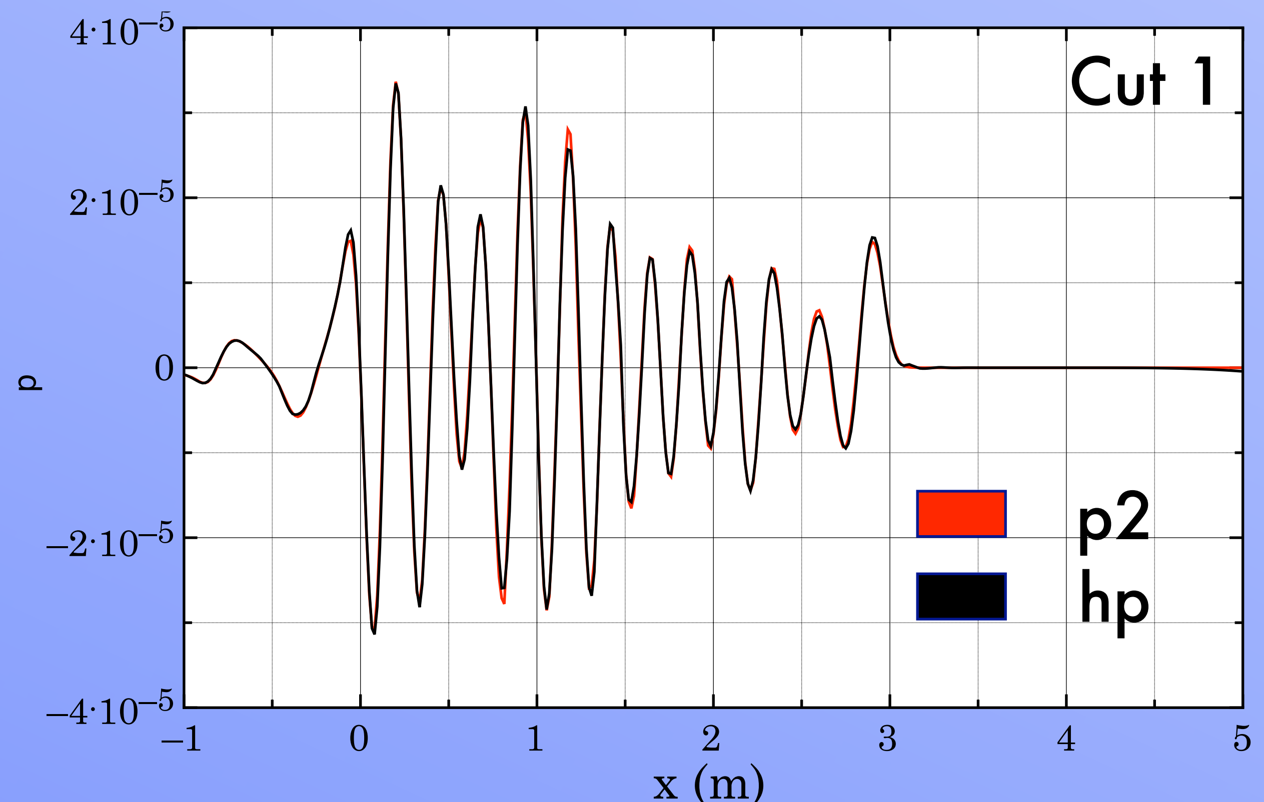
iter	1456
dt	4,46 μs
t	6504 μs
CPU	580″
RAM	300 MO

