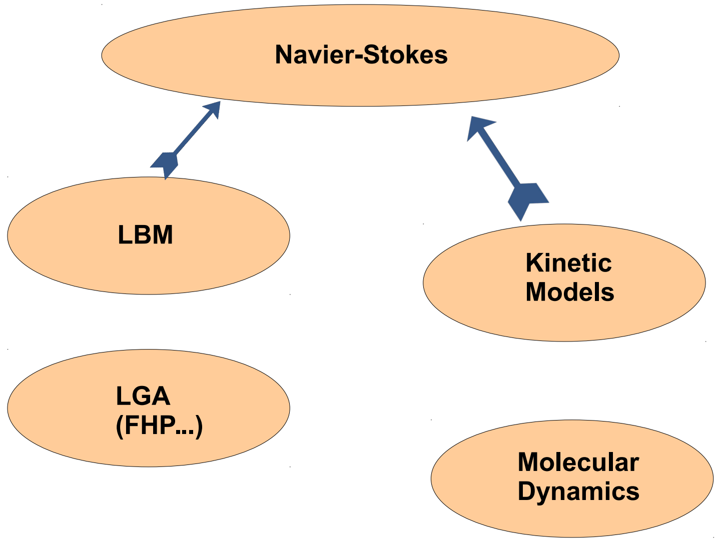
Familles de modèles : des fluides au transport routier, piétonnier, ou intracellulaire

Cécile Appert-Rolland*

Laboratoire de Physique Théorique
CNRS / University Paris-Sud / University Paris-Saclay

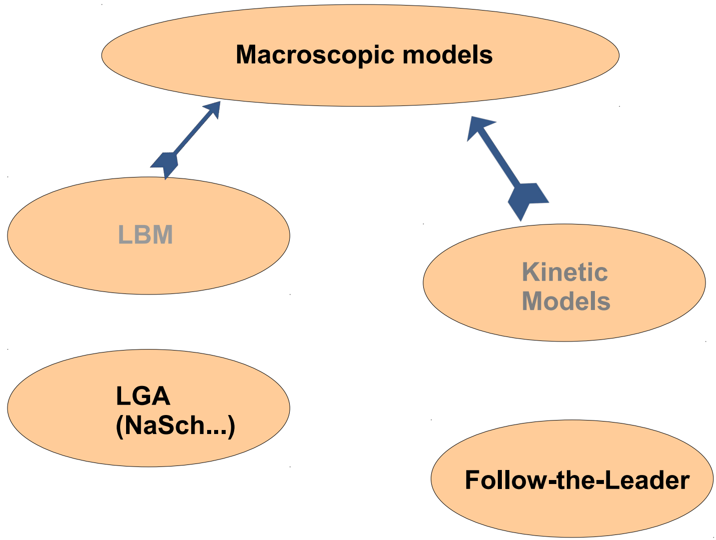
Plan

Modèles pour les fluides



ROAD TRAFFIC

Modèles pour le traffic routier



Cellular automata simulations

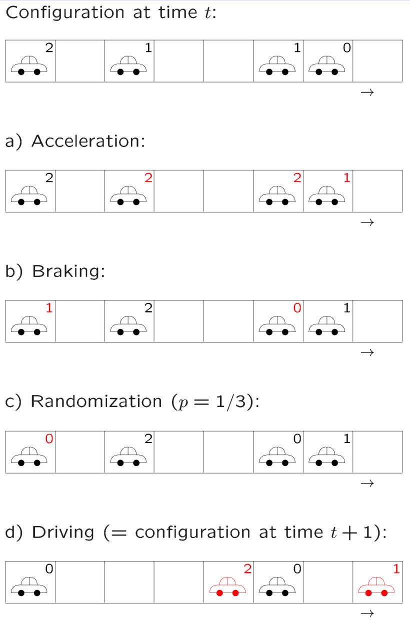
Road = divided into cells

Particle = vehicle

State = speed (between 0 and v_{MAX})

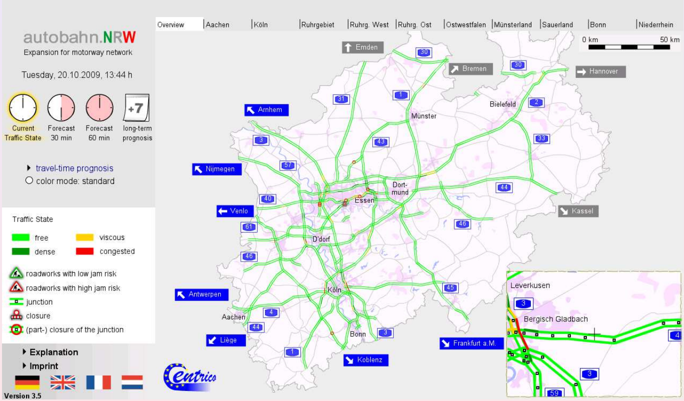
Evolution rules = acceleration and deceleration + propagation

- Pionnering work [Nagel & Schreckenberg (1992)]
- Model by [Knospe et al (2000)]
  - finite braking capacity
  - anticipation
  - slow-to-start rule -> metastability

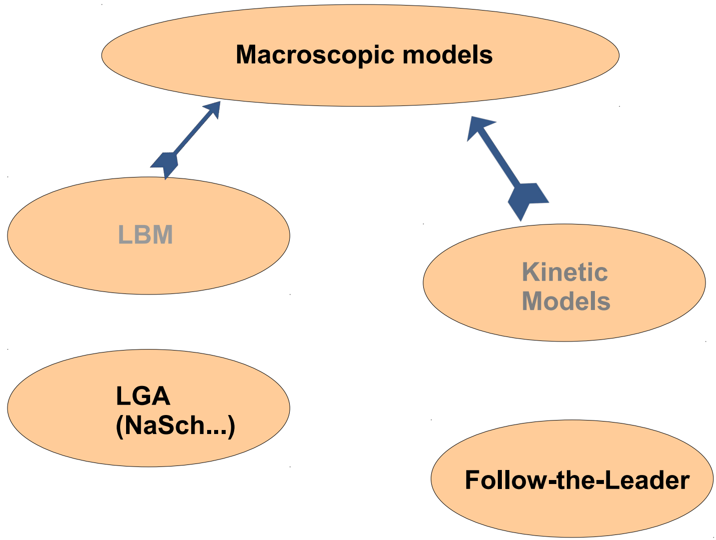


Road traffic by cellular automata

Many improvements, real life applications



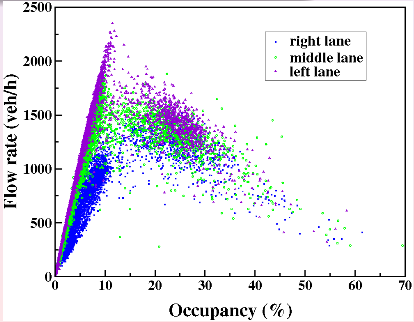
Modèles pour le traffic routier



Mass conservation

Model LWR (1955-1956)

$$\begin{aligned} \partial_t \rho + \partial_x(\rho u) &= 0 \\ u &= V(\rho) \end{aligned}$$

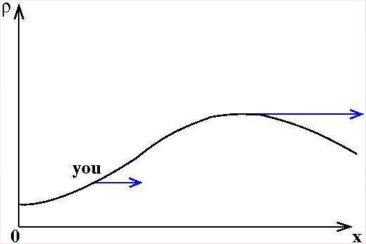


Macroscopic model for car traffic

Mass conservation

Payne-Whitham model (1971)

$$\begin{aligned}\partial_t \rho + \partial_x(\rho u) &= 0\\ \partial_t u + u\partial_x u &= -\frac{1}{\rho}p'(\rho)\partial_x\rho + \frac{1}{\tau}\left(V(\rho)-u\right)\end{aligned}$$



Macroscopic model for car traffic

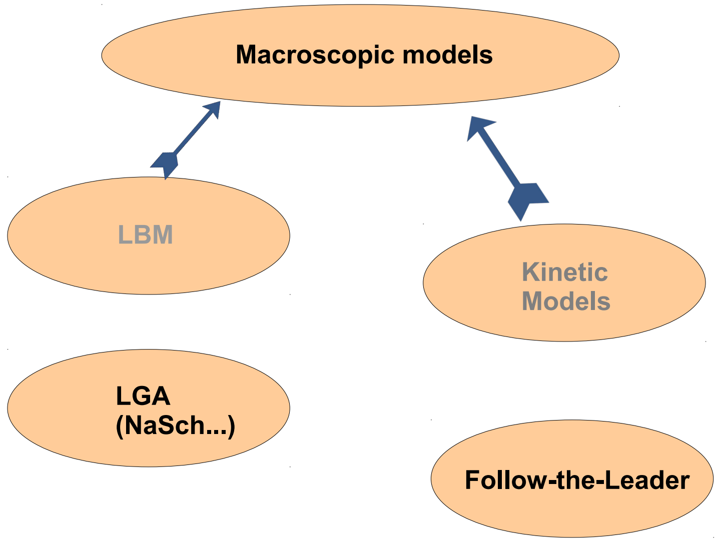
Aw-Rascle model (2000)

$$
\begin{aligned}
\partial_t \rho + \partial_x(\rho u) &= 0 \\
\partial_t(\rho u) + \partial_x(\rho u u) &= -\rho \frac{dp}{dt} + \frac{1}{\tau}\left(V(\rho) - u\right)
\end{aligned}
$$

where

$$d/dt = \partial_t + u\partial_x \tag{1}$$

Modèles pour le traffic routier



Follow-the-leader Model

Un exemple :

[Aw et al (2000)]

