Passivité et Systèmes Hamiltoniens à Ports

Applications en audio et acoustique musicale

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Séminaire du GT "Schémas de Boltzmann sur réseau à l'IHP"
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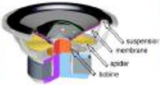
Institut Henri Poincaré, Paris, France










Motivation

(PHS=Port Hamiltonian System)

Why PHS for musical audio/acoustic applications ?

Instruments involve & PHS support:

1. **Multi-physics**: mechanics, acoustics, electronics, thermodynamics, etc.
2. **Power balance**: conservative/dissipative/external parts = passivity
   (+ time causality, irreversibility, natural symmetries)
3. **Nonlinearities:** amplitude-dependent timbre, self-oscillations, regime bifurcation, chaos, etc.
4. **Non-ideal dissipation:** crucial for realism
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Objectives

1. **Modelling**: Component-based approach
2. **Numerics**: power-balanced/passive schemes
   (accuracy, reject aliasing due to nonlinearities+sampling, etc.)
3. **Computational cost**: solvers in view of real-time sound synthesis
4. **Code generator**: component netlists → equations → C++ code
5. **Control**: power-balanced reprogrammed physics to reach behaviours
   (transducer correction, acoustic absorbers, hybrid instruments, etc.)

Outline

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1. Motivation
2. **PREAMBLE:** reminders on dynamical systems and Lyapunov analysis
3. **MODELLING**: Input-State-Output representations of PHS
4. **NUMERICS** with sound applications
5. **STATISTICAL PHYSICS** and Boltzmann principle for PHS
6. **CONTROL**: digital passive controller for hardware
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Stability and passivity in **nonlinear dynamical systems**

- Stability of an equilibrium point (autonomous system)
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$\rightarrow$ **Lyapunov analysis**

Preamble (1/4): autonomous systems

$$\begin{aligned} \dot{x}(t) &= f\big(x(t)\big), \quad \text{for } t \geq 0, \quad \text{with } f : \mathbb{R}^n \to \mathbb{R}^n \quad (n \in \mathbb{N}^*) \\ x(0) &= x_0 \in \mathbb{R}^n \end{aligned}$$

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Cauchy-Lipschitz theorem: *f locally Lipschitz $\Rightarrow \exists! t \mapsto x(t)$*

x can be defined on $J_{x_0} \subseteq \mathbb{R}$, an open maximal interval that contains 0, or on interval $J_{x_0}^+ := J_{x_0} \cap \mathbb{R}_+$, for its restriction to positive times.

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Stabilities of x^* (L: local, A: asymptotic, G: global)

(LS) if: $\forall R > 0, \exists r(R) > 0$ such that $\forall x_0 \in \mathbb{R}^n$,
$\|x_0 - x^*\| < r(R) \Rightarrow \|x(t) - x^*\| < R, \; \forall t \in J_{x_0}^+$

Lemma: if $\|x_0 - x^*\| < r(R)$, then $J_{x_0}^+ = \mathbb{R}^+$

(LAS) if: (LS) and $\exists r > 0$ s.t. $\|x_0 - x^*\| < r \Rightarrow \lim_{t \to +\infty} x(t) = x^*$

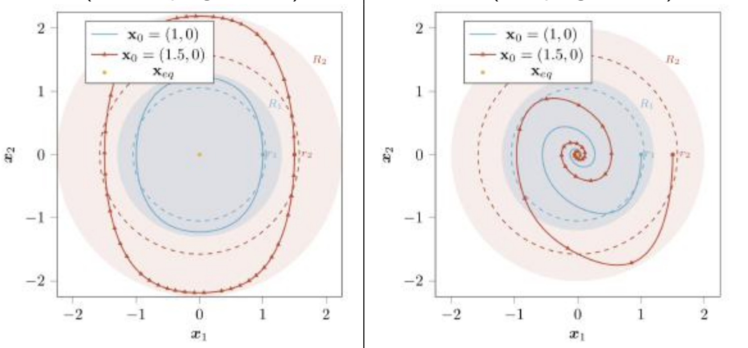
(GAS) if: (LAS) for all $r > 0$

Preamble (2/4): the Duffing oscillator

$$\ddot{y} + \alpha\dot{y} + (1+\beta y^2)y = 0$$

$$\dot{x}(t) = f\big(x(t)\big), \quad \text{with } x = [y, \dot{y}]^T,$$
$$\text{and } f(x) = [x_2,\ -\alpha x_2 - (1+\beta x_1^2)x_1]^T$$

LS (no damping: $\alpha = 0$) | LAS (damping: $\alpha > 0$)



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Definition (Hyp.: $x^* = 0$ and $\Omega \subseteq \mathbb{R}^n$ open set)

$V : \Omega \longrightarrow \mathbb{R}$ is a Lyapunov function of $\mathcal{S}$ if:

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($\Leftrightarrow \nabla V(x)^T f(x) \leq 0$, for all x in Ω)

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Let $\mathcal{I}$ be the **largest subset of** $\{x \in \Omega \text{ s.t. } \nabla V(x)^T f(x) = 0\}$ (points leaving V invariant) that is **invariant under the flow** in positive time.
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Usual difficulty: find a Lyapunov function for a given nonlinear f

Preamble (4/4): Passivity (input/output systems)

Input/output system (u: input, y: output, $\dim u = \dim y \geq 1$)

$$\mathcal{S} : \quad \dot{x} = f\big(x, u\big), \quad y = h(x, u) \quad \text{and } x(0) = x_0$$

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Passivity: $\mathcal{S}$ is passive if V satisfies (i-ii) and if (iii) is replaced by

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Strict passivity: $\frac{\mathrm{d}}{\mathrm{d}t} V \circ x(t) \leq y(t)^T u(t) - \psi\big(x(t)\big)$

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- $\rightarrow$ Stability for $u = 0$
- $\rightarrow$ Stabilization for dissipative feedback-loop laws: $(u = -Ry \Rightarrow y^T u = -R\|y\|^2 \leq 0)$
- $\rightarrow$ In physics, a natural Lyapunov function is the energy

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Port-Hamiltonian Systems

with

a component-based approach

(finite-dimensional case $\equiv$ *ODEs)*

A physical system is made of. . .

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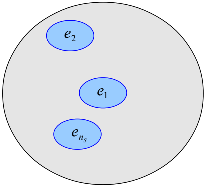

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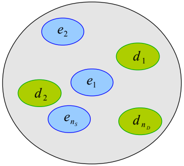

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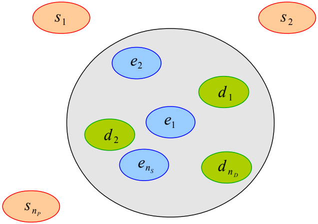

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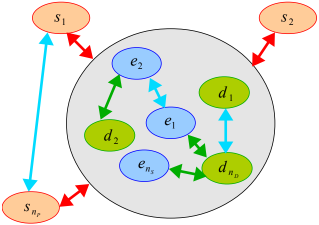

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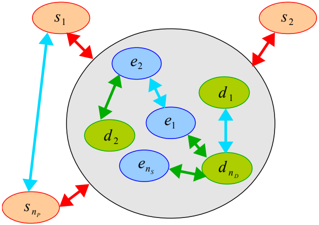

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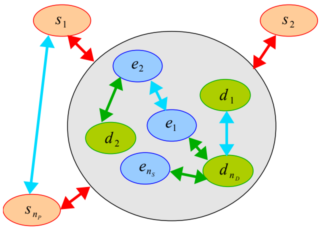

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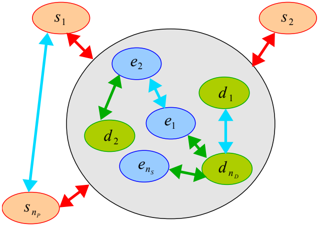

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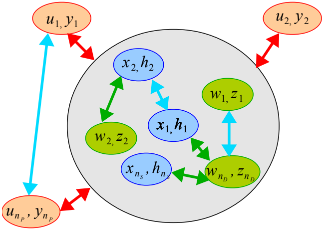

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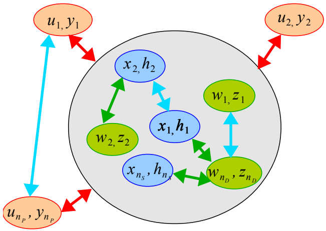

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PHS: Input-State-Output representation ($\boldsymbol{S}$: interconnection matrix)

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \underbrace{\begin{bmatrix} \boldsymbol{S}_{\mathbf{xx}} & \boldsymbol{S}_{\mathbf{xw}} & \boldsymbol{S}_{\mathbf{xu}} \\ * & \boldsymbol{S}_{\mathbf{ww}} & \boldsymbol{S}_{\mathbf{wu}} \\ * & * & \boldsymbol{S}_{\mathbf{yu}} \end{bmatrix}}_{\text{with } \boldsymbol{S} = -\boldsymbol{S}^{\mathsf{T}}} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \qquad (1)$$

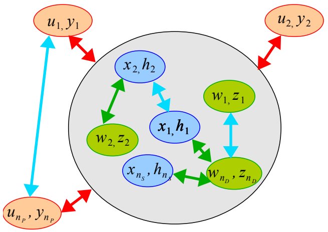
(*i*) storage → differential eq.