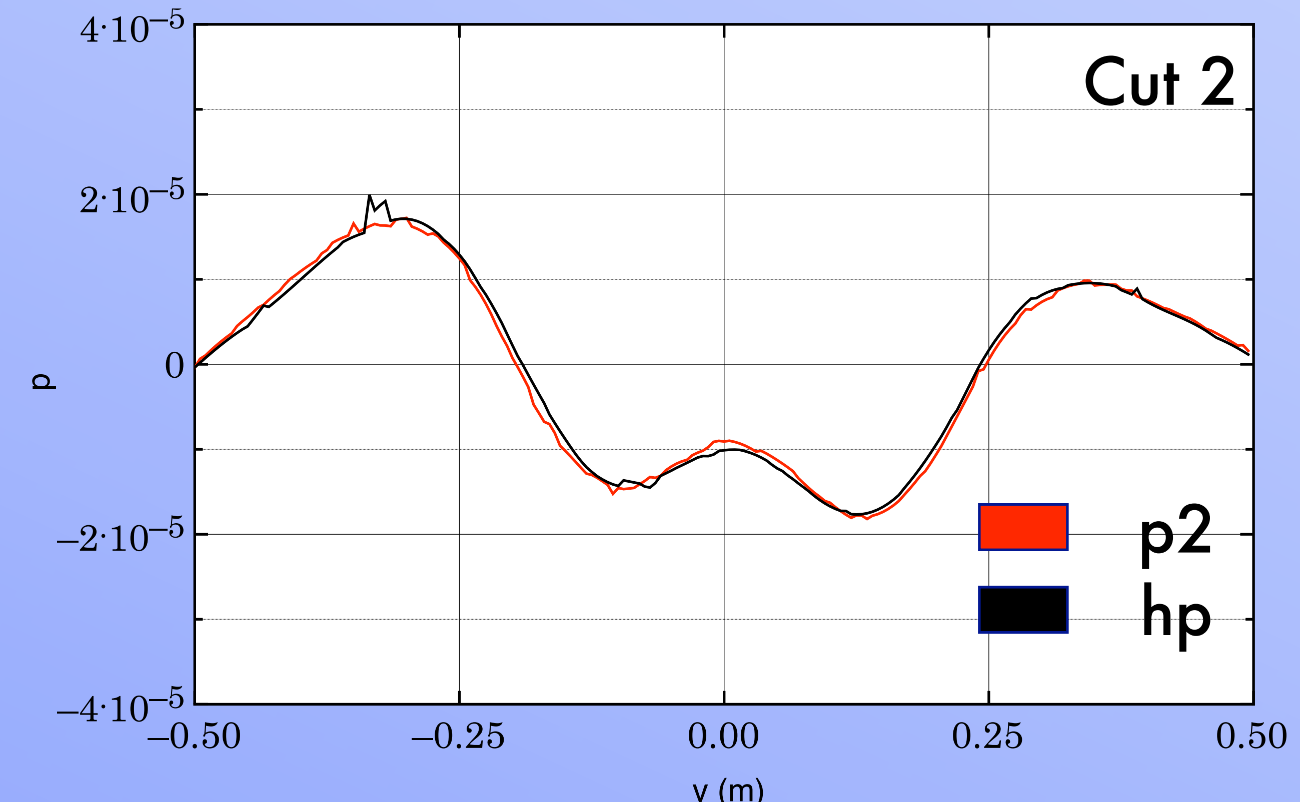


iter	1200
dt	5,42 μs
t	6507 μs
CPU	195″
RAM	200 MO

Optimal hp DGM CAA Validations

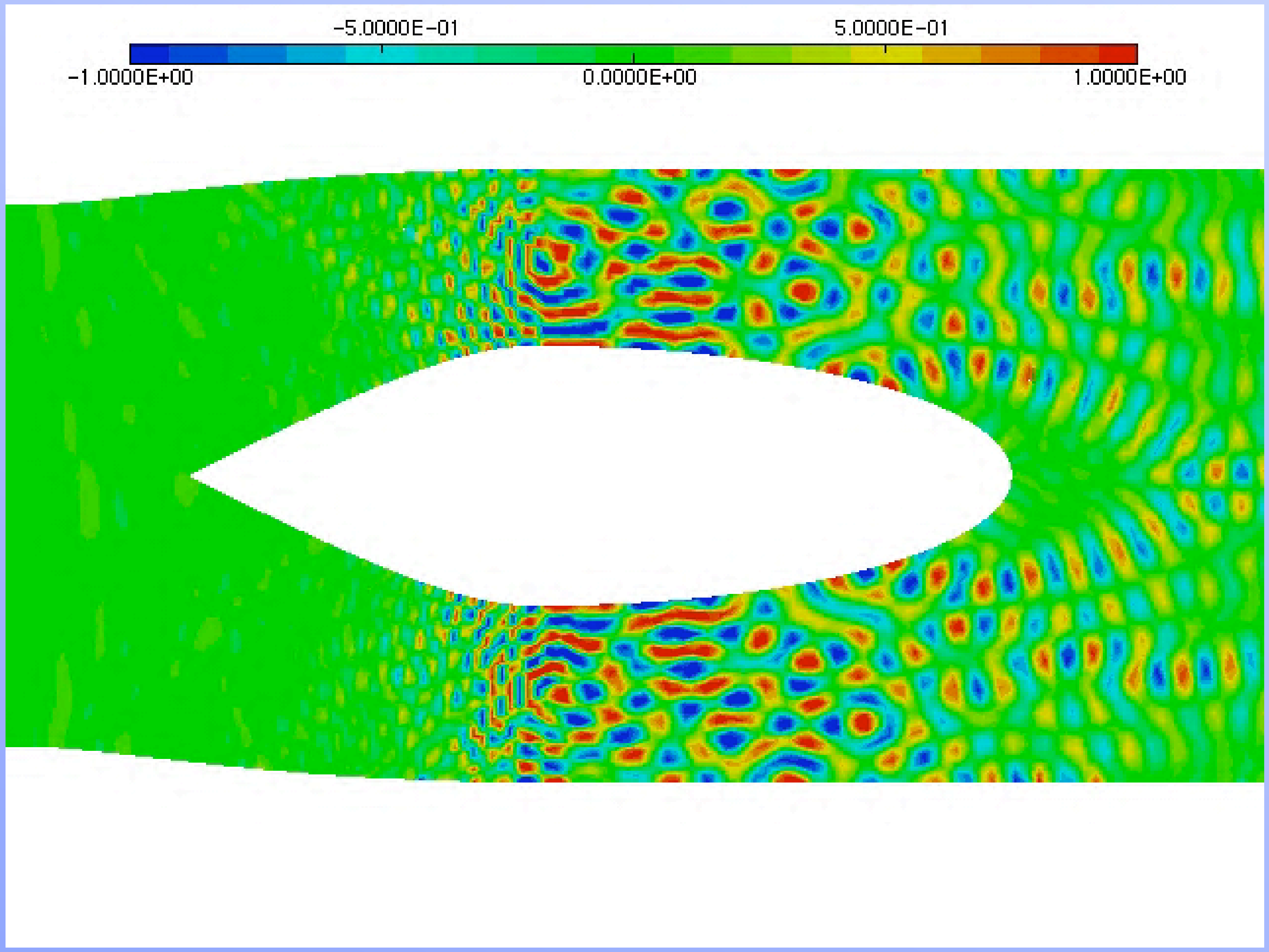
$$\ell_2 = \frac{\sum_{i=1}^{n} \left(\bar{p_1}(i) - \tilde{p_1}(i) \right)^2}{\sum_{i=1}^{n} \left(\bar{p_1}(i) \right)^2} = 0.83\%$$