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= C_\gamma \frac{(v_{i+1} - v_i)}{(x_{i+1} - x_i)^{\gamma+1}} + A\frac{1}{T_r}\left(V(\rho_i) - v_i\right) \end{aligned}$$

where

$$\rho_i = \frac{l}{(x_{i+1} - x_i)}$$

Micro-Macro derivation

Pas de chaos moléculaire

Systèmes homogènes

$$\rho_i = \frac{l}{(x_{i+1} - x_i)}$$

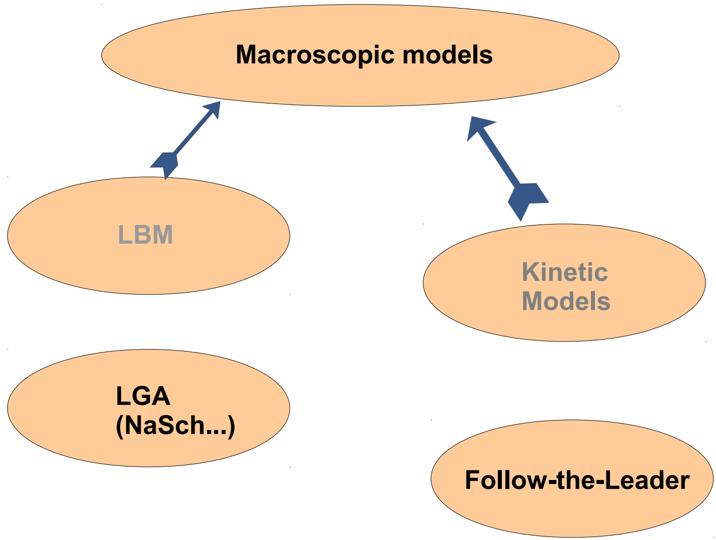
[Berg, Mason, Woods, PRE (2000)]

Systèmes inhomogènes

$$\int_0^{x_{i+1}-x_i} \rho(x+y)dy = 1$$

Expansion in powers of y

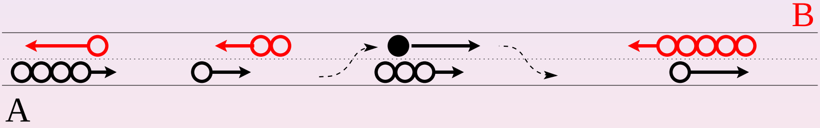
Modèles pour le traffic routier



Bidirectional road

[C. Appert-Rolland, H.J. Hilhorst and G. Schehr : *Spontaneous symmetry breaking in a two-lane model for bidirectional overtaking traffic*, J. Stat. Mech. (2010) P08024]

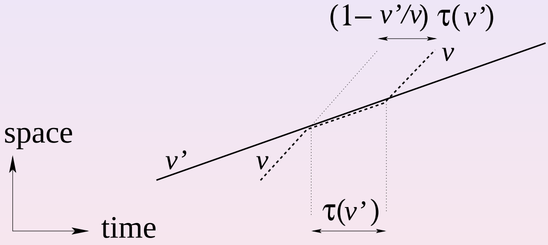
- Continuous space and time



- Distribution of desired velocities $P(v)$
  (Minimum v_0)
- Need a delay τ_0 to take over

For a given lane...

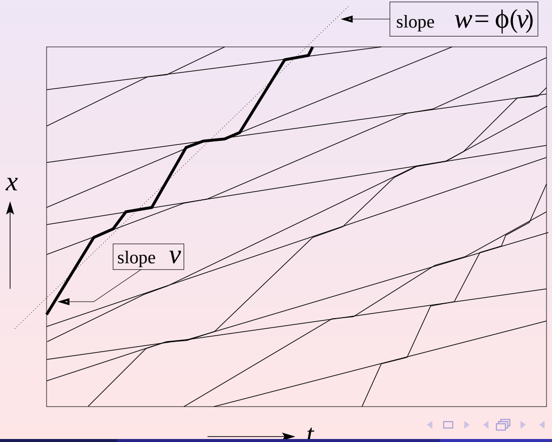
A vehicle with desired velocity v has to wait for a queuing time $\tau(v')$ to take over a vehicle of velocity $v' < v$.



For a given lane...

A vehicle with desired velocity v has to wait for a queuing time $\tau(v')$ to take over a vehicle of velocity $v' < v$.

➥ possible to compute the effective velocity $\phi(v)$ of each vehicle having a desired velocity v



For a given lane...

A vehicle with desired velocity v has to wait for a queuing time $\tau(v')$ to take over a vehicle of velocity $v' < v$.

➥ possible to compute the effective velocity $\phi(v)$ of each vehicle having a desired velocity v

- Open boundary conditions

  Injection with rate $\bar{\omega}$

$$\frac{1}{\phi(v)} = \frac{1}{v_0} - \int_{v_0}^{v} dv' \left[v' + \int_{v_0}^{v'} dv'' \, (v' - v'') \bar{\omega} P(v'') \tau(v'') \right]^{-2} .$$

➥ In particular $\phi(v_0) = v_0$

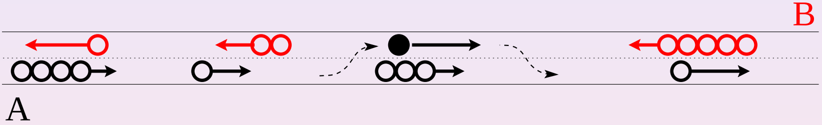
- Similar expressions for periodic boundary conditions

➥ Density of platoons of a certain length

➥ Density of free vehicles, etc ...

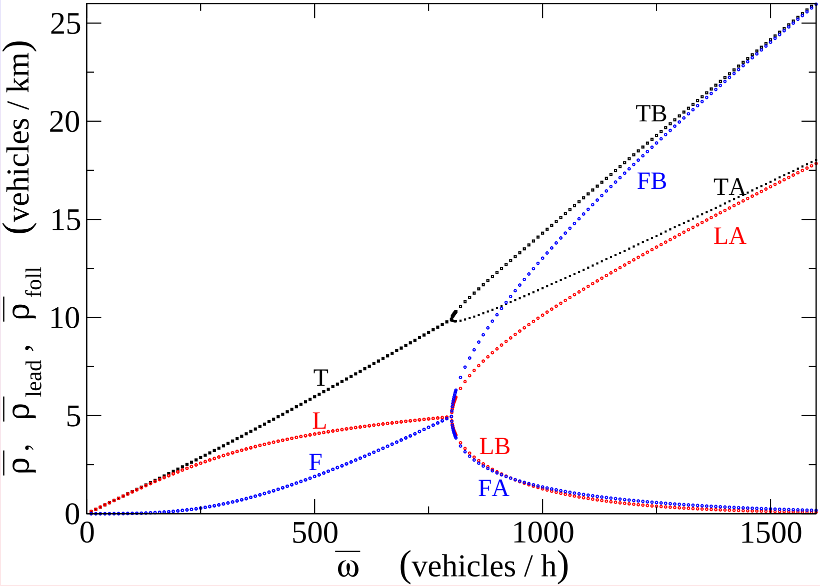
Mean-field coupling between 2 lanes

The waiting time $\tau(v_{slow})$ is computed from the configuration on the other lane (distribution of holes).
How long does it take to meet in the opposite lane a hole of duration greater than τ_0?



- ➥ Mean-field coupling between the lanes
- ➥ Two coupled equations to solve numerically

Mean-field coupling between 2 lanes

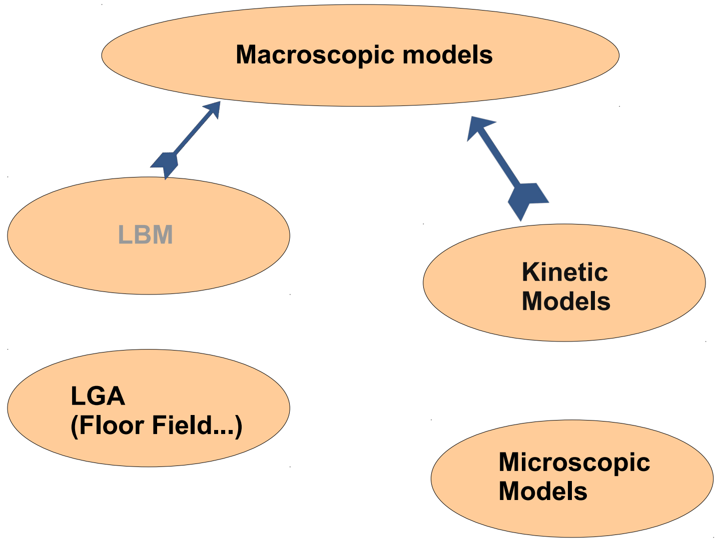


Modèle microscopique de route bidirectionnelle

- Spontaneous symmetry breaking in a mean-field description of a bidirectional road
- Microscopic model: asymmetry also observed between the lanes
- Size of the vehicles negligeable if $\bar{\rho} << 40$ veh/km;
  - Transition around $\bar{\rho} = 5$ veh/km, observable on real data?

PEDESTRIANS

Modèles pour le traffic piétonnier



Modèles pour le traffic piétonnier

- Mass conservation
- Transport in 2D space
- Destination for each pedestrian
- Less inertial effects

Pedestrians : Microscopic Models

First Generation Models

- Boids
- Rule models
- Force models

Boids

[Craig W. Reynolds, Computer Graphics (1987)]

- Flocks, Herds, and Schools

Pedestrians : Microscopic Models

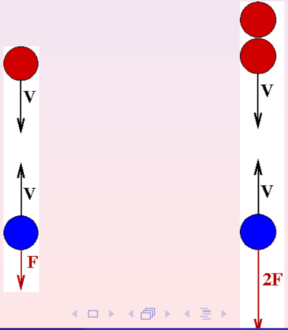
First Generation Models

- Boids
- Rule models
- Force models

Social force model

[D. Helbing & P. Molnár, PRE (1995)]

- Position Based Model
- Multiple interactions: Sum of forces



Pedestrians : Microscopic Models

First Generation Models

- Boids
- Rule models
- Force models

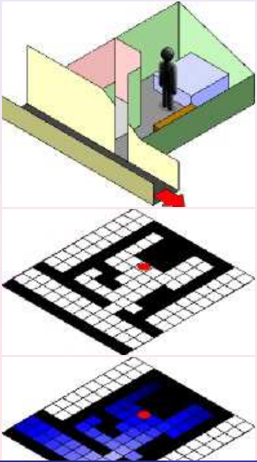
Cellular automata model

- Floor field model

➥ isotropy pbl

[C. Burstedde et al, Physica A 295 (2001) 507-525]