(*ii*) memoryless → algebraic eq.

(*iii*) ports → physical signals

A physical system is made of...

receiver convention



(i) Energy-storing components → *store energy*

$E = H(\mathbf{x}) = \sum_{n=1}^{N} H_n(x_n) \geq 0$

(ii) Memoryless passive components → *receive power*

$P_{\text{diss}} = \mathbf{z}(\mathbf{w})^{\mathsf{T}}\mathbf{w} = \sum_{m=1}^{M} z_m(w_m)\, w_m \geq 0$

(effort × flow : force × velocity, voltage × current, etc)

(iii) External components → *receive power*

$P_{\text{ext}} = \mathbf{u}^{\mathsf{T}}\mathbf{y} = \sum_{p=1}^{P} u_p y_p$

\+ Conservative connections → *sum of received powers is zero*

$$\underbrace{\nabla H(\mathbf{x})^{\mathsf{T}}\dot{\mathbf{x}}}_{P_{\text{stored}}=\mathrm{d}E/\mathrm{d}t} + \underbrace{\mathbf{z}(\mathbf{w})^{\mathsf{T}}\mathbf{w}}_{\geq 0} + \mathbf{u}^{\mathsf{T}}\mathbf{y} = 0 \qquad \textbf{(power balance)}$$

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(*i*) storage → differential eq.
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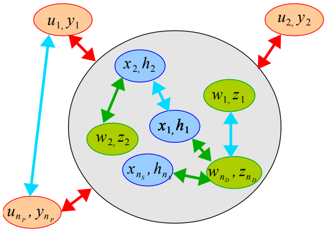
(*iii*) ports → physical signals

Power balance: $\mathbf{e}^{\mathsf{T}}\mathbf{f} \overset{(1)}{=} \mathbf{e}^{\mathsf{T}}\boldsymbol{S}\mathbf{e} = 0$ as $\boldsymbol{S} = -\boldsymbol{S}^{\mathsf{T}}$ $\Rightarrow \mathbf{e}^{\mathsf{T}}\boldsymbol{S}\mathbf{e} = (\mathbf{e}^{\mathsf{T}}\boldsymbol{S}\mathbf{e})^{\mathsf{T}} = -(\mathbf{e}^{\mathsf{T}}\boldsymbol{S}\mathbf{e})$

A physical system is made of...

receiver convention



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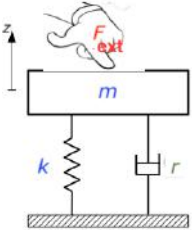
(*i*) storage → differential eq.
(*ii*) memoryless → algebraic eq.
(*iii*) ports → physical signals

→ **Differential-Algebraic Formulation** *(with no constraint: PH-DAE [Maschke,Schaft])*

Example: damped mechanical oscillator excited by F_{ext} $(m\ddot{z} + r\dot{z} + kz = F_{\mathrm{ext}})$

(no gravity)



- **4 separate components**

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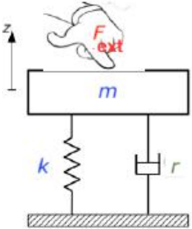
(no gravity)



- **4 separate components**

(i_1) mass $\boxed{\mathrm{m}}$ of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),

	state	energy H_n	flow **f**	effort **e**
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H_1'(x_1) = x_1/m$

blue : force

red : velocity

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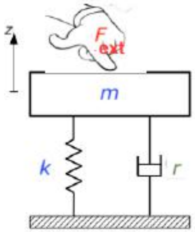

- **4 separate components**

(i_1) mass [m] of momentum $\pi = mv$ (energy: $\frac{1}{2}mv^2 = \frac{\pi^2}{2m}$),

(i_2) spring [sp] of elongation ξ

	state	energy H_n	flow $\mathbf{f}$	effort $\mathbf{e}$
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blue : force

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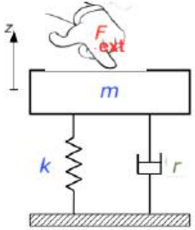
Example: damped mechanical oscillator excited by $F_{\rm ext}$ $(m\ddot{z} + r\dot{z} + kz = F_{\rm ext})$

(no gravity)



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(ii) damper [dp] of velocity $V_{\rm dp}$

	state	energy H_n	flow **f**	effort **e**
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dp			$w := V_{\rm dp}$	$z(w) := r\,w$

blue : force

red : velocity

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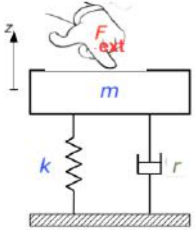
(no gravity)



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(ii) damper dp of velocity $V_{\rm dp}$

(iii) actuator ext applying a force F_{ext}

	state	energy H_n	flow **f**	effort **e**
m	$x_1 := \pi$	$\pi^2/(2m)$	$\dot{x}_1 = \dot{\pi}$	$H_1'(x_1) = x_1/m$
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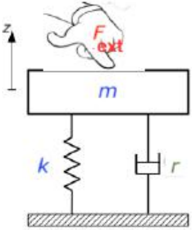
Example: damped mechanical oscillator excited by F_{ext} $(m\ddot{z} + r\dot{z} + kz = F_{\mathrm{ext}})$

(no gravity)



- **4 separate components**

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(i₂) spring $\boxed{\text{sp}}$ of elongation ξ

(ii) damper $\boxed{\text{dp}}$ of velocity V_{dp}

(iii) actuator $\boxed{\text{ext}}$ applying a force F_{ext} ($\rightarrow$your finger experiences $-F_{ext}$)

	state	energy H_n	flow **f**	effort **e**
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Example: damped mechanical oscillator excited by $F_{\rm ext}$ $(m\ddot{z} + r\dot{z} + kz = F_{\rm ext})$

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- **4 separate components**

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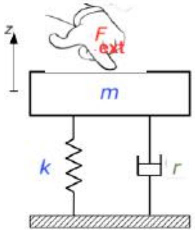

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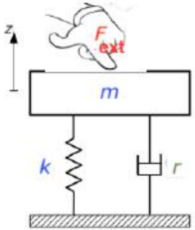
(no gravity)



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- **assembled with rigid connections**

$$
\underbrace{\begin{array}{r} \dot{\pi} = F_{\text{m}} \\ \dot{\xi} = V_{\text{sp}} \\ \hline V_{\text{dp}} \\ \hline V_{\text{ext}} \end{array} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ w \\ y \end{bmatrix}}_{\mathbf{f}} = \underbrace{\left[\begin{array}{c|c|c} & & \\ \hline & & \\ \hline & & \end{array}\right]}_{S} \underbrace{\begin{bmatrix} H_1'(x_1) \\ H_2'(x_2) \\ z(w) \\ u \end{bmatrix} \begin{array}{ll} V_{\text{m}} = & \pi/m \\ F_{\text{sp}} = & k\xi \\ \hline F_{\text{dp}} = & r\,V_{\text{dp}} \\ \hline -F_{\text{ext}} & \end{array}}_{\mathbf{e}}
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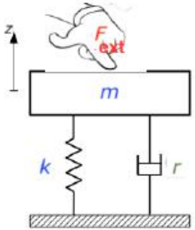
(no gravity)



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  - internal forces are balanced $F_{\mathrm{m}} + F_{\mathrm{sp}} + F_{\mathrm{dp}} + (-F_{\mathrm{ext}}) = 0$

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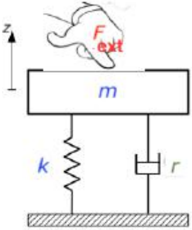

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  - internal forces are balanced $F_{\text{m}} + F_{\text{sp}} + F_{\text{dp}} + (-F_{\text{ext}}) = 0$
  - all velocities are equal $V_{\text{m}} = V_{\text{sp}} = V_{\text{dp}} = V_{\text{ext}}$

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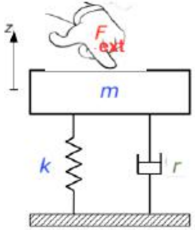
(no gravity)



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  - internal forces are balanced $F_{\rm m} + F_{\rm sp} + F_{\rm dp} + (-F_{\rm ext}) = 0$
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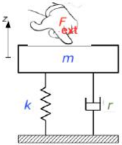
→ Formulation (1) with $H(\mathbf{x}) = H_1(x_1) + H_2(x_2)$ *(ODE: with $z = \xi$)*
→ $\boldsymbol{S} = -\boldsymbol{S}^\intercal$ is canonical *(no mechanical coefficients)*

Some variations: nonlinear components (modifying H or $\mathbf{z}$) and also...



$$\begin{pmatrix} F_{\mathrm{m}} \\ V_{\mathrm{sp}} \\ \hline V_{\mathrm{dp}} \\ \hline V_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} V_{\mathrm{m}} \\ F_{\mathrm{sp}} \\ \hline F_C \\ \hline -F_{\mathrm{ext}} \end{pmatrix}$$