Optimal hp DGM CAA

$f = 3\,\mathrm{kHz}$

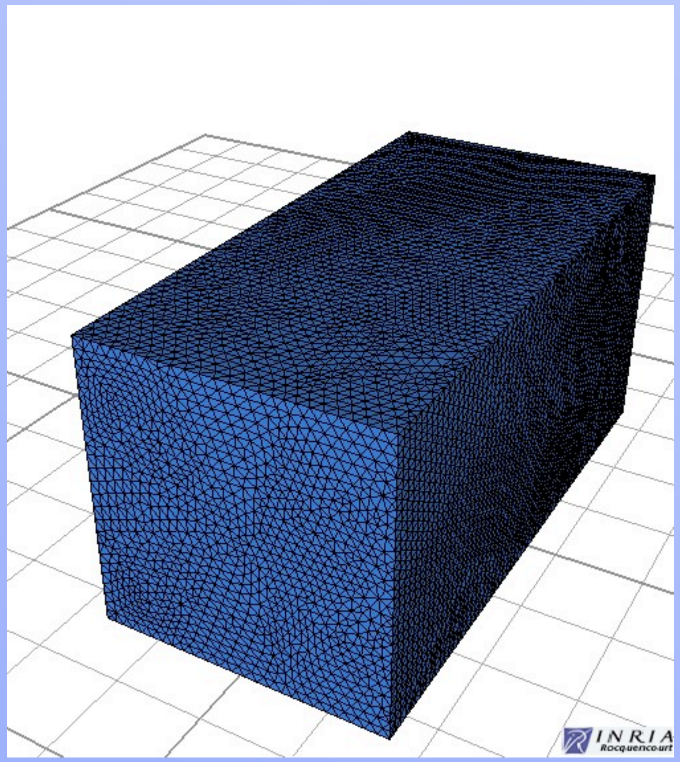


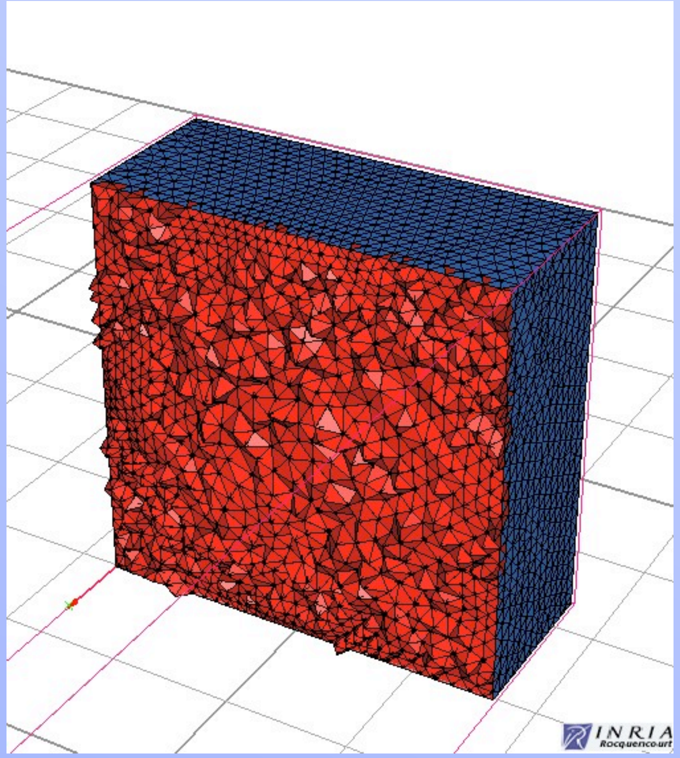
Part Two
High Performance Computing

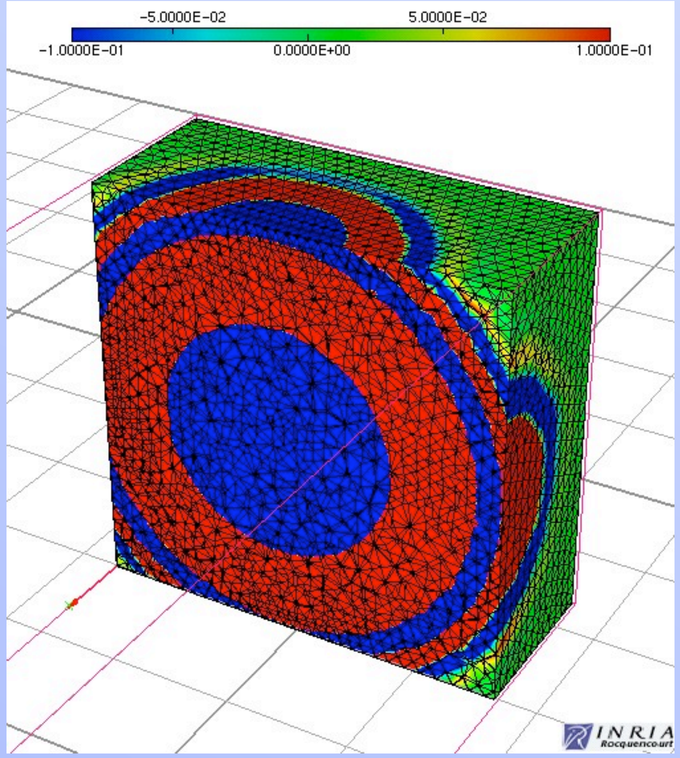
HPC = High Performance Computing

ONERA

Simulations for 3D geometries







500 000 Tetraedra Mesh

4 GBytes

First Idea for 3D Simulation:
HPC or HCC

Formal Calculation

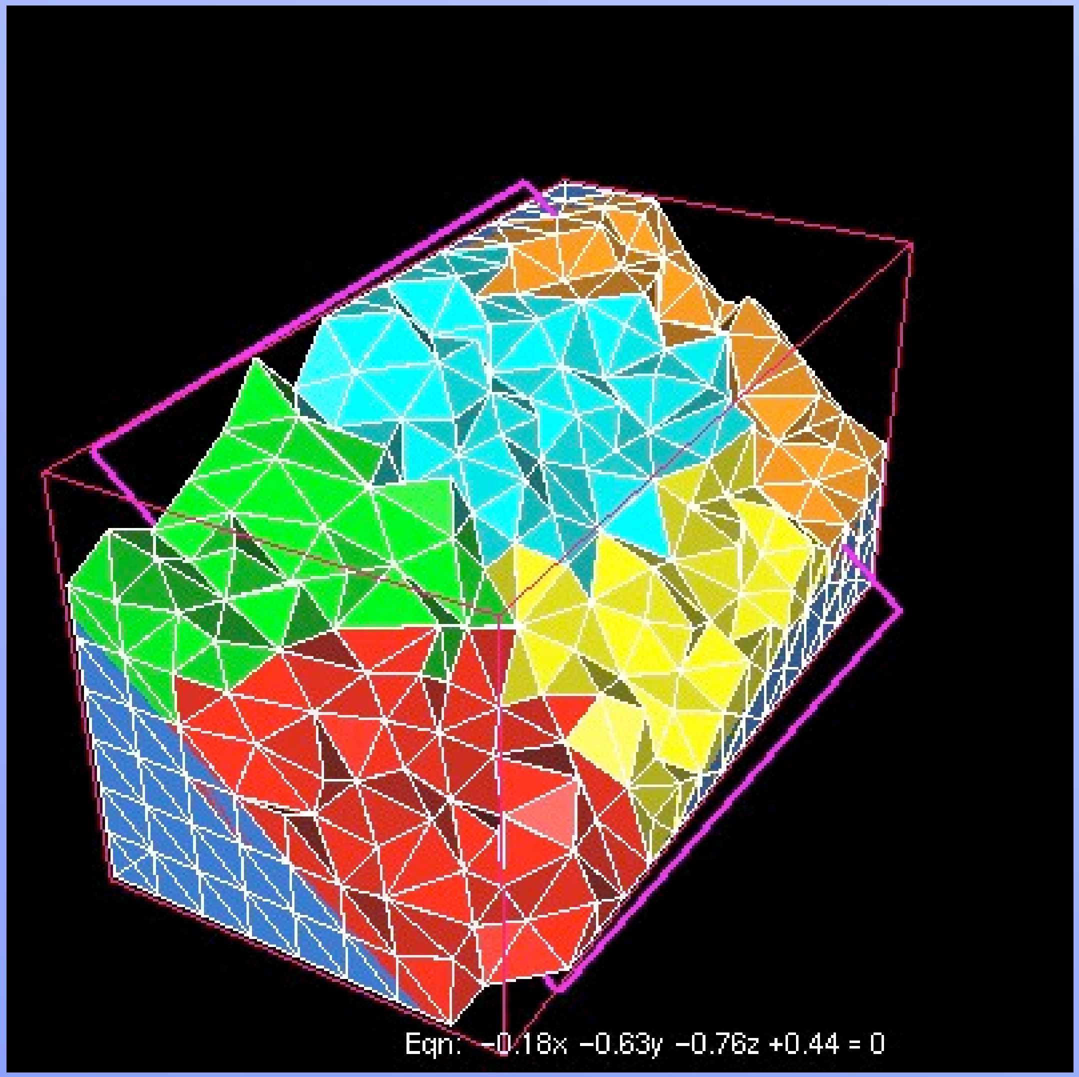
Vectorization

Massively Parallel Computation (MPI+OMP)