- PEDGO Software

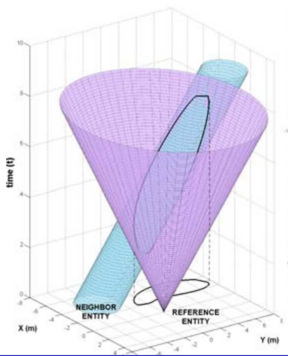


Pedestrians : Microscopic Models

Velocity based models

- [Paris, Pettré, Donikian (2007)]
- [RVO (2008)]
- [Pettré *et al* (2009)]
- [Ondrej et al (2010), Moussaïd et al (2011)]

- Determination of admissible velocities (to avoid collision in the next few seconds)
- Optimal choice among this set of velocity

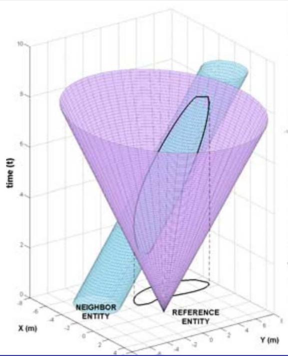


[Paris *et al* (2007)]

Pedestrians : Microscopic Models

- [Paris, Pettré, Donikian (2007)]
- [RVO (2008)]
- [Pettré *et al* (2009)] ➥ Velocities are gradually evaluated
- [Ondrej et al (2010), Moussaïd et al (2011)] ➥ Decoupling of velocity modulus and angle

- Determination of admissible velocities (to avoid collision in the next few seconds)
- Optimal choice among this set of velocity
- Automatic composition of interactions



[Paris *et al* (2007)]

Pedestrians : Microscopic Models

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- [Ondrej et al (2010), Moussaïd et al (2011)]

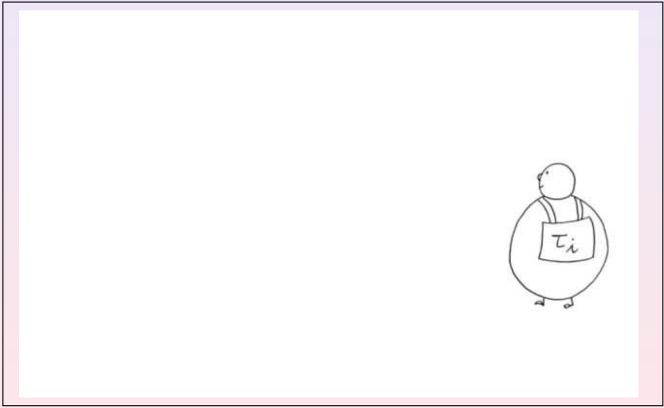
- Determination of velocities ?
- ➥ visual information
- ➥ cognitive process

- Vision based model [Ondrej et al, SIGGRAPH 2010]

Vision based model

[Ondrej et al, SIGGRAPH 2010]
[Cutting et al, 1995]

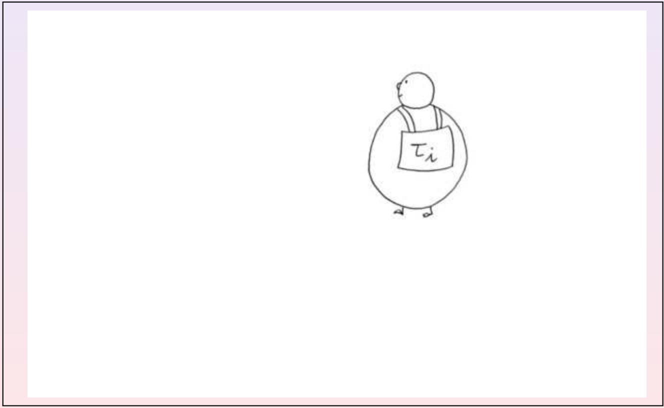
- Movement



Vision based model

[Ondrej et al, SIGGRAPH 2010]
[Cutting et al, 1995]

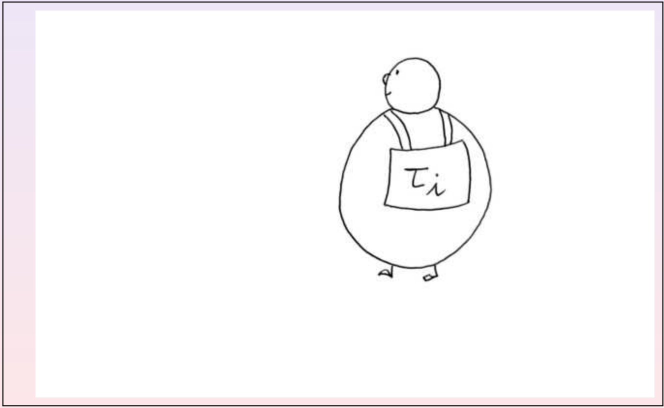
- Movement
- Size



Vision based model

[Ondrej et al, SIGGRAPH 2010]
[Cutting et al, 1995]

- Movement
- Size



Vision based model: Perception

- Movement

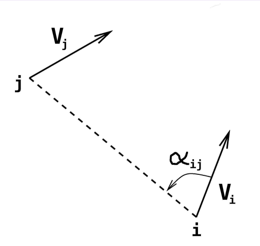
➥ time Derivative of the Bearing Angle (DBA) $\dot{\alpha}_{ij}$

  ☆ Future collision if $\dot{\alpha}_{ij} = 0$

- Size

➥ time to interaction (tti) τ_{ij}

  ☆ Soon if τ_{ij} small

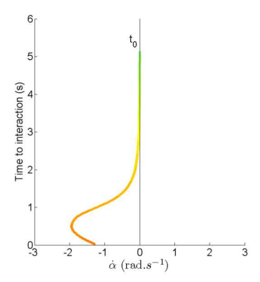


Vision based model: Reaction

[Ondrej et al, SIGGRAPH 2010]

Velocity can change in

- modulus
- direction

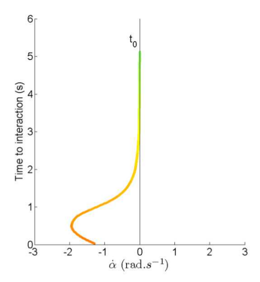


Vision based model: Reaction

[Ondrej et al, SIGGRAPH 2010]

Velocity can change in

- modulus
- direction ✔



Vision based model: Reaction

[Degond, A-R, Pettré, Theraulaz (2013) Kinetic and Related Models]

How threatening is the collision?

$$\Phi(|\dot{\alpha}_{ij}|, |\tau_{ij}|) = \Phi_0 \max\{\sigma - |\dot{\alpha}_{ij}|, 0\} \quad \text{with} \quad \sigma = a + \frac{b}{(|\tau_{ij}| + \tau_0)^c}$$

- Parameters a, b, and c can be evaluated from experiments
- τ_0 will bound the angular speed of pedestrians

Vision based model: Reaction

[Degond, A-R, Pettré, Theraulaz (2013) Kinetic and Related Models]

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$$\Phi(|\dot{\alpha}_{ij}|, |\tau_{ij}|) = \Phi_0 \max\{\sigma - |\dot{\alpha}_{ij}|, 0\} \quad \text{with} \quad \sigma = a + \frac{b}{(|\tau_{ij}| + \tau_0)^c}$$

- One pair interaction: Φ = angular velocity
- In general:
  - Multiple interactions
  - Target ξ

➔ $\Phi_c(\mathbf{x}, \mathbf{u}, \xi)$ = cost function of most threatening collision $= \max_j \Phi$

➔ $\Phi_t(\mathbf{x}, \mathbf{u}, \xi)$ = cost function for deviating from the target

$\min(\Phi_c + \Phi_t)$ ➥ optimal velocity $\mathbf{u}_i(t)$ $(||\mathbf{u}_i|| = 1)$

Mean-field kinetic model

Probability distribution $f(\mathbf{x}, \mathbf{u}, \xi, t)$

$$\partial_t f + c\mathbf{u} \cdot \nabla_x f + \nabla_u \cdot (\omega_f \, \mathbf{u}^\perp f) = d\Delta_u f.$$

Determination of $\omega_f(\mathbf{x}, \mathbf{u}, \xi, t)$:

Extremum of cost function is ill-defined when a probability distribution is considered

- ➥ $\Phi_c(\mathbf{x}, \mathbf{u}, t) =$ weighted average of the cost functions Φ
- ➥ $\Phi_t(\mathbf{x}, \mathbf{u}, \xi, t)$ is the same as for the IBM

$$\Rightarrow \omega_f(\mathbf{x}, \mathbf{u}, \xi, t)\mathbf{u}^\perp = -\nabla_u \left[\Phi_c(\mathbf{x}, \mathbf{u}, \xi, t) + \Phi_t(\mathbf{x}, \mathbf{u}, \xi, t)\right]$$

Macroscopic model

$$\rho(\mathbf{x}, \xi, t) = \int_{\mathbf{u} \in \mathbb{S}^1} f(\mathbf{x}, \mathbf{u}, \xi, t) \, d\mathbf{u},$$

$$\mathbf{U}(\mathbf{x}, \xi, t) = \frac{1}{\rho(\mathbf{x}, \xi, t)} \int_{\mathbf{u} \in \mathbb{S}^1} f(\mathbf{x}, \mathbf{u}, \xi, t) \, \mathbf{u} \, d\mathbf{u}$$

Moment method

Multiply the kinetic equation by the moments of **u**: $(1, \mathbf{u}, \cdots)$
Integrate over **u**

➥ Closure problem ➔ Need for a closure relation

Macroscopic model: Closure relations

Monokinetic closure

$$f(\mathbf{x}, \mathbf{u}, \xi, t) = \rho(\mathbf{x}, \xi, t)\delta_{\mathbf{U}(\mathbf{x},\xi,t)}(\mathbf{u}).$$

(no noise)

$$\partial_t \mathbf{U} + c\mathbf{U} \cdot \nabla_x \mathbf{U} = \omega_{\rho\delta_U}(\mathbf{x}, \mathbf{U}(\mathbf{x}, \xi, t), \xi, t)\, \mathbf{U}^{\perp}(\mathbf{x}, \xi, t)$$

where again $\omega_{\rho\delta_U}$ is determined from a cost function.