Hamiltonian systems (conservative, autonomous)

$$\begin{pmatrix} F_{\mathrm{m}} \\ V_{\mathrm{sp}} \\ \hline . \\ \hline . \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & . & . \\ +1 & 0 & . & . \\ \hline . & . & . & . \\ \hline . & . & . & . \end{array}\right) \cdot \begin{pmatrix} v_M \\ F_{\mathrm{sp}} \\ \hline . \\ \hline . \end{pmatrix}$$

"Mass+Damper+Excitation" (spring removed)

$$\begin{pmatrix} F_{\mathrm{m}} \\ . \\ \hline V_{\mathrm{dp}} \\ \hline V_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & . & -1 & -1 \\ \hline . & . & . & . \\ \hline +1 & . & 0 & 0 \\ \hline +1 & . & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} V_{\mathrm{m}} \\ . \\ \hline F_C \\ \hline -F_{\mathrm{ext}} \end{pmatrix}$$

"Mass+Excitation"

$$\begin{pmatrix} F_{\mathrm{m}} \\ \hline . \\ \hline . \\ \hline V_{\mathrm{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & . & . & -1 \\ \hline . & . & . & . \\ \hline . & . & . & . \\ \hline +1 & . & . & 0 \end{array}\right) \cdot \begin{pmatrix} V_{\mathrm{m}} \\ . \\ \hline . \\ \hline -F_{\mathrm{ext}} \end{pmatrix}$$

PHS shifting

PHS shifting

① (PHS) $$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}(t)} = \boldsymbol{S} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}(t)}$$

PHS shifting

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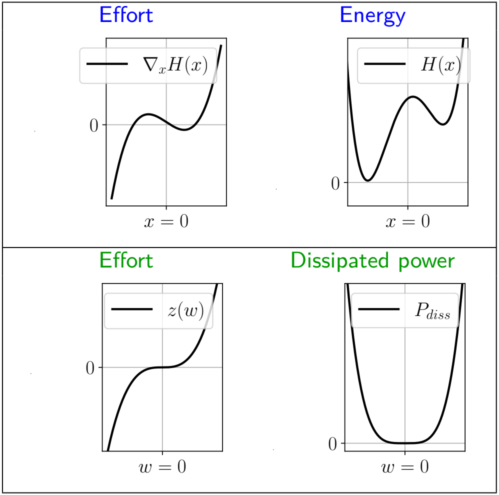

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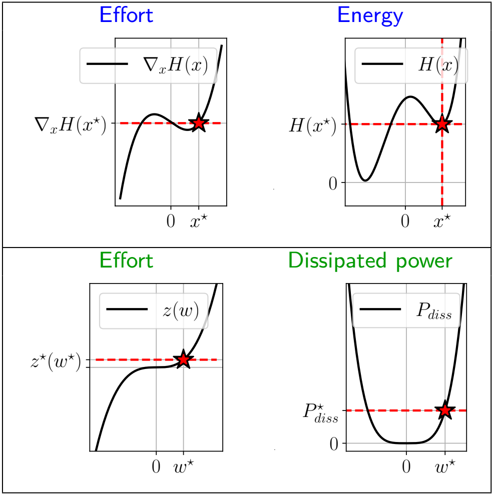
② Equilibrium var$^\star = \{\mathbf{u}^\star, \mathbf{x}^\star, \mathbf{w}^\star, \mathbf{y}^\star\}$

(PHS)* $\underbrace{\begin{bmatrix} \dot{\mathbf{x}}^\star = \mathbf{0} \\ \mathbf{w}^\star \\ \mathbf{y}^\star \end{bmatrix}}_{\mathbf{f}^\star} = \boldsymbol{S} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}^\star) \\ \mathbf{z}(\mathbf{w}^\star) \\ \mathbf{u}^\star \end{bmatrix}}_{\mathbf{e}^\star}$



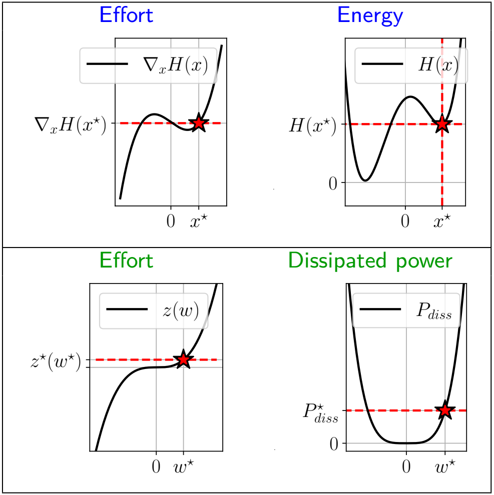
PHS shifting

① (PHS) $\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}(t)} = \boldsymbol{S} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}(t)}$

② Equilibrium $\text{var}^\star = \{\mathbf{u}^\star, \mathbf{x}^\star, \mathbf{w}^\star, \mathbf{y}^\star\}$

$(\text{PHS})^\star$ $\underbrace{\begin{bmatrix} \dot{\mathbf{x}}^\star = \mathbf{0} \\ \mathbf{w}^\star \\ \mathbf{y}^\star \end{bmatrix}}_{\mathbf{f}^\star} = \boldsymbol{S} \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}^\star) \\ \mathbf{z}(\mathbf{w}^\star) \\ \mathbf{u}^\star \end{bmatrix}}_{\mathbf{e}^\star}$

③ Fluctuations $\widetilde{\text{var}}(t) = \text{var}(t) - \text{var}^\star$



PHS shifting

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② Equilibrium var*=$\{\mathbf{u}^\star, \mathbf{x}^\star, \mathbf{w}^\star, \mathbf{y}^\star\}$

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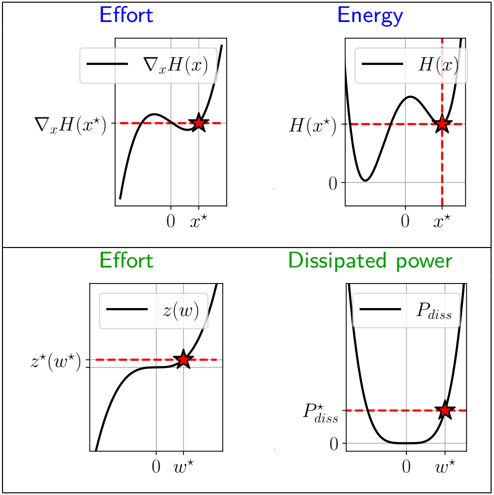
③ Fluctuations $\widetilde{\text{var}}(t)$ = var(t) - var*

$\widetilde{\text{(PHS)}} \equiv$ (PHS) - (PHS)*

$\mathbf{f}(t) - \mathbf{f}^\star = \boldsymbol{S}\,(\mathbf{e}(t) - \mathbf{e}^\star)$



PHS shifting

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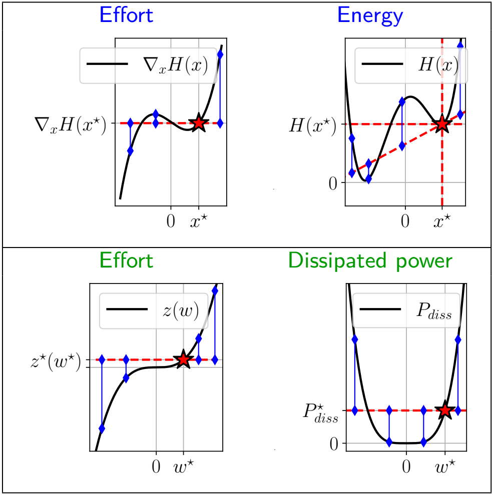
$$\underbrace{\begin{bmatrix}\widetilde{\dot{\mathbf{x}}}\\ \widetilde{\mathbf{w}}\\ \widetilde{\mathbf{y}}\end{bmatrix}}_{\widetilde{\mathbf{f}}(t)} = \boldsymbol{S}\underbrace{\begin{bmatrix}\nabla H(\mathbf{x}^\star + \widetilde{\mathbf{x}}) - \nabla H(\mathbf{x}^\star)\\ \mathbf{z}(\mathbf{w}^\star + \widetilde{\mathbf{w}}) - \mathbf{z}(\mathbf{w}^\star)\\ \widetilde{\mathbf{u}}\end{bmatrix}}_{\widetilde{\mathbf{e}}(t)}$$



PHS shifting

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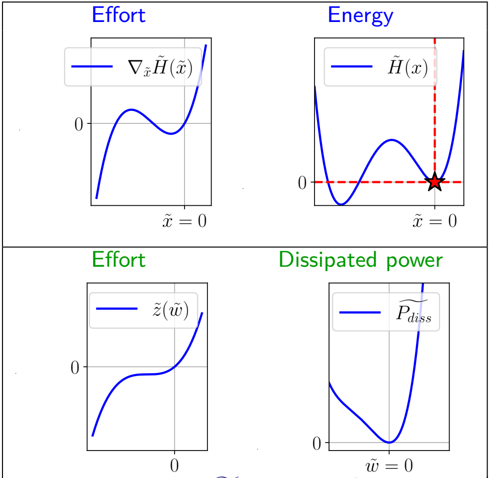