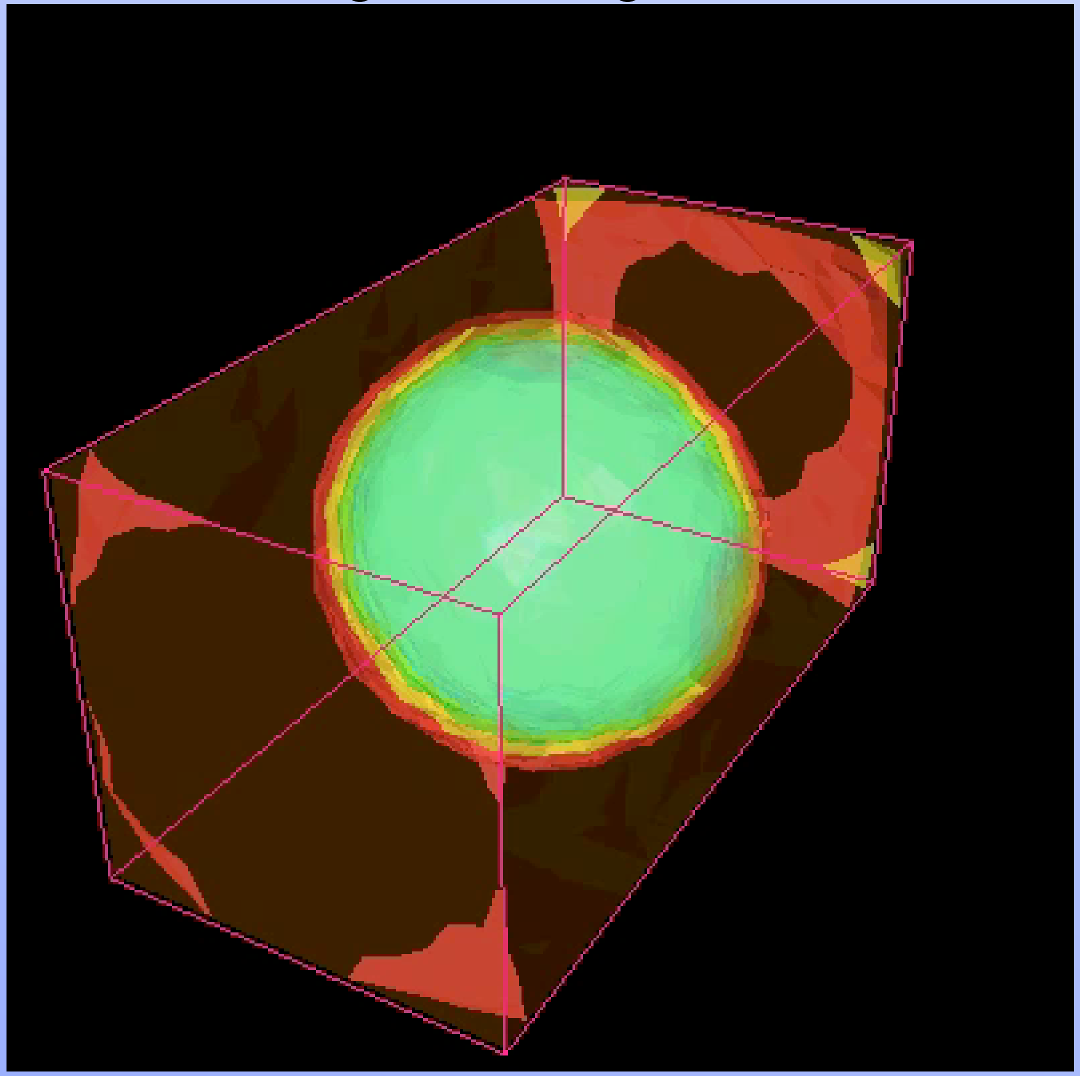
High Performance Computing

Massively Parallel Computation

ParMetis: Parallel Graph Partitioning

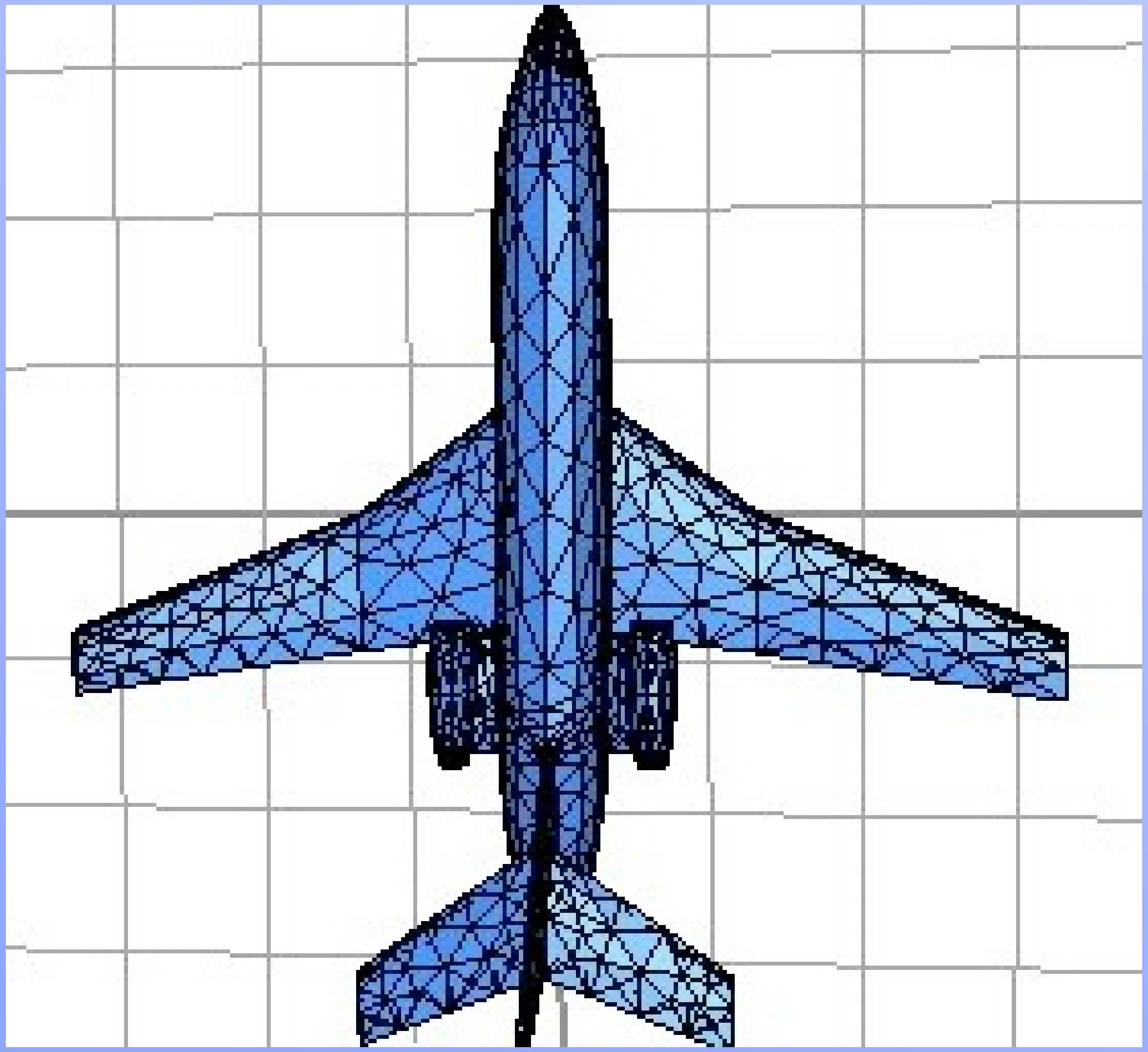


open mpi 1.0.1
