# Adaptive multiresolution-based lattice Boltzmann schemes and their accuracy analysis *via* the equivalent equations

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**Abstract**

Multiresolution provides a fundamental tool based on the wavelet theory to build adaptive numerical schemes for Partial Differential Equations and time-adaptive meshes, allowing for precise error control [4]. This strategy is used to build adaptive lattice Boltzmann methods with this desirable feature [3, 2]. Furthermore, these schemes allow for an effective memory compression of the solution when spatially localized phenomena – such as shocks – are involved. Nevertheless, the peculiar way of modeling the desired physical phenomena in the lattice Boltzmann schemes calls, besides the possibility of controlling the error introduced by the mesh adaptation, for a deeper understanding of how this new scheme could alter the physics approximated by the numerical strategy. This issue is studied by writing the equivalent equations [5] of the adaptative method after having put the scheme under an adapted formalism [1]. It provides an essential tool to master the perturbations introduced by the adaptive numerical strategy, which can thus be devised in such a way as to preserve the desired features of the reference scheme. The theoretical considerations are corroborated by numerical experiments in both the 1D and 2D context, showing the relevance of the asymptotic analysis. In particular, we show that our numerical method outperforms traditional approaches, whether or not the solution of the reference scheme converges to the solution of the target equation. Furthermore, we discuss the influence of different collision strategies for non-linear problems, showing that they have only a marginal impact on the quality of the solution.

## References

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