Other closure relations are possible

Macroscopic model

Hydrodynamic limit

Force and diffusion dominate ➥ Development around a Local Thermodynamical Equilibrium solution f^0

$$\nabla_u \cdot (\omega_f^0 \, \mathbf{u}^\perp \, f^0) = d \Delta_u f^0.$$

First order macroscopic model

$$\partial_t \rho_{(x,t)}(\xi) + \nabla_x \cdot (c \rho_{(x,t)}(\xi) U_{x,[\rho_{(x,t)}]}(\xi)) = 0$$

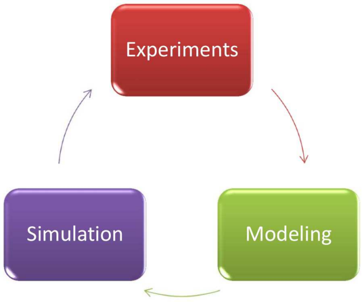
supplemented by a relation giving $U_{x,[\rho_{(x,t)}]}(\xi)$

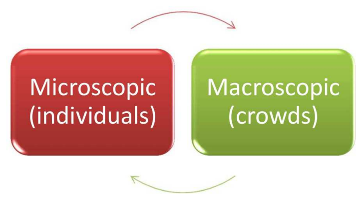
Perspective

- Compare macroscopic models
- Can macroscopic model reproduce pattern formation?

PEDIGREE Project

PEDIGREE = PEDestrian GRoups: EmErgence of collective behavior through experiments, modelling and simulation





PEDIGREE Project

<u>IMT, Toulouse</u>

P. Degond

J. Fehrenbach

J. Hua   S. Motsch

J. Narski

L. Navoret

<u>INRIA-Rennes</u>

J. Pettré

S. Lemercier

S. Donikian

<u>CRCA, Toulouse</u>

G. Théraulaz

M. Moussaïd

M. Moreau

<u>LPT, Paris-Sud</u>

C. Appert-Rolland

A. Jelic

J. Cividini

Experiments

Aim:

- Well-controlled experiments
- Reference data
- Multi-scale data

High precision motion capture: VICON system

Experiments with pedestrians

Two experimental campaigns (250 persons),
with the help from M2S (Univ. Rennes 2)

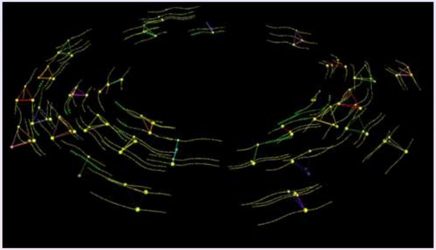
Ring corridor
Mono- or bi-directional flow

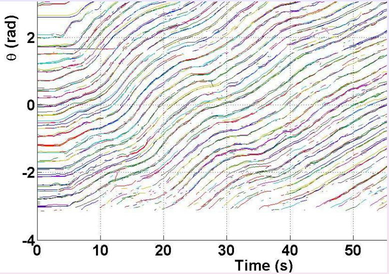
➥ lane formation, jamming

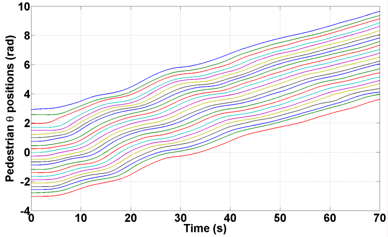
One-dimensional circle
No passing

➥ longitudinal interactions

Reconstruction of trajectories

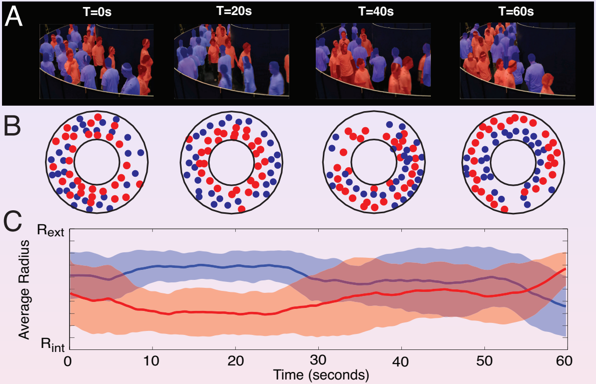






- From raw data to 3D markers' trajectories
- From markers to pedestrians
- Interpolating for missing data

Ring



[M. Moussaïd, E. Guillot, M. Moreau, J. Fehrenbach, O. Chabiron, S. Lemercier, J. Pettré, C. Appert-Rolland, P. Degond and G. Theraulaz, *Traffic Instabilities in Self-organized Pedestrian Crowds*, PLoS Computational Biology (2012)]

Ring

Ring

Macroscopic model for pedestrians in a corridor

$$
\begin{aligned}
&\partial_t \rho_+ + \partial_x(\rho_+ u_+) = 0, \\
&\partial_t \rho_- + \partial_x(\rho_- u_-) = 0, \\
&\partial_t(\rho_+ u_+) + \partial_x(\rho_+ u_+ u_+) = -\rho_+ \left(\frac{d}{dt}\right)_+ [p(\rho_+, \rho_-)], \\
&\partial_t(\rho_- u_-) + \partial_x(\rho_- u_- u_-) = \rho_- \left(\frac{d}{dt}\right)_- [p(\rho_-, \rho_+)],
\end{aligned}
$$

where

$$(d/dt)_\pm = \partial_t + u_\pm \partial_x$$

[C. A-R, P. Degond, and S. Motsch. *Two-way multi-lane trafic model for pedestrians in corridors*. Networks and Heterogeneous Media, **6**:351, (2011).]

Ring

Macroscopic model for pedestrians in a corridor

$$\begin{aligned} u_+ &= w_+ - p(\rho_+, \rho_-) \\ -u_- &= w_- - p(\rho_-, \rho_+) \end{aligned}$$

where w is a Rieman invariant

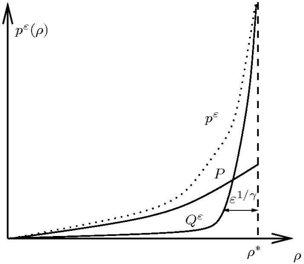
$$\begin{aligned} \partial_t w_+ + u_+ \partial_x w_+ &= 0 \\ \partial_t w_- + u_- \partial_x w_- &= 0 \end{aligned}$$

Ring

$$p(\rho_+,\rho_-) = P(\rho) + Q^\varepsilon(\rho_+,\rho_-), \quad \text{with} \quad \rho = \rho_+ + \rho_-$$

$$P(\rho) = M\rho^m, \quad m \geq 1,$$

$$Q^\varepsilon(\rho_+,\rho_-) = \frac{\varepsilon}{q(\rho_+)\left(\frac{1}{\rho} - \frac{1}{\rho^*}\right)^\gamma}, \quad \gamma > 1.$$



Ring

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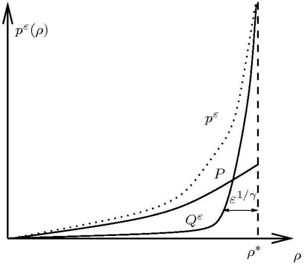

Ring

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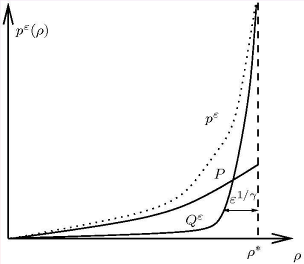
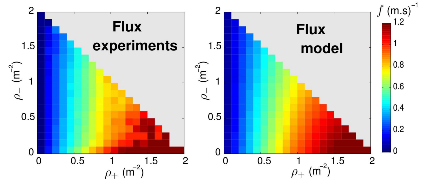
$$P(\rho) = M\rho^m, \quad m \geq 1,$$

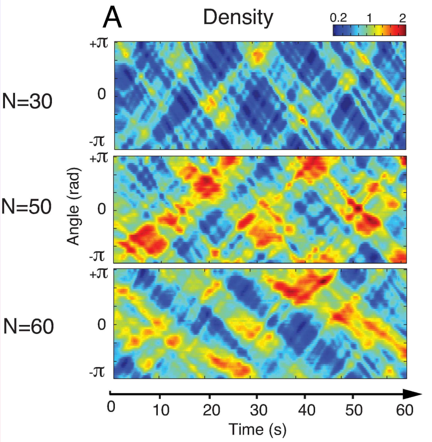
$$Q^\varepsilon(\rho_+,\rho_-) = \frac{\varepsilon}{q(\rho_+)\left(\frac{1}{\rho} - \frac{1}{\rho^*}\right)^\gamma}, \quad \gamma > 1.$$



Ring

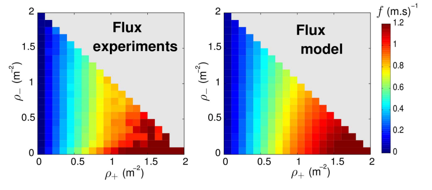


Flux
experiments
Flux
model
ρ_- (m^{-2})
ρ_+ (m^{-2})
f (m.s)$^{-1}$
0 0.2 0.4 0.6 0.8 1 1.2
0 0.5 1 1.5 2

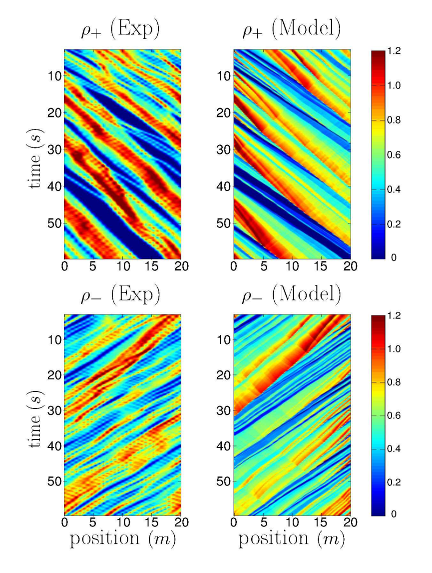


A
Density
0.2 1 2
N=30
N=50
N=60
Angle (rad)
+π
0
-π
0 10 20 30 40 50 60
Time (s)

Ring

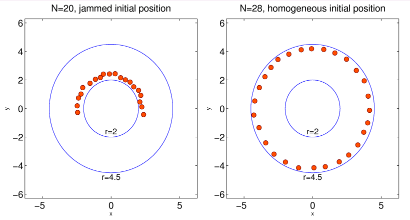


Flux experiments
Flux model
ρ_- (m^{-2})
ρ_+ (m^{-2})
f (m.s)$^{-1}$



ρ_+ (Exp)
ρ_+ (Model)
ρ_- (Exp)
ρ_- (Model)
time (s)
position (m)

1D Circle

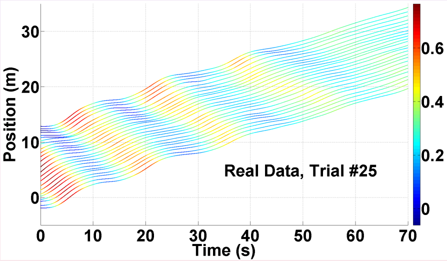


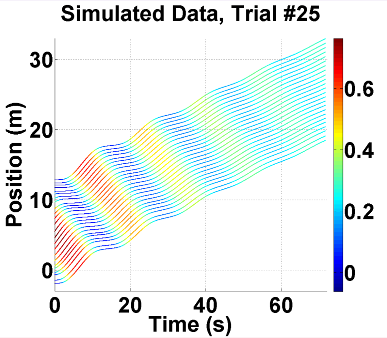
Density varying from 0.31 to 1.86 ped/m.