Message Passing Interface

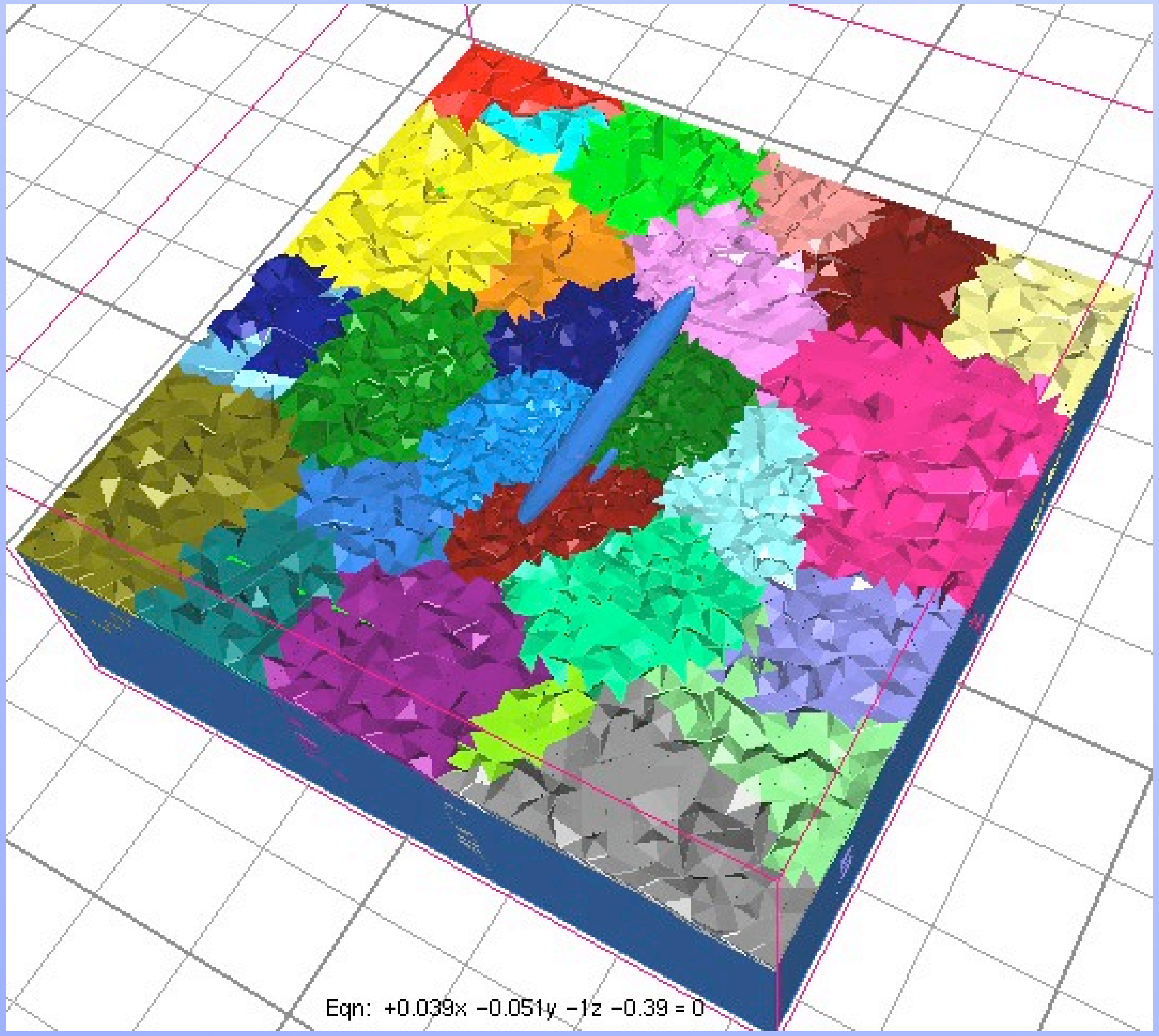


ONERA

Falcon CAA HPC Computation

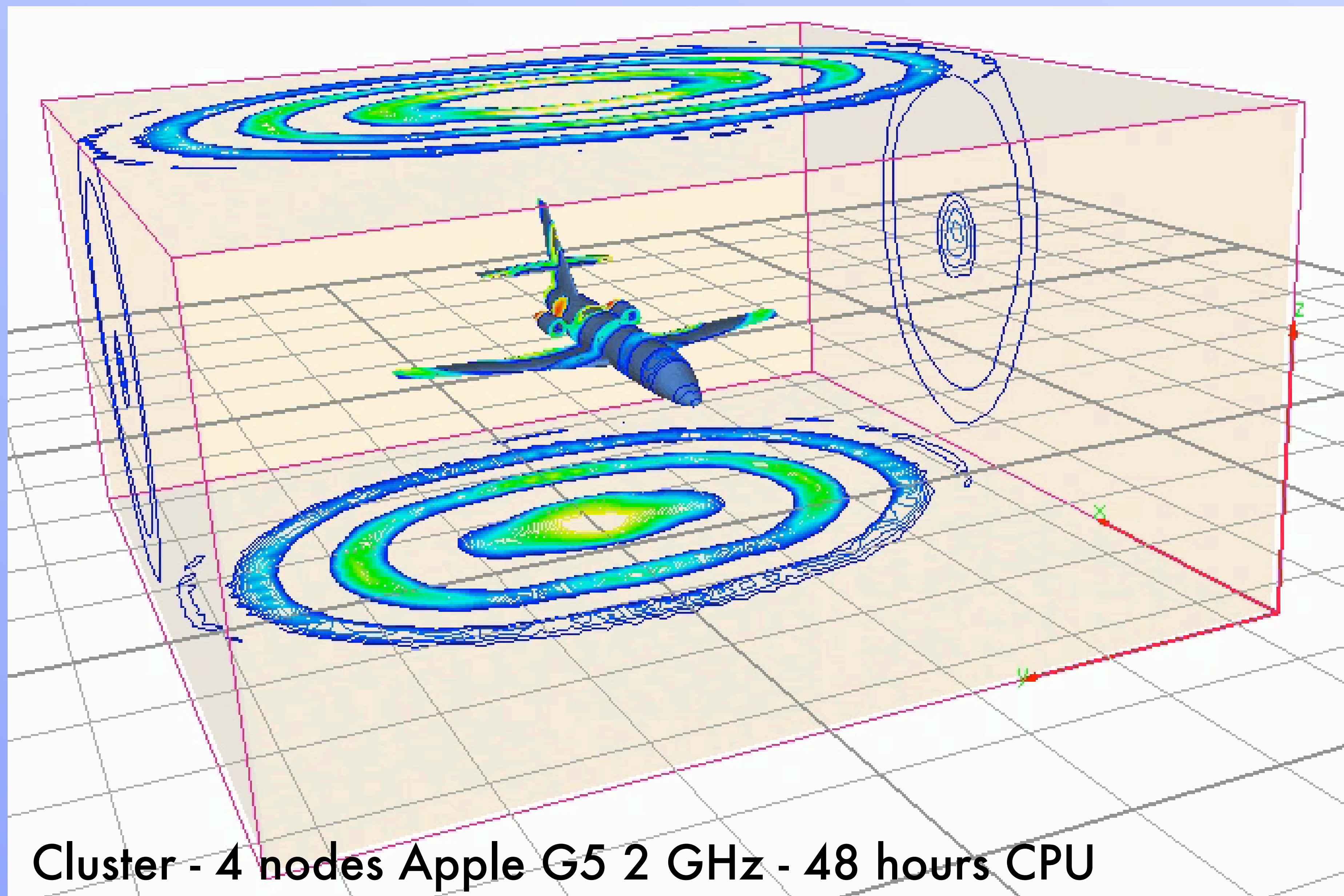


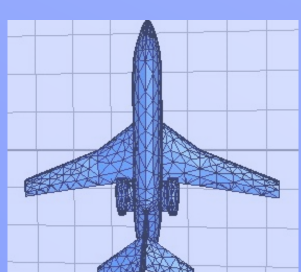
TetMesh (INRIA/Simulog)