($\widetilde{\text{PHS}}$ is passive if $\widetilde{\mathbf{z}_{\mathbf{w}^\star}}(\widetilde{\mathbf{x}})^\mathsf{T}\,\widetilde{\mathbf{w}} \geq 0$)

Shifted pHs with

$$\widetilde{H_{\mathbf{x}^\star}}(\widetilde{\mathbf{x}}) := H(\widetilde{\mathbf{x}} + \mathbf{x}^\star) - \nabla H(\mathbf{x}^\star)^\mathsf{T}\,\widetilde{\mathbf{x}} - H(\mathbf{x}^\star)$$

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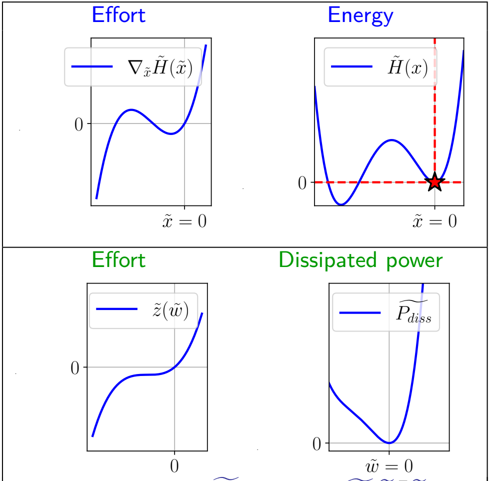

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Examples: gravity ($F_{\text{ext}} = \widetilde{F_{\text{ext}}} - g$), battery, etc.

Differential formulation

Differential formulation

$$\underbrace{\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{bmatrix}}_{\mathbf{f}} = \left(\underbrace{\begin{bmatrix} \boldsymbol{J}_{\mathbf{xx}} & \boldsymbol{J}_{\mathbf{xu}} \\ * & \boldsymbol{J}_{\mathbf{yu}} \end{bmatrix}}_{=: \ \boldsymbol{J} = -\boldsymbol{J}^\mathsf{T}} - \underbrace{\begin{bmatrix} \boldsymbol{R}_{\mathbf{xx}} & \boldsymbol{R}_{\mathbf{xu}} \\ * & \boldsymbol{R}_{\mathbf{yu}} \end{bmatrix}}_{=: \ \boldsymbol{R} = \boldsymbol{R}^\mathsf{T} \succeq 0} \right) \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \rightarrow$$

power balance with
$P_{\mathrm{diss}} = \mathbf{e}^\mathsf{T} \boldsymbol{R}\, \mathbf{e} \geq 0$

Link with Differential-Algebraic Formulation (1) ?

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{\mathbf{xx}} & \boldsymbol{S}_{\mathbf{xw}} & \boldsymbol{S}_{\mathbf{xu}} \\ * & \boldsymbol{S}_{\mathbf{ww}} & \boldsymbol{S}_{\mathbf{wu}} \\ * & * & \boldsymbol{S}_{\mathbf{yu}} \end{bmatrix} \begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{bmatrix}$$

Differential formulation

$$\underbrace{\begin{bmatrix}\dot{\mathbf{x}}\\ \mathbf{y}\end{bmatrix}}_{\mathbf{f}} = \Bigg(\underbrace{\begin{bmatrix}J_{xx} & J_{xu}\\ * & J_{yu}\end{bmatrix}}_{=:\ J=-J^\mathsf{T}} - \underbrace{\begin{bmatrix}R_{xx} & R_{xu}\\ * & R_{yu}\end{bmatrix}}_{=:\ R=R^\mathsf{T}\succeq 0}\Bigg)\underbrace{\begin{bmatrix}\nabla H(\mathbf{x})\\ \mathbf{u}\end{bmatrix}}_{\mathbf{e}} \rightarrow$$

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Assume that $S_{ww} = \mathbf{0}$

$P := [-S_{xw}^\mathsf{T}, S_{wu}]$ is independent of $\mathbf{w}$

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Differential formulation

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Link with Differential-Algebraic Formulation (1) ?

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Differential formulation

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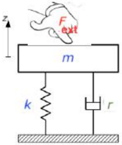
Then, $\mathbf{w} = P \underbrace{\begin{bmatrix} \nabla H(\mathbf{x}) \\ \mathbf{u} \end{bmatrix}}_{\mathbf{e}} \Longrightarrow J = \begin{bmatrix} S_{xx} & S_{xu} \\ * & S_{yu} \end{bmatrix} - P^\top J_\Gamma P$ with $J_\Gamma := \frac{1}{2}(\Gamma - \Gamma^\top)$

$R = P^\top R_\Gamma P \succeq 0$ with $R_\Gamma := \frac{1}{2}(\Gamma + \Gamma^\top)$

Example: damped mechanical oscillator excited by F_{ext}



$$\begin{array}{c} F_{\mathrm{m}} \\ V_{\mathrm{sp}} \\ V_{\mathrm{dp}} \\ V_{\mathrm{ext}} \end{array} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline w \\ \hline y \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} \partial_{x_1} H(\mathbf{x}) \\ \partial_{x_2} H(\mathbf{x}) \\ \hline z(w) = r\,w \\ \hline u \end{pmatrix} \begin{array}{c} V_{\mathrm{m}} \\ F_{\mathrm{sp}} \\ F_C \\ -F_{\mathrm{ext}} \end{array}$$

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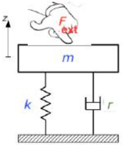

$$\begin{matrix} F_{\mathrm{m}} \\ V_{\mathrm{sp}} \\ V_{\mathrm{dp}} \\ V_{\mathrm{ext}} \end{matrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline w \\ \hline y \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & -1 & -1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} \partial_{x_1} H(\mathbf{x}) \\ \partial_{x_2} H(\mathbf{x}) \\ \hline z(w) = r\,w \\ \hline u \end{pmatrix} \begin{matrix} V_{\mathrm{m}} \\ F_{\mathrm{sp}} \\ F_C \\ -F_{\mathrm{ext}} \end{matrix}$$

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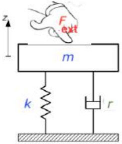

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Recall:

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{S}_{\mathbf{xx}} & \boldsymbol{S}_{\mathbf{xu}} \\ * & \boldsymbol{S}_{\mathbf{yu}} \end{bmatrix} - \boldsymbol{P}^{\top} \boldsymbol{J}_{\Gamma} \boldsymbol{P} \qquad \text{with } \boldsymbol{J}_{\Gamma} := \frac{1}{2}(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}^{\top})$$

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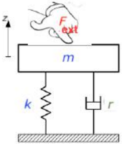

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$\rightarrow \; J_{\Gamma} = 0, \quad R_{\Gamma} = r$

$$\begin{matrix} F_{\mathrm{m}} \\ V_{\mathrm{sp}} \\ V_{\mathrm{ext}} \end{matrix} \quad \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ y \end{pmatrix} = \left(\left(\begin{array}{cc|c} 0 & -1 & -1 \\ +1 & 0 & 0 \\ \hline +1 & 0 & 0 \end{array}\right) - \left(\begin{array}{cc|c} r & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \right) \cdot \begin{pmatrix} \partial_{x_1} H(\mathbf{x}) \\ \partial_{x_2} H(\mathbf{x}) \\ \hline u \end{pmatrix} \quad \begin{matrix} V_{\mathrm{m}} \\ F_{\mathrm{sp}} \\ -F_{\mathrm{ext}} \end{matrix}$$

$\rightarrow$ matrix R combines interconnection routing and mechanical coefficients (r)

Outline

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Power-balanced numerical method

and

non-iterative solver

Power-balanced numerical method : discrete gradient

Classical numerical schemes for $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$:

- efficiently approximate $\frac{\mathrm{d}.}{\mathrm{d}t}$ and exploit f
- *a posteriori* analysis of stability

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A discrete power-balanced method (PHS)

Exploit differentiation chain rule

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_n \frac{\partial H}{\partial x_n}\frac{\mathrm{d}x_n}{\mathrm{d}t} \simeq \sum_n \underbrace{\frac{H_n(x_n[k+1]) - H_n(x_n[k])}{x_n[k+1]-x_n[k]}}_{\left[\nabla_D H\left(x[k],\delta x[k]\right)\right]_n} \underbrace{\frac{x_n[k+1]-x_n[k]}{\delta t}}_{[\delta x[k]/\delta t]_n} = \frac{E[k+1]-E[k]}{\delta t}$$

Jointly substitute $\dot{\mathbf{x}} \to \delta\mathbf{x}/\delta t$ and $\nabla H(\mathbf{x}) \to \nabla_D H(\mathbf{x}, \delta\mathbf{x})$:

$$\underbrace{\begin{pmatrix} \frac{\delta \mathbf{x}}{\delta t} \\ \mathbf{w} \\ -\mathbf{y} \end{pmatrix}}_{\mathbf{f}[k]} = \boldsymbol{S} \underbrace{\begin{pmatrix} \nabla_D H(\mathbf{x}, \delta\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{\mathbf{e}[k]}$$

Simulation : solve $(\delta\mathbf{x}, w)$ at each time step k (e.g. Newton-Raphson algo.)