1D Circle

$$a(t) = C\frac{\Delta v(t-\tau)}{[\Delta x(t)]^{\gamma}}$$

Ex: ρ = 1.59 ped/m



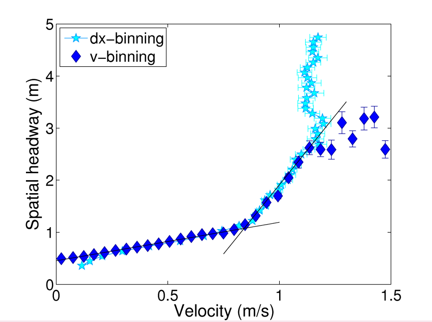


[S. Lemercier et al, *A realistic model of following behavior for crowd simulation*, EUROGRAPHICS (2012)]

1D Circle

1D Circle

1D Circle



Several regimes

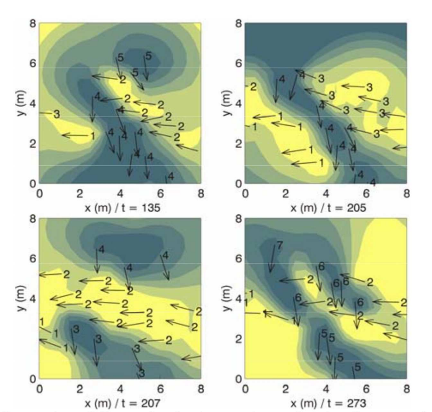
- Free
- Weakly constrained
  - ➥ $t_{adaptation} = 5.32$ s
- Strongly constrained
  - ➥ $t_{adaptation} = 0.74$ s

[A. Jelić, C. A-R, S. Lemercier, J. Pettré,
Properties of pedestrians walking in line – Fundamental diagrams,
Phys. Rev. E, **85** (2012) 036111]

Intersection of two perpendicular pedestrian flows

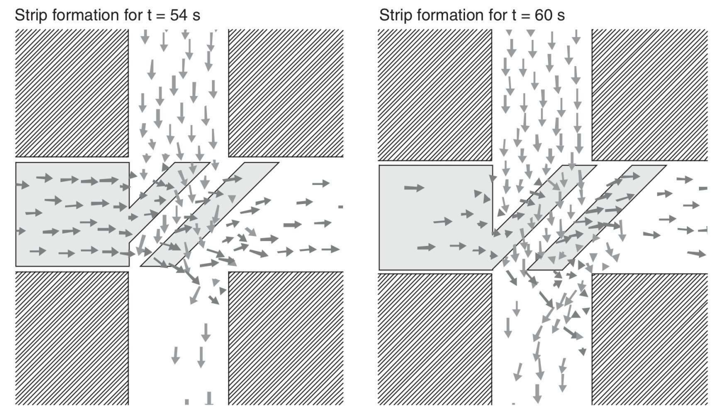
Diagonal instability: • observed in experiments

in [Hoogendoorn & Daamen, TGF'03 (Springer) 2005, pp. 121]



Intersection of two perpendicular pedestrian flows

Diagonal instability:
- observed in simulations

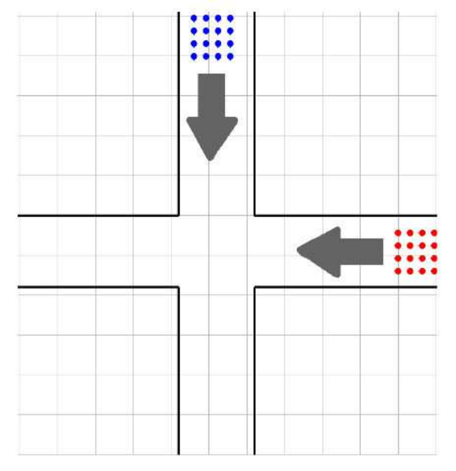


[Hoogendoorn & Bovy, Optim. Control Appl. Meth., 24 (2003) 153]

Intersection of two perpendicular pedestrian flows

Diagonal instability:

- observed in simulations

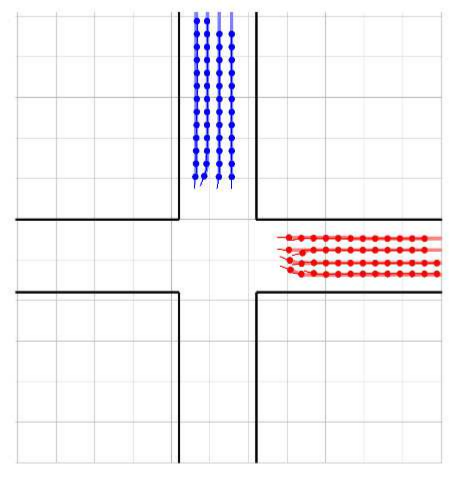


[Ondrej et al, SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows

Diagonal instability:

- observed in simulations



[Ondrej et al, SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows

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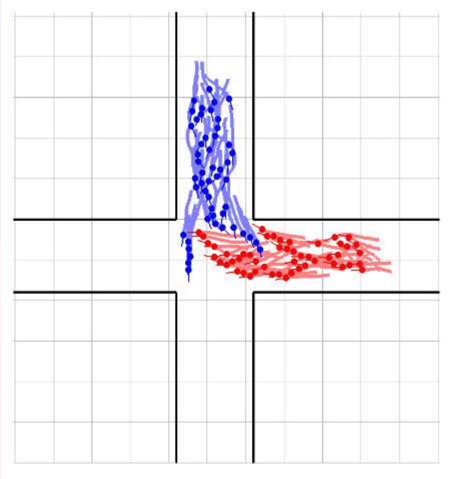
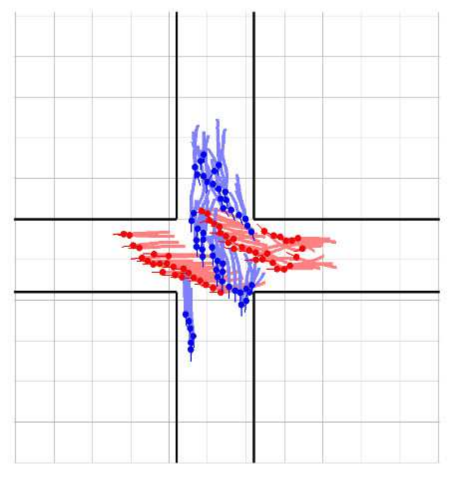


[Ondrej et al, SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows

Diagonal instability:

- observed in simulations

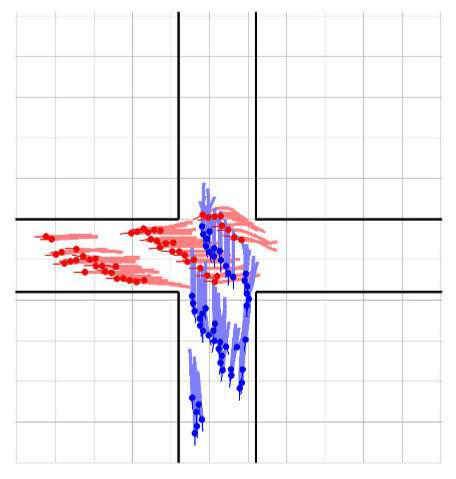


[Ondrej et al, SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows

Diagonal instability:

- observed in simulations

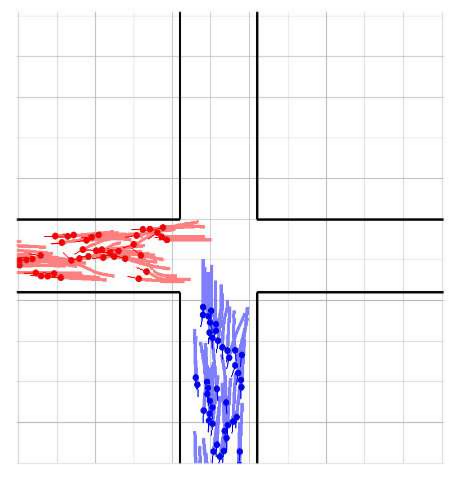


[Ondrej et al, SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows

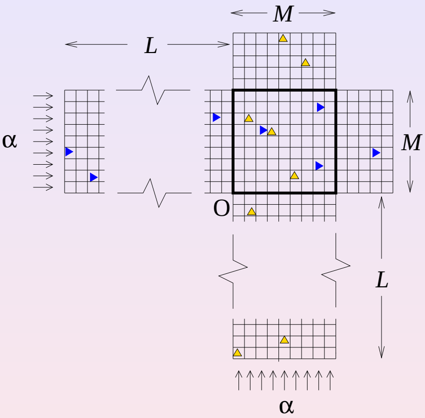
Diagonal instability:

- observed in simulations



[Ondrej et al. SIGGRAPH 2010]

Intersection of two perpendicular pedestrian flows



- $\mathcal{E}$ = Eastbound particles
- $\mathcal{N}$ = Northbound particles

$n^{\mathcal{E}}(\mathbf{r})$, $n^{\mathcal{N}}(\mathbf{r})$ = boolean occupation variables

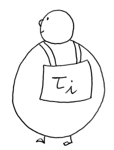
- As α increases: jamming transition
  [H. J. Hilhorst, C. A-R, J. Stat. Mech. (2012) P06009]

➥ Here we consider only the free flow phase.