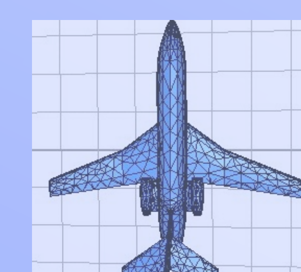


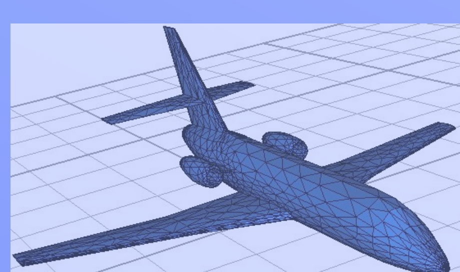
500 000 Tetras
64 domains

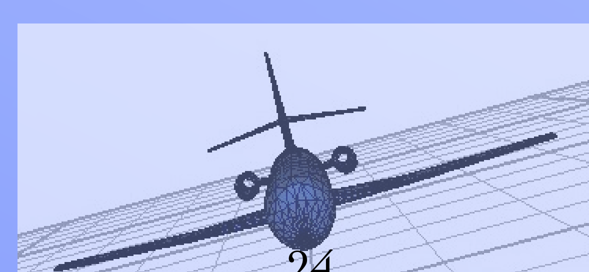
Falcon CAA HPC Computation

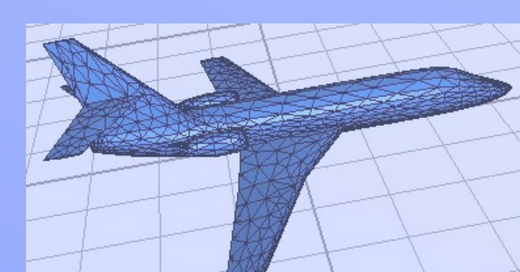




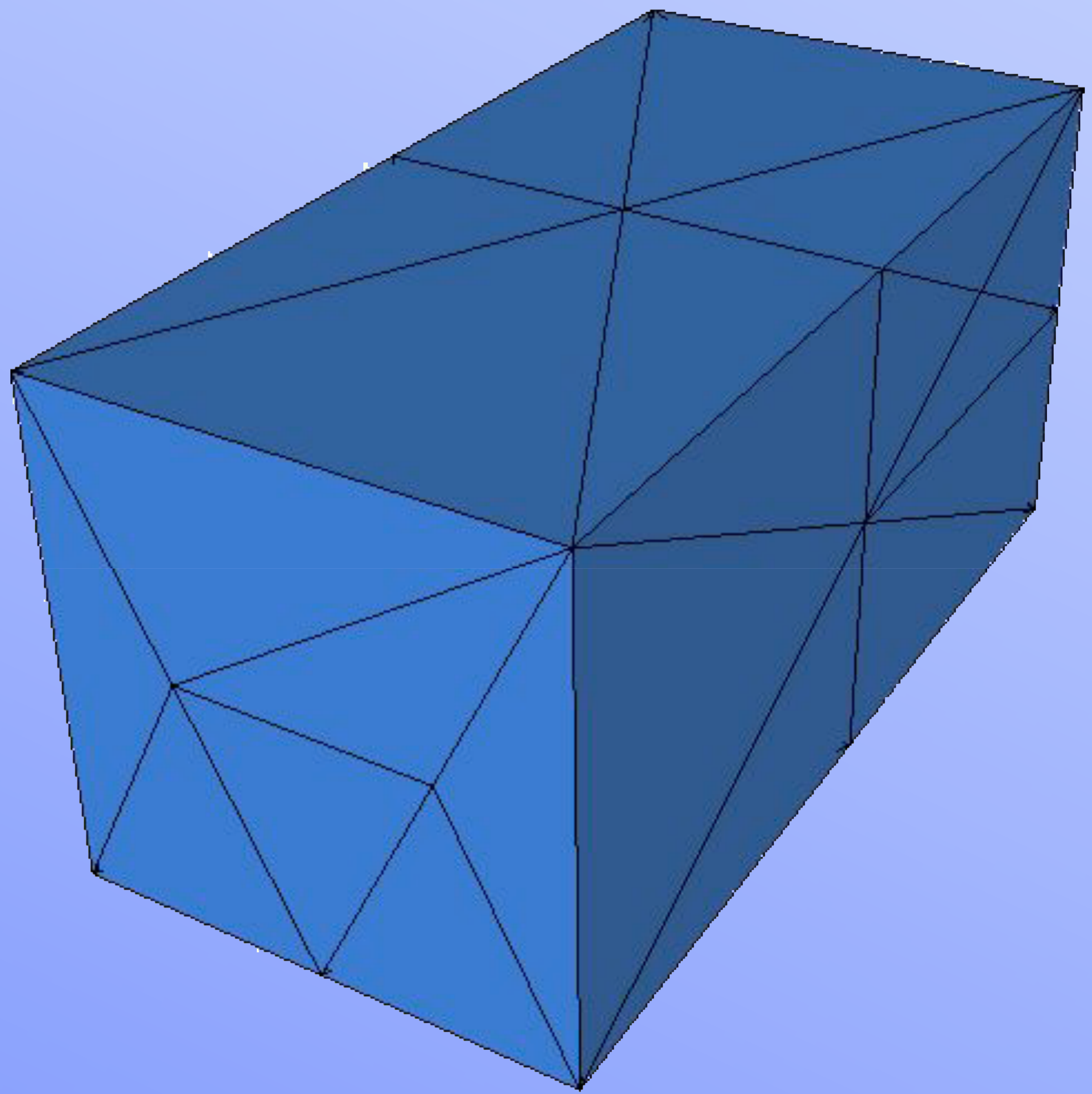