Power-balanced numerical method : discrete gradient

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$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_n \frac{\partial H}{\partial x_n}\frac{\mathrm{d}x_n}{\mathrm{d}t} \simeq \sum_n \underbrace{\frac{H_n(x_n[k+1]) - H_n(x_n[k])}{x_n[k+1]-x_n[k]}}_{\left[\nabla_D H\left(x[k],\delta x[k]\right)\right]_n} \underbrace{\frac{x_n[k+1]-x_n[k]}{\delta t}}_{[\delta x[k]/\delta t]_n} = \frac{E[k+1]-E[k]}{\delta t}$$

Jointly substitute $\dot{\mathbf{x}} \to \delta\mathbf{x}/\delta t$ and $\nabla H(\mathbf{x}) \to \nabla_D H(\mathbf{x}, \delta\mathbf{x})$:

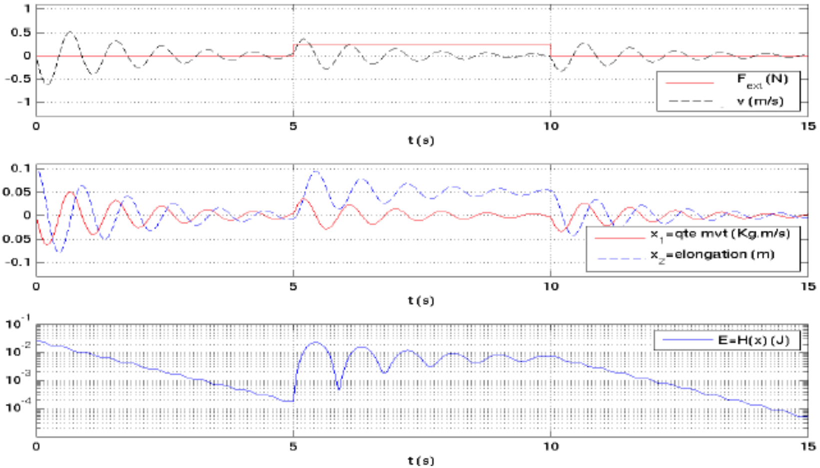
$$\underbrace{\begin{pmatrix} \frac{\delta \mathbf{x}}{\delta t} \\ \mathbf{w} \\ -\mathbf{y} \end{pmatrix}}_{\mathbf{f}[k]} = \boldsymbol{S} \underbrace{\begin{pmatrix} \nabla_D H(\mathbf{x},\delta\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}}_{\mathbf{e}[k]}$$

Simulation : solve $(\delta\mathbf{x}, w)$ at each time step k (e.g. Newton-Raphson algo.)

- Skew-symmetry of S preserved $\Rightarrow$ $0 = \mathbf{e}^T \boldsymbol{S}\mathbf{e} = \mathbf{e}^{\mathsf{T}}\mathbf{f} = \delta E/\delta t + \mathbf{z}(\mathbf{w})^T\mathbf{w} + \mathbf{u}^T\mathbf{y}$
- For **linear systems**, $\nabla_D H(\mathbf{x},\delta\mathbf{x}) = \nabla H(\mathbf{x}+\delta\mathbf{x}/2)$ restores the **mid-point scheme**.
- Method also applies to nonlinear components and non separate Hamiltonian
- Power-balanced Runge-Kutta scheme (non iterative) [Lopes et al., LHMNC'2015]

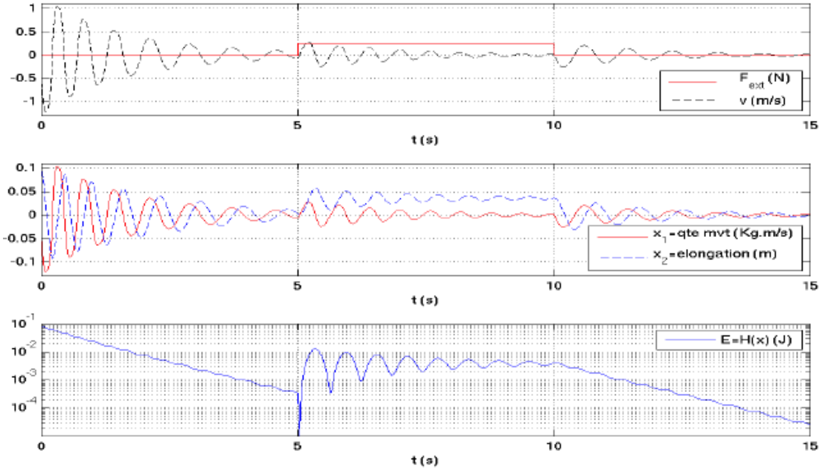
Simulation 1: mass-spring-damper

- **Parameters:** $M = 100\,\text{g}$, $K = 5\,\text{N/m}$, $C = 0.1\,\text{N.s/m}$ et $\delta t = 5\,\text{ms}$
- **Initial conditions:** $x_0 = \left[mv_0 = 0,\ \ell_0 = 10\,\text{cm}\right]^T$
- **Excitation:** $F_{\text{ext}}(t) = F_{\max}\ \mathbf{1}_{[5\text{s},10\text{s}]}(t)$ with $F_{\max} = K\ell_0/2 = 0.25\,\text{N}$



Simulation 2: idem with a hardening spring

- **Potential energy:** $H_2^{\mathrm{NL}}(x_2) = K\,L^2 \left[\cosh(x_2/L) - 1\right] \qquad \left(\sim k\,x_2^2/2\right)$
- **Physical law:** $F_2 = \left(H_2^{\mathrm{NL}}\right)'(x_2) = K\,L\,\sinh(x_2/L) \quad \left(\sim K\,x_2\right)$
- **Reference elongation:** $L = \ell_0/4 = 25\,\mathrm{mm}$



Quadratisation method *(goal: non-iterative solver)*

Numerical method: solve $\boldsymbol{\delta x}$ at each step k → implicit scheme

$$\begin{bmatrix} \boldsymbol{\delta x}/\delta t \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{\mathbf{xx}} & \boldsymbol{M}_{\mathbf{xu}} \\ \boldsymbol{M}_{\mathbf{yx}} & \boldsymbol{M}_{\mathbf{yu}} \end{bmatrix} \begin{bmatrix} \nabla_D H\left(\mathbf{x}, \boldsymbol{\delta x}\right) \\ \mathbf{u} \end{bmatrix} \quad \text{with } \boldsymbol{M} = \boldsymbol{J} - \boldsymbol{R}.$$

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Quadratic Hamiltonian $H(\mathbf{x}) = \frac{1}{2}\mathbf{x}\boldsymbol{L}\mathbf{x}^{\mathsf{T}} \Rightarrow \nabla_D H(\mathbf{x}, \boldsymbol{\delta x}) = \boldsymbol{L}(\mathbf{x} + \frac{1}{2}\boldsymbol{\delta x})$

Linear solver: $\boldsymbol{\delta x}/\delta t = \boldsymbol{\Delta}^{-1}(\boldsymbol{A}\mathbf{x} + \boldsymbol{B}\mathbf{u})$,

with $\boldsymbol{A} := \boldsymbol{M}_{\mathrm{xx}}\boldsymbol{L}$, $\boldsymbol{B} := \boldsymbol{M}_{\mathrm{xu}}$, and $\boldsymbol{\Delta} := \boldsymbol{I} - \frac{\delta t}{2}\boldsymbol{A}$ (invertible)

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① **Principle**: if H is non quadratic, make it quadratic !

+ benefit from the passive interconnection matrices $\boldsymbol{J} = -\boldsymbol{J}^\mathsf{T}$, $\boldsymbol{R} = \boldsymbol{R}^\mathsf{T} \succ 0$

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Transform the PHS on $\mathbf{x}$ into the $\widehat{\text{PHS}}$ on $\boldsymbol{q}$ *(use $\boldsymbol{X}$ & Jacobian of $\boldsymbol{Q}$)*

$$\boldsymbol{J}(\mathbf{x}) = -\boldsymbol{J}(\mathbf{x})^{\mathsf{T}},\ \boldsymbol{R}(\mathbf{x}) = \boldsymbol{R}(\mathbf{x})^{\mathsf{T}} \succeq 0 \ \overset{\boldsymbol{Q}}{\longrightarrow}\ \widehat{\boldsymbol{J}}(\boldsymbol{q}) = -\widehat{\boldsymbol{J}}(\boldsymbol{q})^{\mathsf{T}},\ \widehat{\boldsymbol{R}}(\boldsymbol{q}) = \widehat{\boldsymbol{R}}(\boldsymbol{q})^{\mathsf{T}} \succeq 0$$

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③ If $H(\mathbf{x}) = \sum_{n=1}^{N} H_n(x_n)$ ($\mathcal{C}^1$, strictly quasi-convex, $H_n(x_n) \geq 0$ and $\underset{0}{\sim} \frac{k_n}{2}x_n^2$)