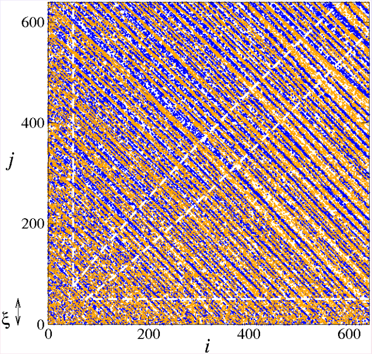
Updates

Cellular automaton = geometry + rules + update

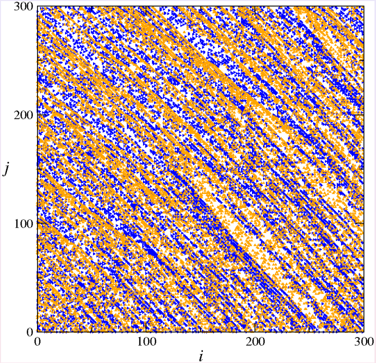
- Frozen shuffle update
  - Each particle has a phase $\tau_i \in [0, 1[$
  - At each time step, update in the order of increasing phase.
- Alternating parallel update
  - $\mathcal{E}$ particles are updated in parallel at integer times t
  - $\mathcal{N}$ particles are updated in parallel between integer times, at $t + \frac{1}{2}$



Observations



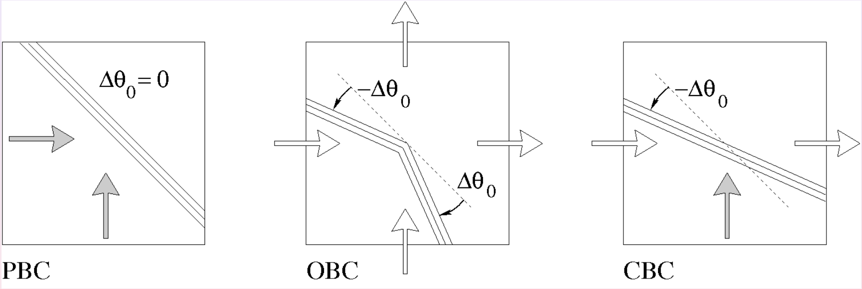
- frozen shuffle update
- $M = 640$



- alternating parallel update
- $M = 300$

Observations

Summary: pattern depends on the boundary conditions



Mean field equations

We postulate some mean-field equations:

$$\begin{aligned}\rho^{\mathcal{E}}_{t+1}(\mathbf{r}) &= [1-\rho^{\mathcal{N}}_t(\mathbf{r})]\rho^{\mathcal{E}}_t(\mathbf{r}-\boldsymbol{e}_x)+\rho^{\mathcal{N}}_t(\mathbf{r}+\boldsymbol{e}_x)\rho^{\mathcal{E}}_t(\mathbf{r})\\ \rho^{\mathcal{N}}_{t+1}(\mathbf{r}) &= [1-\rho^{\mathcal{E}}_t(\mathbf{r})]\rho^{\mathcal{N}}_t(\mathbf{r}-\boldsymbol{e}_y)+\rho^{\mathcal{E}}_t(\mathbf{r}+\boldsymbol{e}_y)\rho^{\mathcal{N}}_t(\mathbf{r})\end{aligned}$$

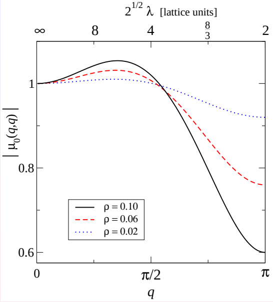
- pair correlations $\langle n^{\mathcal{E}} n^{\mathcal{N}} \rangle$ have been factorized
- interaction terms $\langle n^X n^X \rangle$ between same-type particles have been neglected (low density)

Simulations: same patterns as for the particle model

Mean field equations

- PBC
  - Linear stability analysis $\rho_t^{\mathcal{E},\mathcal{N}}(\mathbf{r}) = \overline{\rho} + \delta\rho_t^{\mathcal{E},\mathcal{N}}(\mathbf{r})$

➥ Most unstable mode traveling in the $(1,1)$ direction with wavelength



$$\lambda_{\max} = 2\pi/|\mathbf{q}|_{\max} \quad = \quad 3\sqrt{2}[1-(\sqrt{3}/\pi)\overline{\rho}] + \mathcal{O}(\overline{\rho}^2),$$

Mean field equations

- OBC
  - Linear stability analysis:

  ➥ Calculation of Green function
  [Cividini & Hilhorst (2014) arXiv:1406.5394]

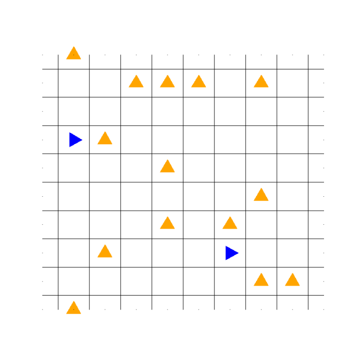
  ➥ diagonals, but no sign of the chevron effect

Chevron effect = non linear effect

Effective interactions

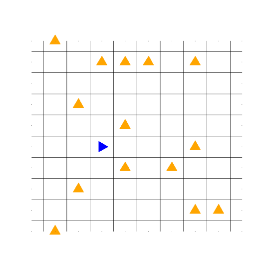
From which microscopic mechanism does the (tilted) diagonal pattern emerge?

➥ effective interaction between two $\mathcal{E}$ particles crossing a flow of $\mathcal{N}$ particles

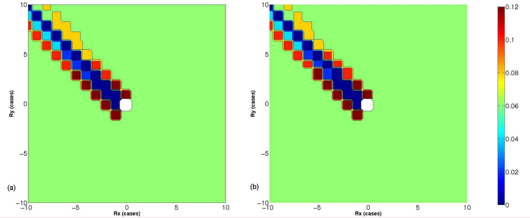


Wake of a single $\mathcal{E}$ particle

Ensemble averaged wake



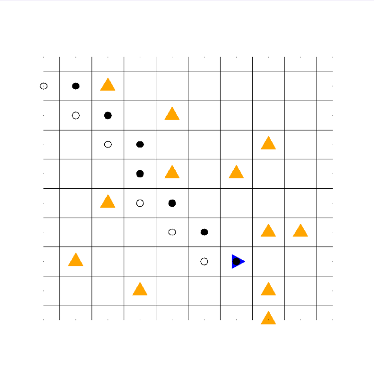
Frozen shuffle update



Theory — Simulation

Wake of a single $\mathcal{E}$ particle

Microscopic structure of the wake:



Central part of the wake : the shadow

Construction:

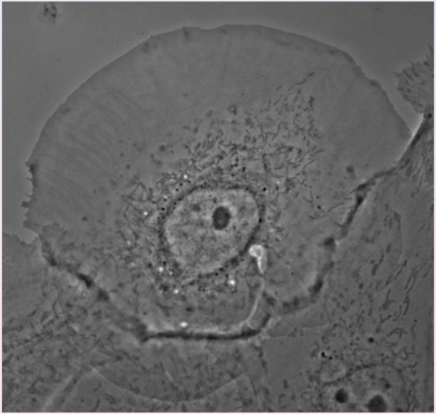
- Before move: white dot
- After move: black dot

At low density, $tan\theta \simeq 1 - \rho^{\mathcal{N}}$

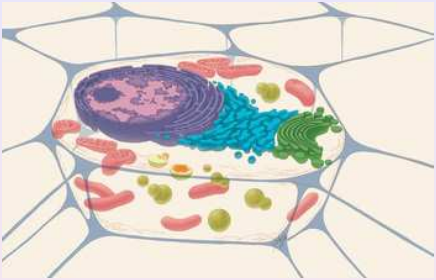
INTRACELLULAR TRANSPORT

Intra-cellular transport

- Need for transport



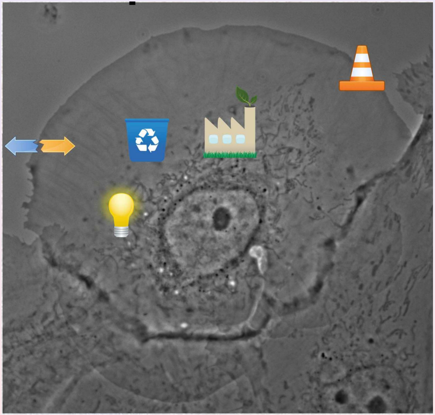
From [Wittmann et al, J. Cell Biol. 161:845 (2003)]



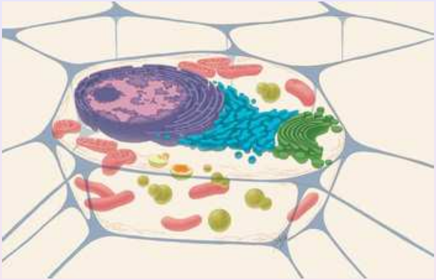
From [Judith Stoffer, NIGMS]

Intra-cellular transport

- Need for transport



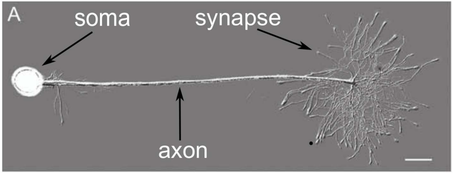
From [Wittmann et al, J. Cell Biol. 161:845 (2003)]



From [Judith Stoffer, NIGMS]

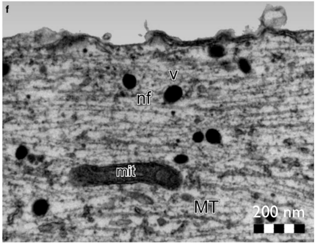
Intra-cellular transport

Shemesh et al., Traffic **9**, 458 (2008)