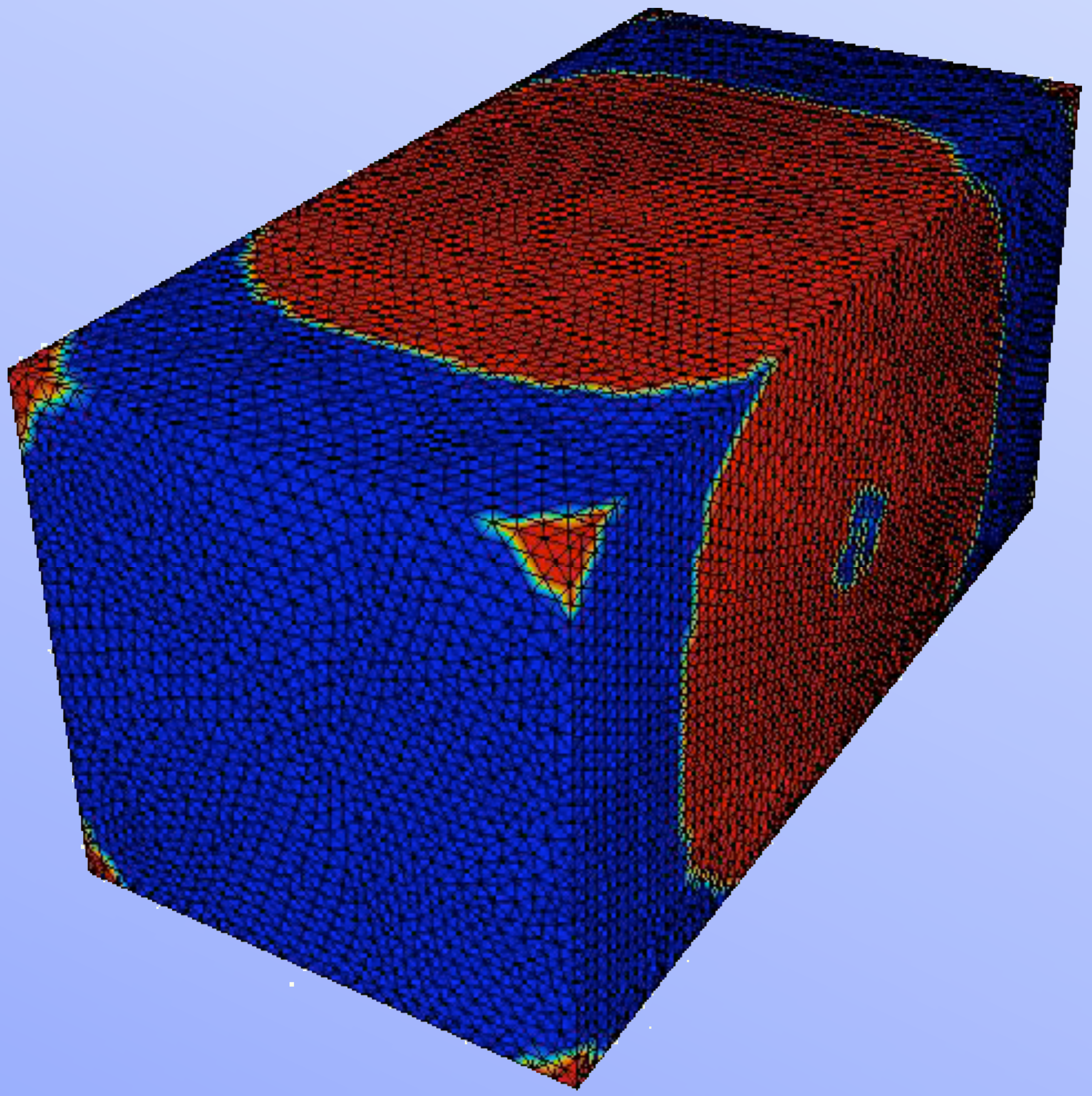




HPC Optimal hp DGM



Mesh for Computation
72 Tetra with P5 approx



Mesh for Visualization
with solution

ONERA

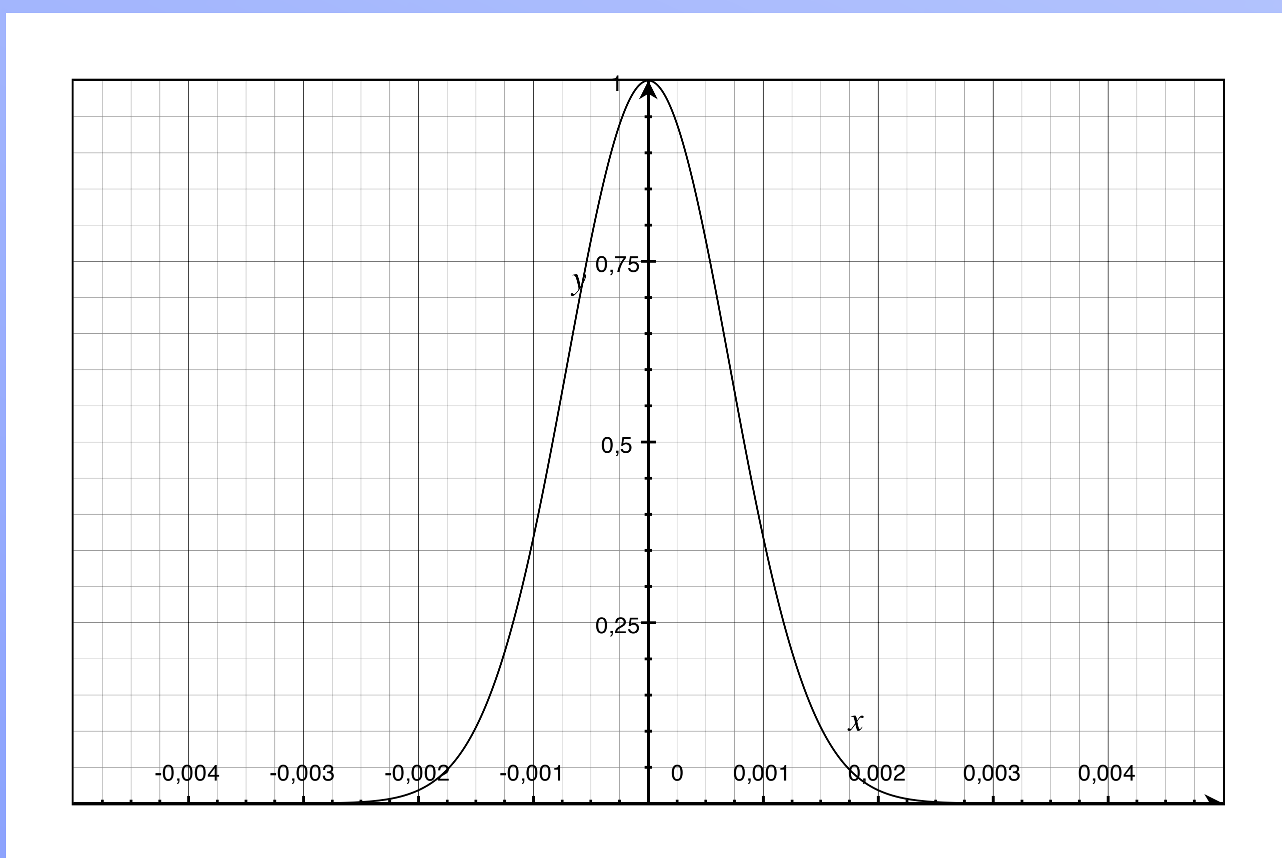
Part Three
Optimal h Adaptation Principles