Then $Q_n(x_n) = \text{sign}(x_n)\sqrt{2H_n(x_n)}$ → exercise 2

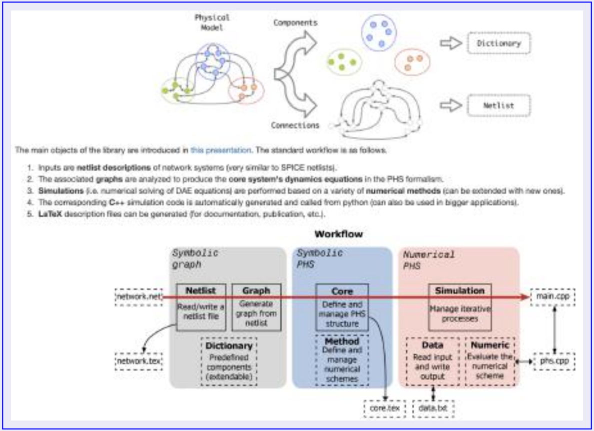
Outline

Automatic generation of code: the PyPHS Python library [Antoine Falaize]

https://pyphs.github.io/pyphs/

2012-16 : First version **[Falaize, PhD]**

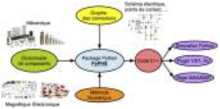
2016−− : Opensource library with periodic releases **[Falaize & contributors]**



→ exercice 4 (tutorial: see links in the references)

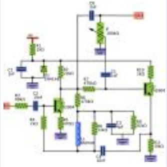
PhD, 2016: Antoine Falaize

Passive modelling, simulation, code generation
and correction of audio multi-physical systems



Two examples

Wah pedal (CryBaby): netlist → **PyPHS** → LateX eq. & C code



Components	Number
Storage	7 linear
Dissipative	18 (5 NL, 2 modulated)
Ports	3 (IN, OUT, battery)

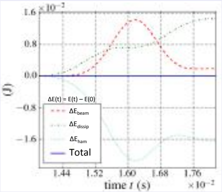
Audio PlugIn:

Sound 1a: dry

Sound 1b: wah

A simplified Fender-Rhodes Piano Sound 2

Components	Hammer	1 beam	Pickup/RC-circuit
Storage	2 NL	$2M$ lin.	2 lin. (+ NL connection)
Dissipative	1 NL	M lin.	1 lin.
Ports	2	1	1



Real-time simulation of Ondes Martenot

[Najnudel et al., AES2018]

Ondes Martenot (created by Maurice Martenot in 1928)

Controls

Circuit

Diffuseurs

→ Video 3 [Thomas Bloch, improvisation, 2010]

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→ Video 3 [Thomas Bloch, improvisation, 2010]

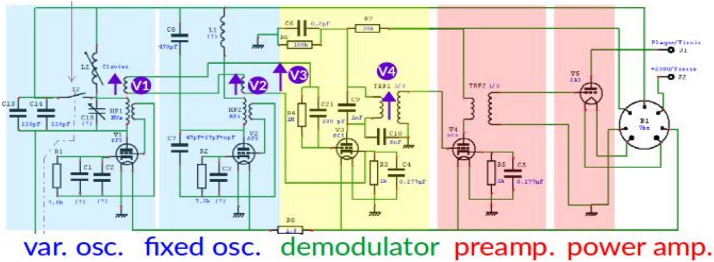
Context/Problem (Musée de la Musique, Philharmonie de Paris)

Technological obsolescence of a musical instrument:
70/281 remaining instruments *(handmade)*, **1200 pieces** *(Varèse, Maessian, etc)*

Objective (Collegium Musicae-Sorbonne Université)

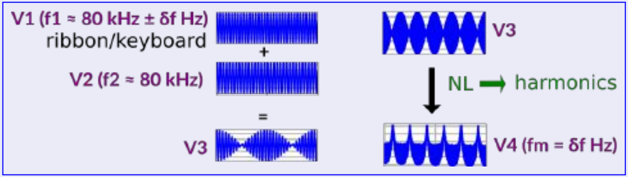
Real-time simulation of the circuit based on physics → PHS approach

Ondes Martenot: 5 stages circuit



Specificities: heterodyne oscillators (1930's)

- 2 High frequencies ($\approx$ 80kHz$\pm\delta f$) $\rightarrow$ demodulator $\rightarrow$ audio range ($\delta f, 2\delta f, \dots$)

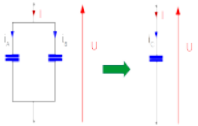


- Vacuum tubes: $w =$ [grid and plate currents]^T, $z(w) =$ associated voltages

  (passive parametric model [Cohen'12])

- **Pb:** ribbon-controlled oscillator involving **time-varying capacitors in parallel**

Ondes Martenot: capacitors in parallel



Problem:

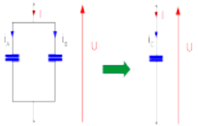
Capacitors	$(n = A, B)$
State *(charge)*:	q_n
Energy :	$H_n(q_n)$
Flux *(current)*:	$i_n = \mathrm{d}q_n/\mathrm{d}t$
Effort *(voltage)*:	$v_n = H_n'(q_n)$

$v_C = v_A = v_B$ &

$$\begin{bmatrix} i_A \\ i_B \\ v_C \end{bmatrix} = \begin{bmatrix} \text{not} \\ \text{realisable} \end{bmatrix} \begin{bmatrix} v_A = H_A'(q_A) \\ v_B = H_B'(q_B) \\ i_C \end{bmatrix}$$

$\rightarrow$ Build the equivalent component $C = A//B$

Ondes Martenot: capacitors in parallel



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→ Build the equivalent component $C = A//B$

Hyp: $q_n \mapsto v_n = H_n'(q_n)$ bijective (increasing law)

Find the total energy $H_C(q_C)$ for the total charge $q_C = q_A + q_B$

1. Charge as a function of the voltage $v_n = v_C$: $q_n = [H_n']^{-1}(v) := Q_n(v_C)$
2. Total charge (idem): $q_C = [Q_A + Q_B](v_C) =: Q_C(v_C)$
3. Total energy function: $H_C(q_C) = \sum_{n=A,B} H_n \circ Q_n \circ Q_C^{-1}(q_C)$

Also available if H_n depends on additional states (ribbon position ℓ)

Power-balanced simulation with $H(q, \ell) = q^2/(2C_{\mathrm{Martenot}}(\ell))$

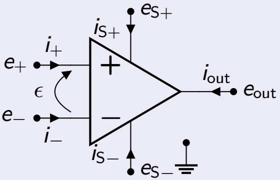
→ video 4 (sound=circuit output voltage, without the *diffuseurs*)

Operational Amplifier

[Muller et al., DAFx'19]

Idealised component

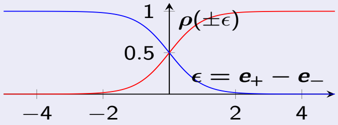
- **5 ports**



- **Algebraic conservative law**

$$\underbrace{\begin{bmatrix} i_+ \\ i_- \\ i_{S+} \\ i_{S-} \\ e_{\text{out}} \end{bmatrix}}_{z_{OPA}(w)} = \underbrace{\left[\begin{array}{cc|ccc} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot & -\rho(+\epsilon) \\ \cdot & \cdot & \cdot & \cdot & -\rho(-\epsilon) \\ \cdot & \cdot & \rho(\epsilon) & \rho(-\epsilon) & \cdot \end{array}\right]}_{\mathbf{J}(w_{OPA}) = -\mathbf{J}(w_{OPA})^T} \underbrace{\begin{bmatrix} e_+ \\ e_- \\ e_{S+} \\ e_{S-} \\ i_{\text{out}} \end{bmatrix}}_{w_{OPA}} .$$

- **Modulation factor**



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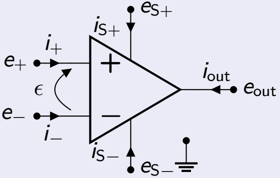

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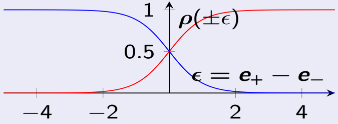
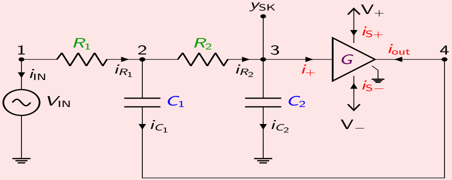
- **Modulation factor**



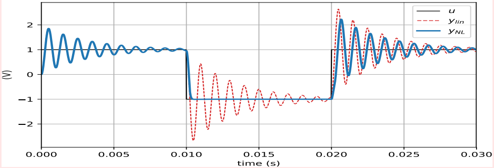
Typical analog filters (Sallen-Key)

- **Circuit:**



- **Nonlinear PHS:** $\begin{bmatrix} \dot{x} \\ w \\ w_{OPA} \\ y \end{bmatrix} = S \begin{bmatrix} \nabla H(x) \\ z(w) \\ z_{OPA}(w_{OPA}) \\ u \end{bmatrix}$

- **Sounds 5 (simulations: linear / nonlinear)**



Motivation

1. Theoretical issues

Given a linear conservative mechanical system,

- find **damping models** that preserve the eigen modes (with eigen structure)
- design **nonlinear** damping in such a class
- provide a **power balanced formulation** that is preserved in **simulations**

2. Application in musical acoustics

Build **physical models** to produce:

- a **variety of beam sounds** (glokenspiel, xylophone, marimba, etc)
- **morphed sounds** through some extrapolations based on **physical grounds** *(e.g. meta-materials with damping depending on the magnitude)*

Damping models for $M\ddot{q} + C\dot{q} + Kq = f$ (finite-dimensional case)

Conservative problem (C=0)

- $\ddot{q} + (M^{-1}K)q = M^{-1}f$
- Eigen-modes e_i: $(M^{-1}K)e_i = \omega_i^2 e_i$ (ω_i: angular freq.)