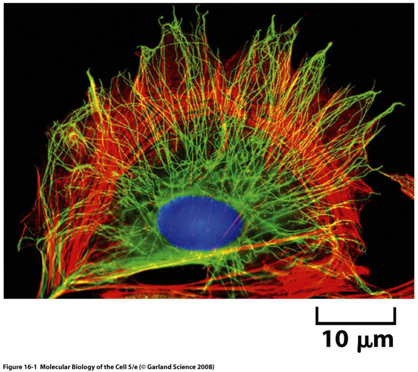
- Particular case: the axon
  - up to 1 m in human beings, a few microns for the diameter
  - crowded environment

v: vesicle
nf: neurofilament
mit: mitochondrion
MT: microtubule



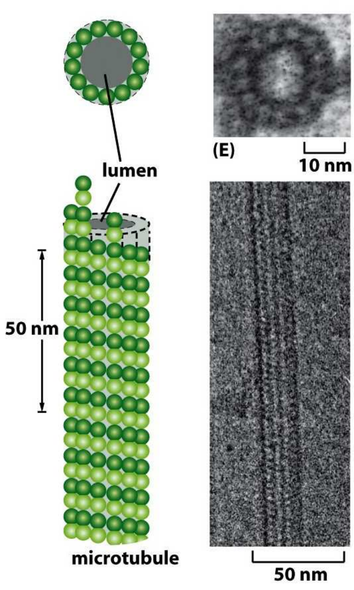
Shemesh & Spira, Acta Neuropathol **120**, 209 (2010)

Cytoskeleton



From [Alberts et al, *Molecular Biology of the Cell,* 5th ed. (2008)]

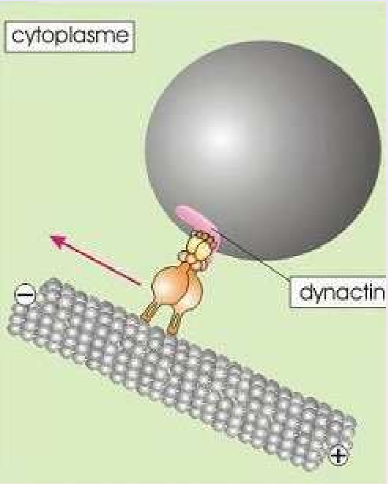
- Green = Microtubule
- Red = Actin
- Blue = DNA



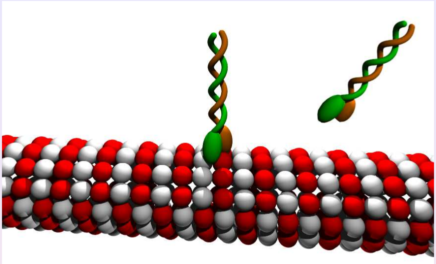
Molecular Motors

[National Institute on Aging - NIH]

Molecular Motors



[From www.ulysse.u-bordeaux.fr/atelier/ikramer/biocell_diffusion]

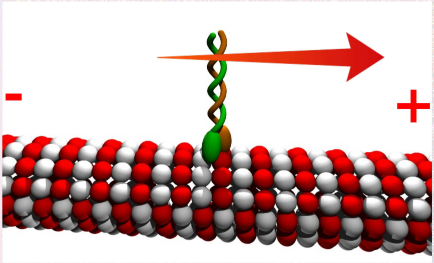


[Image crée à partir d'une image de wikipedia de Kebes]

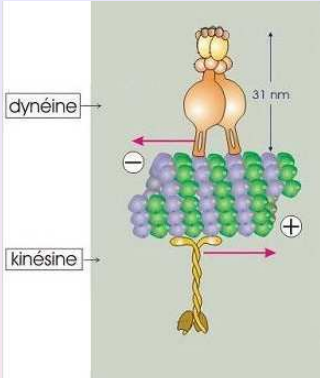
Molecular motors can attach and detach from the MT
➥ Processive and diffusive phases

Molecular Motors

Microtubules are polarized

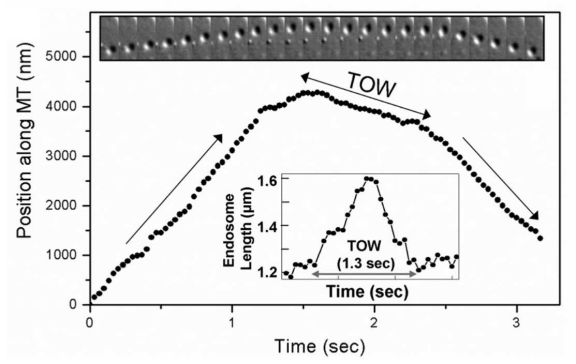


[Image crée à partir d'une image de wikipedia de Kebes]



[Modified from www.ulysse.u-bordeaux.fr/atelier/ikramer/biocell_diffusion]

Tug-of-war



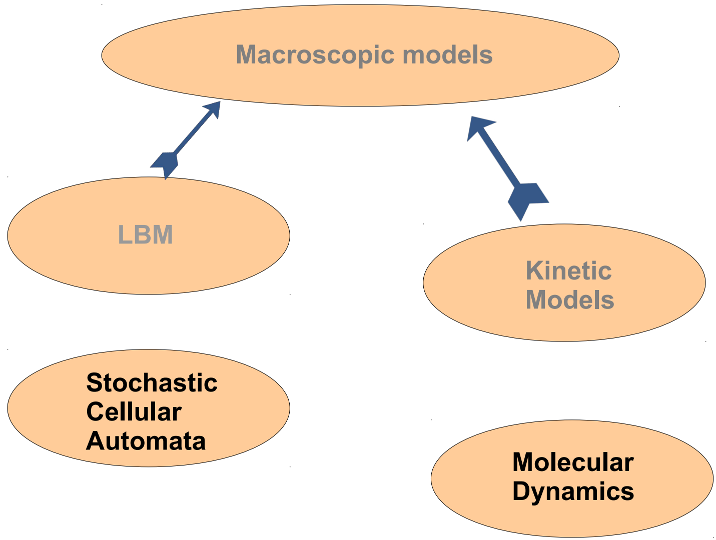
Endosome inside Dictyostelium cells.

[Soppina et al (2009) PNAS]

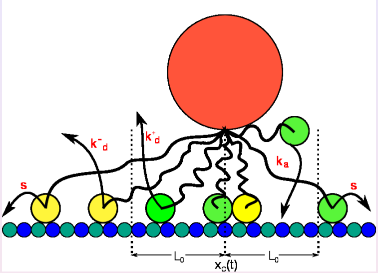
Tug-of-war

[LEX Commons]

Modèles pour le transport intracellulaire



Explicit Position Based Model



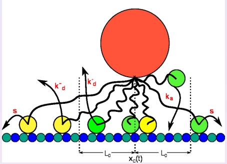
Stochastic Motor Dynamics:

- attachment rate $\tilde{\omega}$
- stepping rate $p = p(F_i)$
- detachment rate $\omega = \omega(F_i)$

Cargo dynamics

$$m\frac{\partial^2 x_C(t)}{\partial t^2} = -\beta\frac{\partial x_C(t)}{\partial t} + F(x_C, \{x_i\}) \quad \text{where } F = \sum_i F_i$$

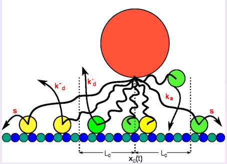
Tug-of-war, asymmetric motors



Asymmetric teams

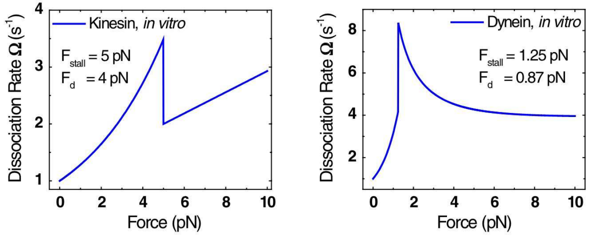
Kinesins and dyneins behave differently

Tug-of-war, asymmetric motors



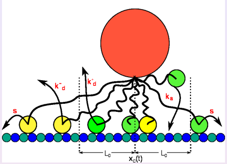
Stochastic motor dynamics

Detachment rate



From [Kunwar et al (2011) PNAS]

Tug-of-war, asymmetric motors



Stochastic motor dynamics

- Stepping rate (for F_i below stall force) :

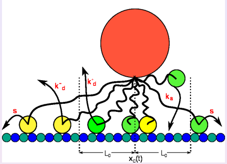
$$s(|F_i|,[ATP]) = \frac{k_{\rm cat}(|F_i|)[ATP]}{[ATP] + k_{\rm cat}(|F_i|)k_{\rm b}(|F_i|)^{-1}},$$

Michaelis-Menten kinetics

From [Schnitzer et al (2000) Nat. Cell Biol.]

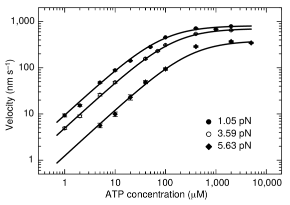
- Stepping rate (for F_i above stall force) :
  backward stepping $s_b = v_b/d$

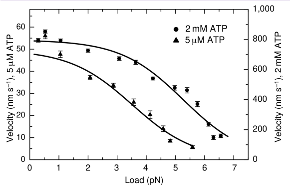
Tug-of-war, asymmetric motors



Stochastic motor dynamics

[ATP] and force dependence



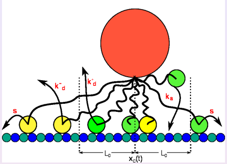


Comparison for kinesin

From [Schnitzer et al (2000) Nat. Cell Biol.]

From [Visscher et al (1999) Nature]

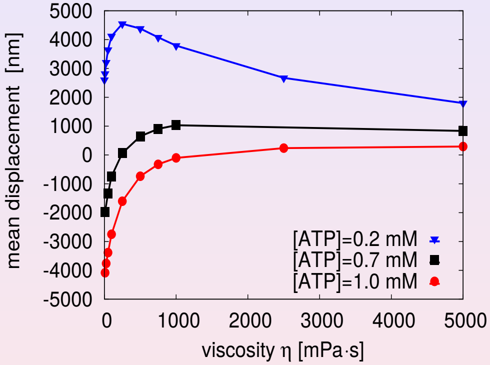
Tug-of-war, asymmetric motors



How does this cargo-motors complex behave?

