Mathematical Model for Coupling Quasi-unidimensional Perfect Flow with Acoustic Boundary Layer □

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• We study a simple model of nonlinear acoustic flow in a cylindric duct. Our objective is to take into account several physical effects such compressibility of the air, viscous dissipation and thermal conduction, expecially in the vicinity of the wall. The proposed model first introduced in [MDDC97] couples perfect fluid in one-dimensional evolution and boundary layer described by the heat equation.

• For a duct with uniform section and classical thermodynamics of perfect gas, the unknowns are density ρ, velocity u, pressure p and internal energy e for time t and abscissa x and on the other hand velocity ξ and temperature θ inside the boundary layer at a distance η from the wall. The originality of our model is due to the fact that velocity is associated to **two** unknowns fields $u(t,\,x)$ and $\xi(t,\,x,\,\eta)$; it is also the case for temperature.

• We first write the equations of conservation of mass, impulse and energy in section x, taking into account viscous friction and thermal flux at the wall :

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho\, u) = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho\, u) + \frac{\partial}{\partial x}(\rho\, u^2 + p) + 2\,\mu\, \frac{\partial \xi}{\partial \eta}\,(t,\,x,\,0) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho\, e + \frac{1}{2}\,\rho\, u^2) + \frac{\partial}{\partial x}(\rho\, u\, e + \frac{1}{2}\,\rho\, u^3 + p\, u) + 2\, k\, \frac{\partial \theta}{\partial \eta}\,(t,\,x,\,0) = 0 \tag{3}$$

and consider also thermostatics state law

$$p(t,x) \equiv (\gamma - 1)\,\rho\, e\,. \tag{4}$$

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Secondly, we consider transfer of impulse and energy inside the boundary layer, neglect nonlinear advective terms and suppose that pressure $p(t,\, x)$ is given by the perfect fluid, i.e. is independent of the distance η to the wall :

$$\rho_0 \, \frac{\partial \xi}{\partial t} \;-\; \mu \frac{\partial^2 \xi}{\partial \eta^2} \;=\; - \frac{\partial p}{\partial x} \tag{5}$$

$$\rho_0 \, C_p \, \frac{\partial \theta}{\partial t} \;-\; k \frac{\partial^2 \theta}{\partial \eta^2} \;=\; \frac{\partial p}{\partial t} \,. \tag{6}$$

The boundary conditions suppose adherence at the wall :

$$\xi(t,\, x,\, 0) \;=\; 0 \,, \qquad \theta(t,\, x,\, 0) \;=\; \theta_0 \tag{7}$$

and uniform transverse flow at the top of the boundary layer :

$$\frac{\partial \xi}{\partial \eta}(t,\, x,\, \eta) \;\rightarrow\; 0 \,, \qquad \frac{\partial \theta}{\partial \eta}(t,\, x,\, \eta) \;\rightarrow\; 0 \quad \text{when } \eta \;\rightarrow\; +\infty \,. \tag{8}$$

• The principal interest of the coupled model (1)-(8) is that the boundary layer thickness does not appear explicitly as an unknown but is a result of the computation. Note the way of coupling perfect fluid equations (1)(2)(3) with boundary-layer dynamics (5)(6) : viscous term $2\,\mu \, \frac{\partial \xi}{\partial \eta} \big(t,\, x,\, 0\big)$ in equation (2) and heat flux $2\,k \, \frac{\partial \theta}{\partial \eta} \big(t,\, x,\, 0\big)$ in equation (3) are data for perfect fluid dynamics ; on the other hand, gradient of pressure $\frac{\partial p}{\partial x}$ inside equation (5) and time derivative of the same field in equation (6) are data for boundary layer dynamics.

• In our contribution, we derive coupled model (1)-(8) from Navier-Stokes and Thin Layer Navier Stokes equations, we solve numerically this coupled model with Lax-Wendroff scheme and appropriate convolution formulae, we present a comparison with the method of characteristics for the integration of simple waves, a validation with Kirchhoff linear theory, combine the effects of visco-thermic dissipation and nonlinear propagation and we show an application to the trombone.

• [MDDC97] R. Msallam, S. Dequidt, F. Dubois, R. Caussé. Modèle et simulations numériques de la propagation acoustique non-linéaire dans les conduits, Congress of the *Société Française d'Acoustique*, Marseille, april 1997.