Damping that preserves eigen-modes ?

- Choose $M^{-1}C$ as a **non-negative polynomial** of matrix $M^{-1}K$

$\rightarrow$ *Caughey class (1960)*: $C = c_0 M + c_1 K + c_2 K M^{-1} K + \ldots$

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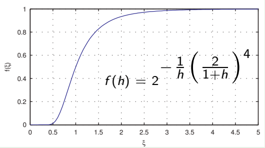
Eigen-modes with nonlinearly-damped dynamics ?

- Make c_n depend on the dynamics

Ex.: damping as a function of energy $H(x)$ (state $x = [q, p = M\dot{q}]^T$)

$c_n(x) = \kappa_n\big(H(x)\big) \in [c_n^-, c_n^+]$ with $c_n^- \geq 0$

- Increasing: $\kappa_n(h) = c_n^- + (c_n^+ - c_n^-) f\left(\frac{h}{h_0}\right)$
- Decreasing: $\kappa_l(h) = c_n^+ - (c_n^+ - c_n^-) f\left(\frac{h}{h_0}\right)$



Application case: the Euler-Bernoulli beam

1. Pinned beam excited by a distributed force

(H1) Euler-Bernoulli kinematics: straight cross-section after deformation

(H2) linear approximation for the conservative problem

(H3) viscous and structural dampings: only $c_0, c_1 \geq 0$

2. Dimensionless model (w: deflection, $t \geq 0$, $0 \leq \ell \leq 1$)

- **PDE:** $\underbrace{\partial_t^2 w}_{\mathcal{M}\equiv Id} + \underbrace{(c_0 + c_1 \partial_\ell^4)}_{\mathcal{C}} \partial_t w + \underbrace{\partial_\ell^4}_{\mathcal{K}} w = f_{\text{ext}} \quad (-u)$
- **Boundaries** $\ell \in \{0, 1\}$**:** fixed extremities ($w = 0$), no momentum ($\partial_\ell^2 w = 0$)
- **Energy:** $E = \int_0^1 \left(\frac{(\partial_\ell^2 w)^2}{2} + \frac{(\partial_t w)^2}{2} \right) \mathrm{d}\ell$

3. Modal decomposition: $e_m(\ell) = \sqrt{2} \sin(m\pi\ell)$ $(1 \leq m \leq n)$

PHS:

$$\partial_t x = (J - R)\nabla H(x) + Gu \quad \text{with} \quad J = \begin{bmatrix} 0_{n\times n} & I_n \\ -I_n & 0_{n\times n} \end{bmatrix}, \; R = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & C \end{bmatrix}$$

$$y = -G^{\mathsf{T}} \nabla H(X) \qquad G^{\mathsf{T}} = [0_{n\times n}, I_n]$$

with $H\big(x = [q; p = M\dot{q}]\big) = \frac{1}{2} p^{\mathsf{T}} M^{-1} p + \frac{1}{2} q^{\mathsf{T}} K q$

and $q = [q_1, \ldots, q_n]^T$, $u = [u_1, \ldots, u_n]^T$, $y = [y_1, \ldots, y_n]^T$

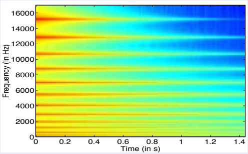
(projections of w, f_{ext}, v_{ext})

where $M = I_n$, $K = \pi^4 \mathrm{diag}(1, \ldots, n)^4$ and $C = c_0 I_n + c_1 K$.

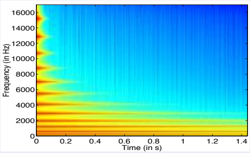
Damping and simulation parameters

Examples of spectrograms for standard linear dampings: $c_0 \sim 10^{-2}$

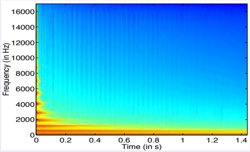
metal ($c_1 \sim 10^{-6}$)



glass ($c_1 \sim 10^{-5}$)



wood ($c_1 \sim 10^{-4}$)



Nonlinear damping (from metal to wood):

$C(x) = c_0(x)I + c_1(x)K$ with
$c_n(x) = \beta_n\big(H(x)\big) \in [c_n^-, c_n^+]$

metal	$c_0^- = 0.02$	$c_1^- = 10^{-6}$
wood	$c_0^+ = 0.04$	$c_1^+ = 10^{-4}$

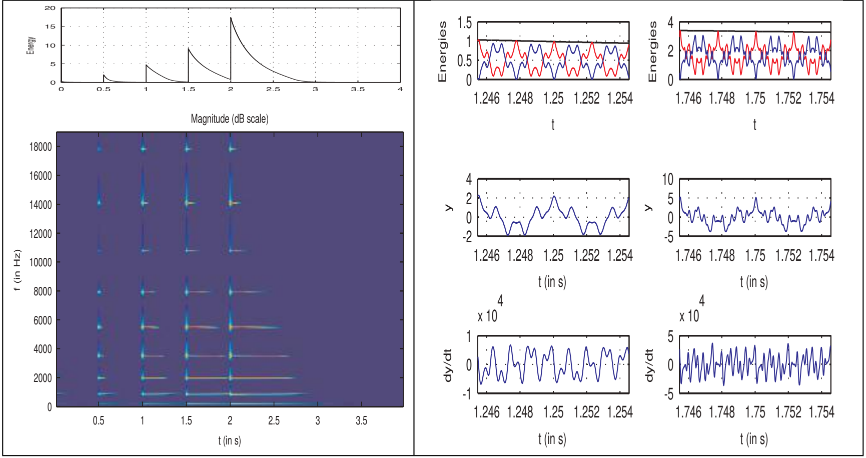
Numerical method preserving the power balance (discrete gradient)

- force distributed close to $z = 0$: $u = [1, \ldots, 1]^T f$
- listened signal: acceleration $[1, \ldots, 1]\dot{y}$
- $n = 9$ modes and time step s.t. $f_1 = 220\,\text{Hz}$ to $f_9 \approx n^2 f_1 = 17820\,\text{Hz}$

Results: $H(x) \ll 1 \longrightarrow$ wood, $H(x) \gg 1 \longrightarrow$ metal

force: 5 piecewise constant pulses (0.1ms) with increasing magnitude →Sound 6

overview	zooms total (black), kinetic (red) and potential (blue)



Outline

From Statistical Physics

to

Macroscopic PHS

STATISTICAL PHYSICS and Boltzmann principle for PHS

PhD, 2022: Judy Najnudel
Power-Balanced Modeling of Nonlinear Electronic Components and Circuits for Audio Effects

From Statistical Physics

to

Macroscopic PHS

METHOD 1: From Statistical Physics to Macroscopic PHS

Motivations

1. **Macro modeling** of systems with **billions** of interacting particles
    - Ferromagnets
    - Gases

    ⋮
2. Formulate as **macroscopic PHS**
    - state = ?
    - ports = ?

METHOD 1: From Statistical Physics to Macroscopic PHS

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A. **Microscopic** description

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A. **Microscopic** description

B. **Experimental** conditions

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C. **Stochastic** setting and averaging of fluctuations

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microstates are all explored

Make information sufficient

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	spin	gas
	$\{-1, 1\}$	$(\boldsymbol{r}, \boldsymbol{p})$

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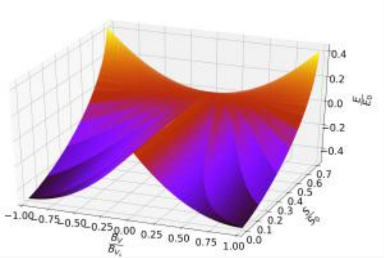
13. **Ports**
    $\boldsymbol{u} \longleftrightarrow (\dot{\overline{\mathcal{S}}}, \dot{\overline{\mathcal{F}}}_k) = \dot{\boldsymbol{x}}$
    $\boldsymbol{y} \longleftrightarrow (T, T\lambda_k) = \nabla E(\boldsymbol{x})$

Ferromagnetic Coils 4/7 - Core Macroscopic Model

$$E_{\text{meanfield}}(T, m) \xrightarrow{\text{change of variable}} E(S, B_V), \quad B_V = m\, B_{V_s} \text{ total magnetic flux}$$

1. State $\boldsymbol{x} = [S, B_V]^\intercal$
2. Energy $E(\boldsymbol{x}) =$

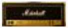


3. Effort $\nabla E(\boldsymbol{x}) = \Big[\underbrace{T}_{\text{internal temperature}}, \underbrace{H}_{\text{internal magnetic field}}\Big]^\intercal$

Ferromagnetic Coils 1/7 - Approach


ferromagnetic coil = ferromagnetic core + winding





Core modeling

Microscopic description
Ising model:
spin $\in \{-1, 1\}$
Heisenberg Hamiltonian $\mathcal{E}$