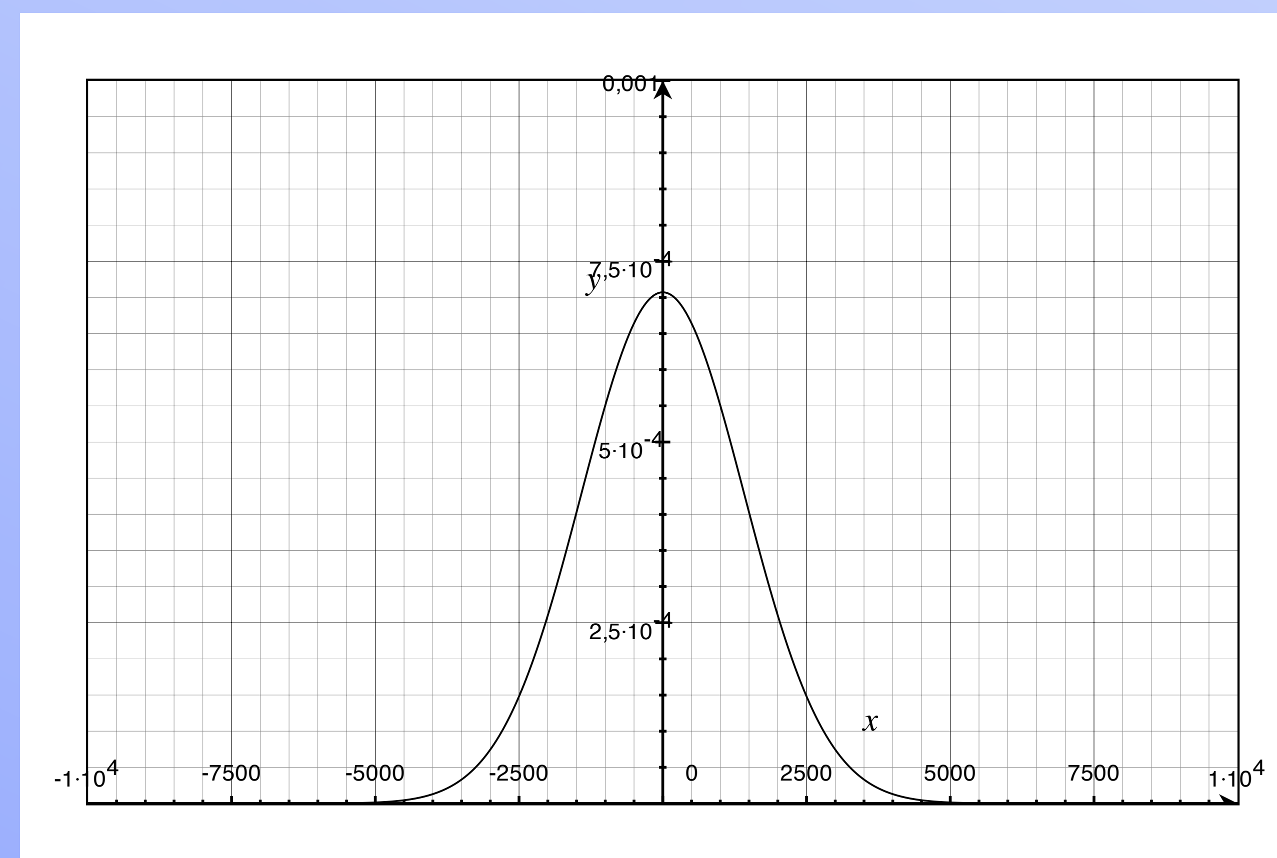
ONERA

Gaussian Distribution of Sources

$$TF(e^{-a^2t^2}) = \frac{1}{a\sqrt{2}}\, e^{\frac{-f^2}{4a^2}}$$

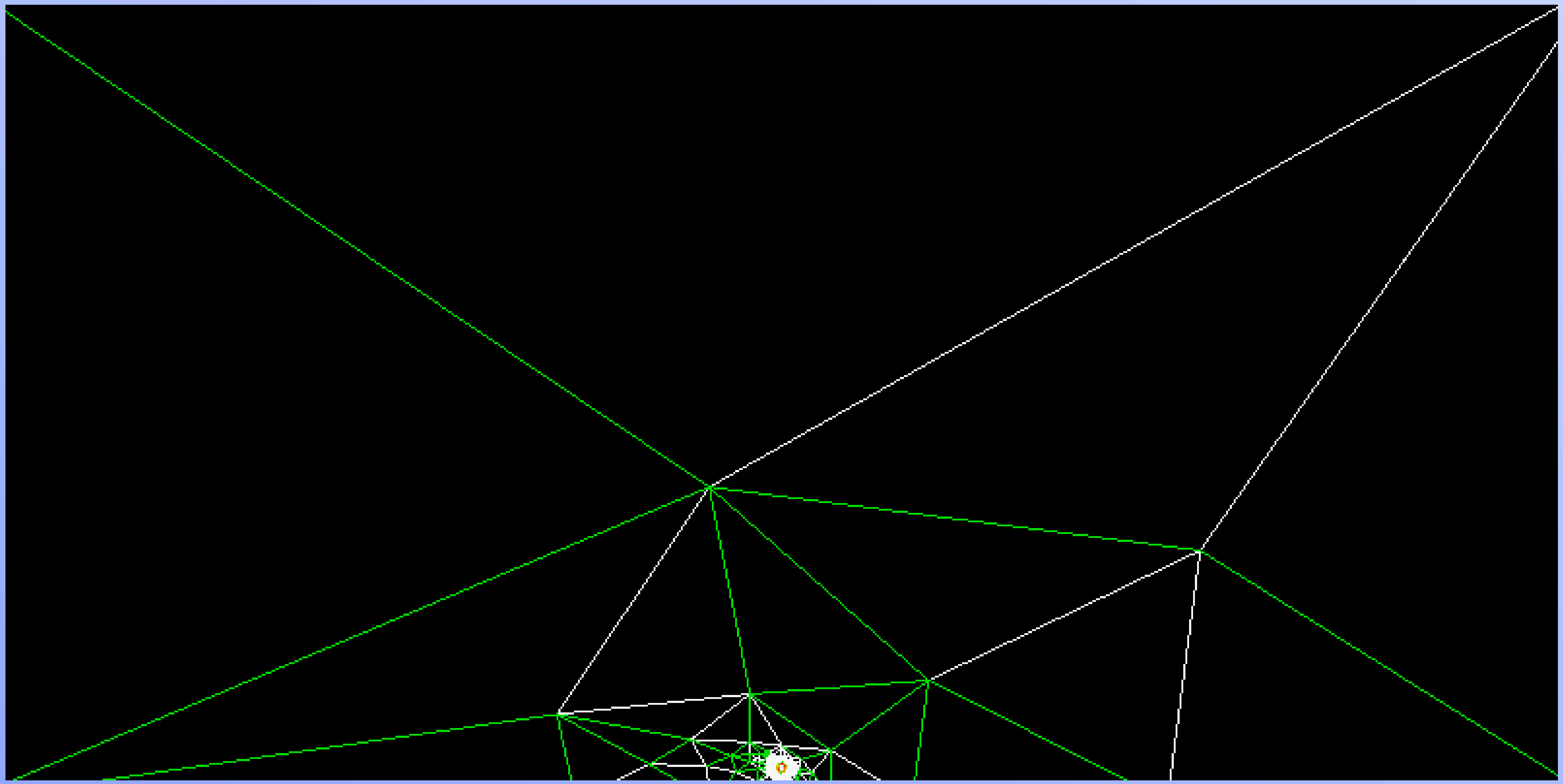
$a = 10^3$



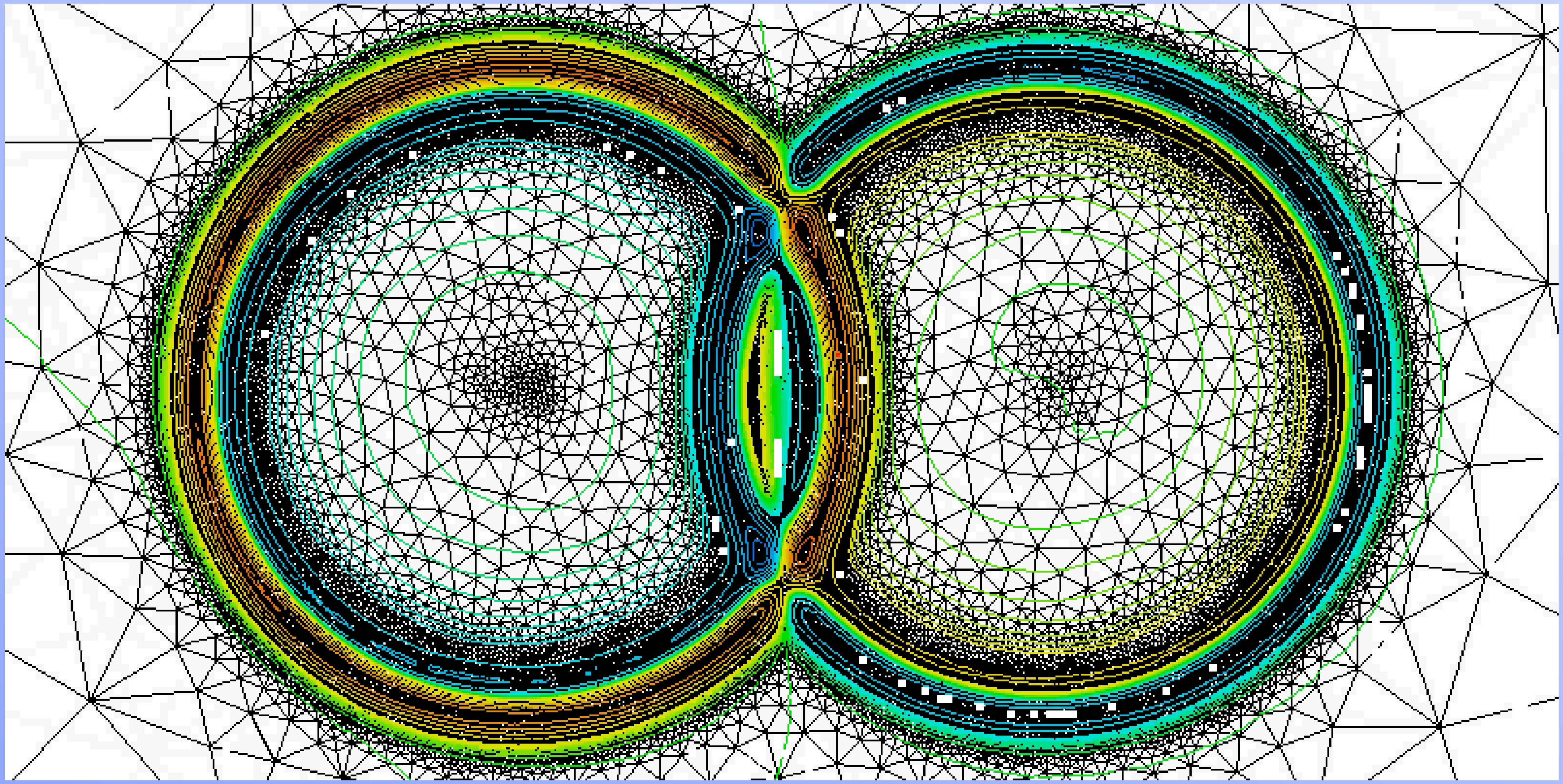


ONERA

Pulse h-adaptation



2 pulses h-adaptation



Conclusion

DGM

DGM is able to solve most CAA problems (and many others)

DGM is expensive (especially for lower order elements)

hp DGM

hp DGM mixes element orders and results a much less expensive cost

With hp DGM, CFD and CAA computations are handled on same mesh

Introduction of the doppler effect when determining local orders

HPC DGM

Computation on clusters make big configurations possible

(hp + HPC) DGM

Balance of the Processes to optimize cluster efficiency