Control by external force

Effective viscosity dependence



$$N_+ = N_- = 5$$

Advantage

Easy control of the cargo-motors complex by a single external parameter

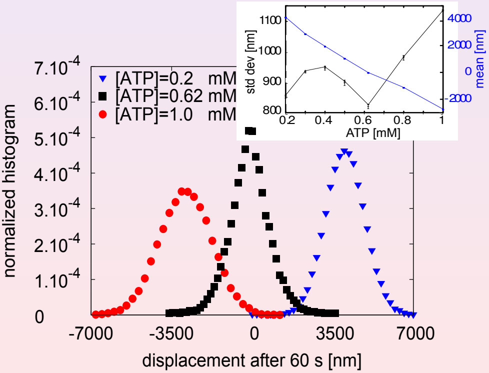
➥ Change of behavior in crowded areas?

From [Klein et al (2014) EPL]

Control by energy supply

Stall force ATP dependance

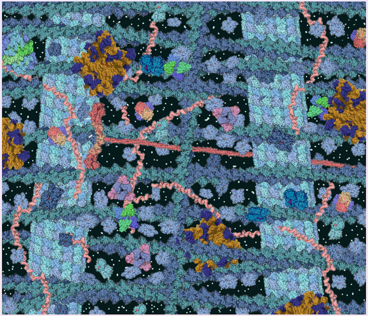
- Dynein: F_s varies linearly from 0.3 pN at vanishing [ATP] to 1.2 pN for saturating [ATP]
- Kinesin: constant $F_s = 2.6$ pN



$$N_+ = N_- = 5$$

From [Klein et al (2014) EPL]

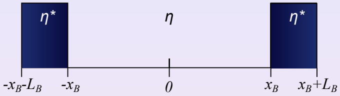
Microtubule based transport



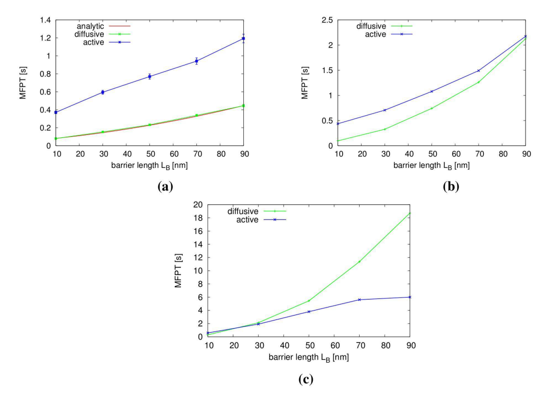
Why pulling by two opposite teams?

- Easy control
- More efficient in a crowded environment

Active transport versus diffusion

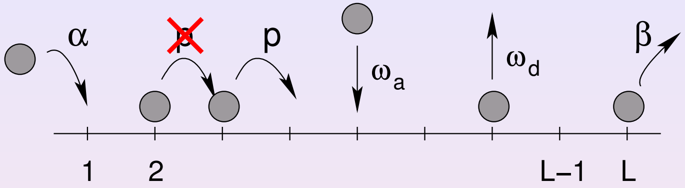


(a) $\eta^* = \eta$,
(b) $\eta^* = 10\eta$,
(c) $\eta^* = 100\eta$



[Klein et al, EPJST (2014)]

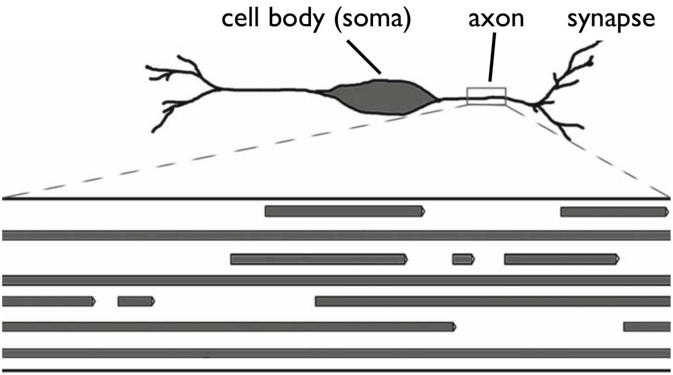
Collective effects



- Cellular automata models with one type of motors
  - [Lipowsky, Klumpp, & Nieuwenhuizen, P.R.L. (2001)]
  - [Parmeggiani, Franosch, & Frey, P.R.L. (2003)]
  - [J. Tailleur, M. Evans, & Y. Kafri, P.R.L. (2009)]

➥ well suited for motility assays (in vitro), predicts the experimentally observed bulk localization of high and low density domains [Nishinari, Okada, Schadschneider, & Chowdhury, P.R.L. (2005)].

Axonal transport



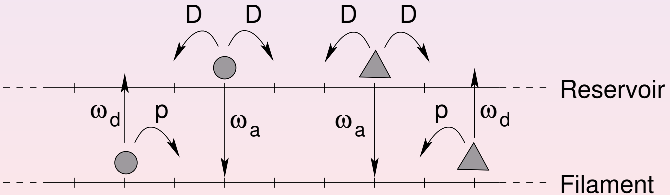
Falnikar & Baas, Res. Prob. Cell. Diff. **48**, 47 (2009)

Bidirectional intracellular traffic

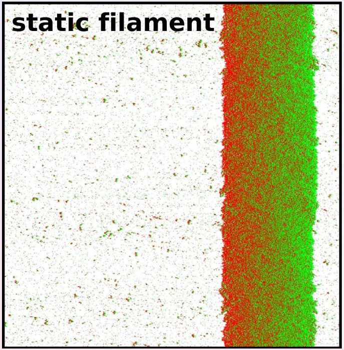
[M. Ebbinghaus and L. Santen, J. Stat. Mech. (2009)]

Ingredients

- Two types of motors going in opposite directions
- Confined diffusion in the surrounding cytoplasm



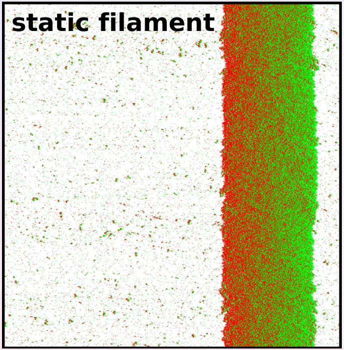
Bidirectional intracellular traffic



- Particles accumulate in a large cluster
- Clustering increases with system size

➥ No transport in thermodynamic limit

Bidirectional intracellular traffic



- Particles accumulate in a large cluster
- Clustering increases with system size

Offering multiple filaments enhances cluster formation.

Intra-cellular traffic

MTs exhibit stochastic switching between a shrinking and a growing state, termed dynamic instability.

[A. Viel, R. A. Lue and J. Liebler, BioVisions project, http://multi media.mcb.harvard.edu]

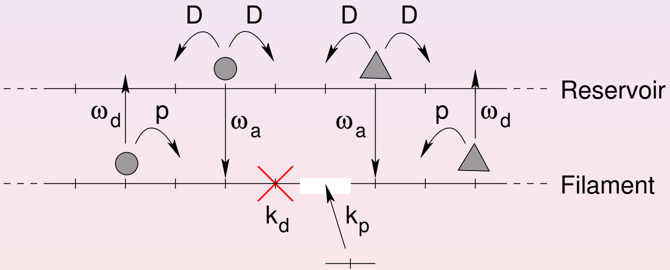
Microtubules seen by fluorescence in S. pombe (yeast)
[M. Erent, D.R. Drummond, R.A. Cross (2012) PLoS ONE 7(2): e30738]

Intra-cellular traffic

Experiment by
[*Shemesh and Spira, Traffic (2008)*]

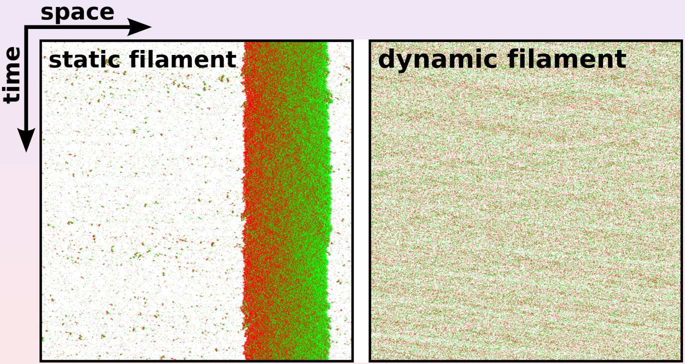
1s (video) = 120s (real time)
Scale bar = 10 μm

- Dynamics of the lattice
  - Some sites of the microtubule are eliminated with rate k_d and recreated with rate k_p.



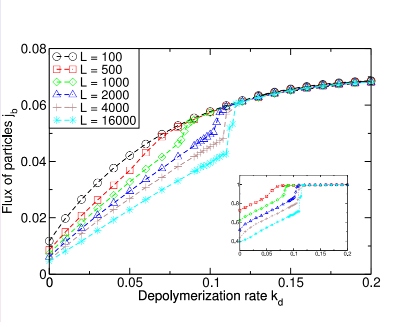
Dynamics of the lattice

- Dynamics of the lattice
  - Some sites of the microtubule are eliminated with rate k_d and recreated with rate k_p.



Bidirectional intracellular traffic

- Recovery of efficient transport through rapid dissolution of emerging clusters (optimal value of k_d).
- Transition to a density-dependent current.



[Ebbinghaus, Appert, Santen, PRE 82 (2010) 040901]

Robust for several types of lattice dynamics

Bidirectional intracellular traffic

☞ Drugs modifying the dynamics of the microtubules induce jams!

- video 1: microtubule dynamics with and without drugs (Paclitaxel)

[*Shemesh and Spira, Acta Neuropathol (2009)*]

Bidirectional intracellular traffic

☞ Drugs modifying the dynamics of the microtubules induce jams!

- video 2: microtubule dynamics and pinocytotic vesicles transport without drugs

[*Shemesh and Spira, Acta Neuropathol (2009)*]

☞ Drugs modifying the dynamics of the microtubules induce jams!

- video 3: microtubule dynamics and pinocytotic vesicles transport with drugs

[*Shemesh and Spira, Acta Neuropathol (2009)*]

THE END

THANK-YOU !!!

For more details:
http://www.th.u-psud.fr/page_perso/Appert/

Thank-you