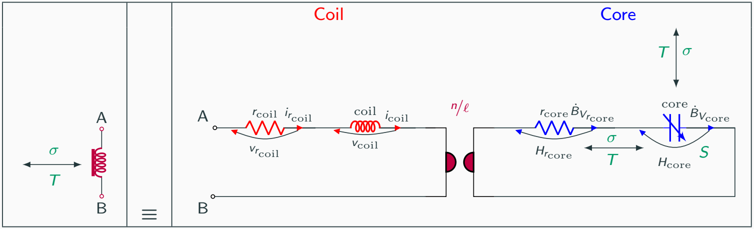
Experimental conditions
Free energy $\mathcal{E}$
Fixed volume V
Fixed number of particles N

$\xrightarrow{\text{Method}}$ Ports $\longleftarrow$

Coil modeling

Linear inductor

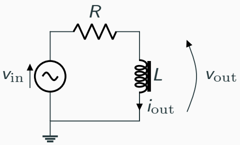
Ferromagnetic Coils 6/7 - Complete PHS Model

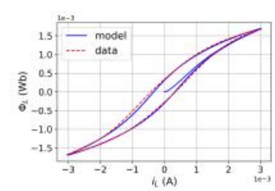


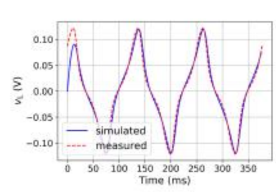
		$\nabla E(\boldsymbol{x})$			$z(\boldsymbol{w})$			$\boldsymbol{u}$
		T_{core}	H_{core}	i_{coil}	σ_i	$\dot B_{V_{r_{core}}}$	$v_{r_{coil}}$	σ_{ext}
$\dot{\boldsymbol{x}}$	$\dot S$	.	.	.	-1	.	.	-1
	$\dot B_{V_{core}}$	.	.	.	.	1	.	.
	v_{coil}	.	.	.	.	$-n/\ell$	-1	.
$\boldsymbol{w}$	T_{core}	1	.	.	.	.	.	.
	$H_{r_{core}}$	.	-1	n/ℓ	.	.	.	.
	$i_{r_{coil}}$	.	.	1	.	.	.	.
$\boldsymbol{y}$	T_{ext}	1	.	.	.	.	.	.

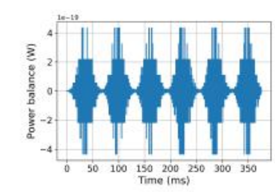
Ferromagnetic Coils 7/7 - Application

Identification of a Fasel inductor









Outline

Passive Control

for

digital hardware devices

CONTROL: digital passive controller (Patent 2019 with T. Lebrun)

Problem statment

- Derive a "discrete-time passive controller" C,

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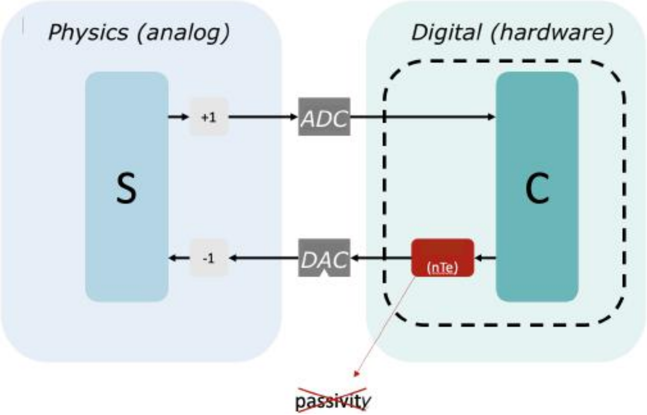
$\rightarrow$ The computational latency breaks passivity !



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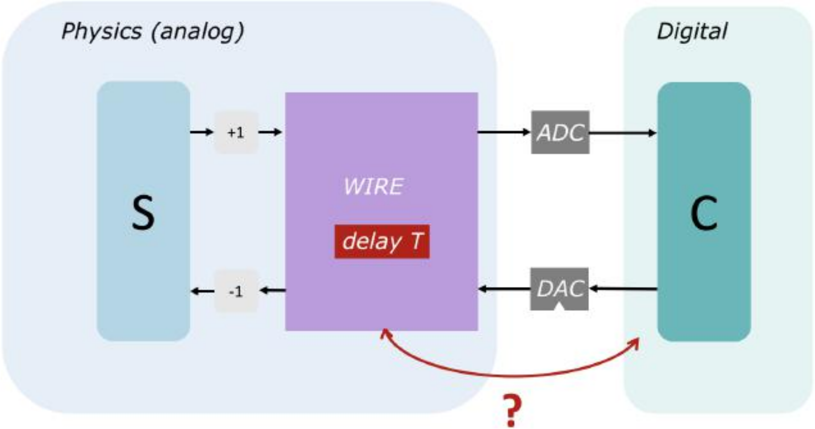
Principle:

- Replace the non-passive delay by a conservative virtual wire



CONTROL: digital passive controller

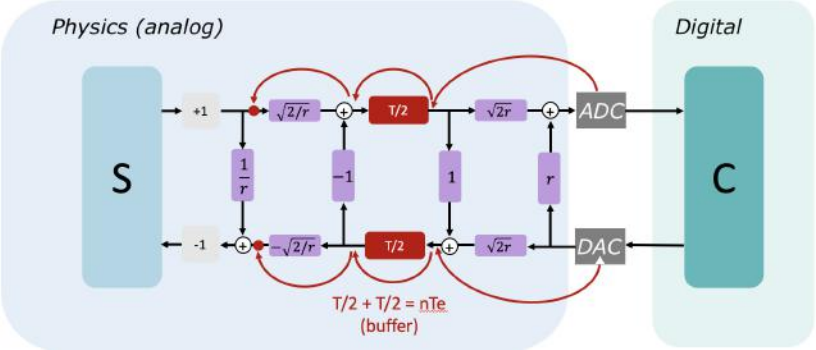
(Patent 2019 with T. Lebrun)

Principle:

- Replace the non-passive delay by a conservative virtual wire

→ Telegraphists equation (r: characteristic impedance)

+ travelling wave decomposition
+ commute the converters (ADC, DAC)



CONTROL: digital passive controller

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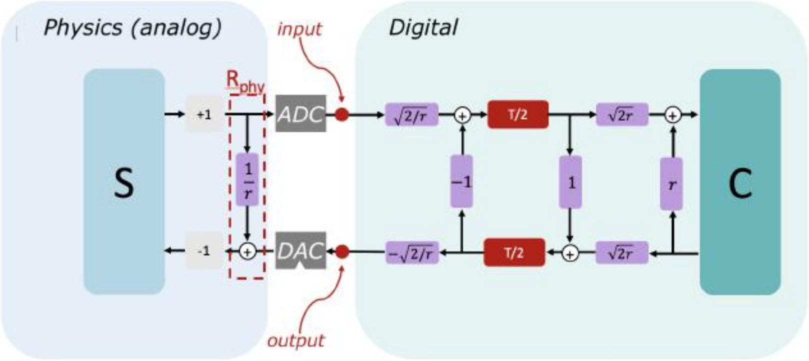
Final result

Half-physical ($R_{\rm phy}$) half-digital (modified controller C) process



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(Patent 2019 with T. Lebrun)

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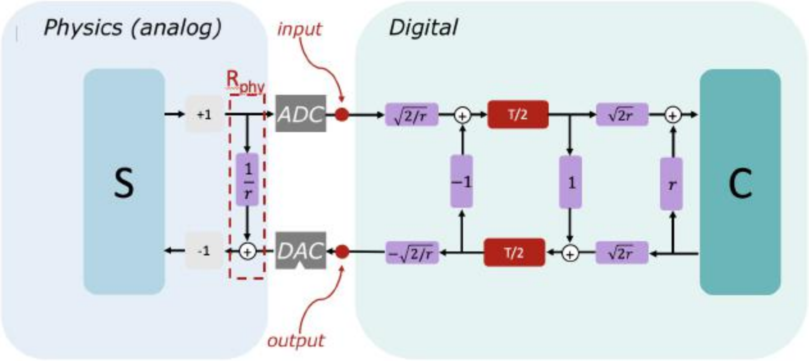

→ Restores passivity without increasing latency

Outline

Conclusion

Recent or ongoing work at STMS lab-IRCAM **[Collaborators]**

1. MODELLING:
   - Vocal apparatus **[Silva, PhD-Wetzel]**
   - Statistical physics (magnets, nonlinear coil)+identification **[PhD-Najnudel]**
   - Boundary-controlled nonlinear mechanical resonators **[PhD-Voisembert]**
   - Nonlinear dissipation class (PDE in in mechanics) **[Matignon]**
   - Bowed instruments (friction model) **[Falaize,Roze]**

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3. CONTROL: **[Boutin, d'Andréa-Novel]**
   - Loudspeaker **[PhD-Lebrun]**
   - Finite-time passive control (tom drum) **[PhD-Wijnand]**
   - Hybrid trombone **[PhD-Martos]**

– The end –

Thank you for your attention

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