A quantum approach for determining a state of the opinion

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Abstract We propose to define a notion of state of the opinion in order to link politician popularity estimations and voting intentions. We present two ways of modelling: a classical approach and quantum modelling. We test these ideas on data obtained during the French presidential election of April 2012.

Keywords Opinion polls, voting.

1) Introduction

Electoral periods are favorable to opinion polls. We keep in mind that opinion polls are intrinsically complex (see *e.g.* Gallup [14] or Tillé [31]) and give an approximates picture of a possible social reality. They are traditionnally of two types: popularity polls for various outstanding political personnalities and voting intentions polls when a list of candidates is known. We remark that in the first case, a grid of appreciation is given by the questionnaire, typically of the type "very good" $\succ$ "good" $\succ$ "no opinion" $\succ$ "bad" $\succ$ "very bad".

• We have two different informations and to construct a link between them is not an easy task. In particular, the determination of the voting intentions is a quasi intractable problem! Predictions of votes classically use of so-called "voting functions". Voting functions have been developed for the prediction of presidential elections in the United States. They are based on correlations between economical parameters, popularity polls and other technical parameters. We refer to Abramowitz [1], Lewis-Beck [27], Campbell [11], Lafay [25] and the survey paper proposed by Auberger [2].

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• We do not detail here the mathematical difficulties associated with the question of voting when the number of candidates is greater than three [3, 7, 10]. They conduct to present-day researches like range voting, independently proposed by Balinski and Laraki [4, 5] and by Rivest and Smith [29, 30]. It is composed by two steps: grading and ranking. In the grading step, all the candidates are evaluated by all the electors. This first step is quite analogous to a popularity investigations and we will merge the two notions in this contribution. The second step of range voting is a majority ranking; it consists of a successive extraction of medians.

• In this contribution, we make the hypothesis that there exists some global state of the opinion associated with a given grid of analysis, denoted by G in the following. We study how voting intentions interact with the state of the opinion. In particular, we propose to determine as much information as possible about this state of the opinion, in the case where voting intentions and popularity polls are both available. In Section 2, we propose a mathematical model founded on a classical framework. The state of the opinion is described by a discrete law of probability and the double information of popularity polls and opinion polls give the input information.

• We adopt afterwards in Section 3 quantum modelling (see *e.g.* Bitbol *et al* [6] for an introduction), in the spirit of authors like Khrennikov and Haven [24], La Mura and Swiatczak [26] and Zorn and Smith [32] concerning voting processes. We recall two voting models developed in previous contributions [12, 13], founded on range voting and first run of an election, having implicitely in mind the case of the French presidential election. Then we propose in Section 4 to link our two quantum models and use for doing this an equivalent candidate and the state of the opinion. We test in Section 5 our previous ideas with three sets of data coming from 2012 French presidential elections and propose numerical results.

2) A classical approach

We consider a grid G of m types of opinions as one of the two following ones. We have $m=5$ for the first grid (1) and $m=3$ for the second one (2):

$$++ \;\succ\; + \;\succ\; 0 \;\succ\; - \;\succ\; -- \tag{1}$$

$$+ \;\succ\; 0 \;\succ\; - \,. \tag{2}$$

These ordered grids are typically used for popularity polls [17, 18, 21, 22]. We assume also that a ranking grid like (1) or (2) is a basic tool to represent a "state of the opinion". If some political personality has a great proportion of "very good" or "++" opinion (as in (1)), we suppose here that this fact is a kind of mirror effect of an existing state of social opinion. The reflection that the opinion is for a certain proportion in a "very good" state.

• We have two type of data, as explained in the introduction. We denote by Γ the set of candidates and we denote by n their number. We suppose also that

$$\text{the number of candidates} \;\equiv\; n \;>\; m \;\equiv\; \text{the size of the grid G}\,. \tag{3}$$

On one side the result of a popularity poll for the n candidates is given. We have a matrix of data $\big(S_{\gamma\,\nu}\big)_{\gamma\in\Gamma,\,\nu\in G}$ with an hypothesis of coherence:

$$S_{\gamma\,\nu} \geq 0\,, \quad \sum_{\nu\in G} S_{\gamma\,\nu} = 1\,, \quad \gamma\in\Gamma\,. \tag{4}$$

On the other side, we have the voting intentions β_γ for each candidate $\gamma\in\Gamma$. We have at our disposal a vector $\beta \equiv \big(\beta_\gamma\big)_{\gamma\in\Gamma}$ with n components and satisfying

$$\beta_\gamma \geq 0\,, \quad \sum_{\gamma\in\Gamma} \beta_\gamma \,\leq\, 1\,, \quad \gamma\in\Gamma\,.$$

In other words,

$$\beta \,\in\, \widetilde{K}_n \,\equiv\, \Big\{ q\in {\rm I\!R}^n,\; q_j \geq 0\,,\; \sum_{j=1}^{n} q_j \,\leq\, 1 \Big\}\,.$$

• We adopt in this section a classical point of view for taking into account the variety of possibles underlyings. We suppose that the opinion ν (with $\nu\in$ G) is present in the entire population with a probability p_ν. So the state of opinion is mathematically modelized by a law of probability $\big(p_\nu\big)_{\nu\in{\rm G}}$. The state of the opinion p satisfies the natural constraints

$$p \,\in\, K_m \,\equiv\, \Big\{ q\in {\rm I\!R}^m,\; q_j \geq 0\,,\; \sum_{j=1}^{m} q_j \,=\, 1 \Big\} \tag{5}$$

that express that we have a discrete law of probability. There are two natural questions when we try to link the vector β of voting intentions with the state of opinions p.

($\mathbf{Q_1}$) If the state of the opinion is known, how to predict the voting intentions ?

($\mathbf{Q_2}$) If the voting intentions are known, how to determine the state of the opinion ?

• The answer to the question (Q_1) is simple if we consider that voting intentions could be determined by the state of the opinion. Then we think coherent to express that the expectation of the family $S_{\gamma\,\nu}$ for ν running in G is equal to the voting intention $\beta\in\widetilde{K}_n$. We can say also that the correlation of the probability vectors p and $s_\gamma \equiv \big(S_{\gamma\,\nu}\big)_{\nu\in{\rm G}}$ is equal to the voting intention β_γ. In algebraic terms,

$$\sum_{\nu\in G} S_{\gamma\,\nu}\; p_\nu \;=\; \beta_\gamma\,, \quad \gamma\in\Gamma\,. \tag{6}$$

The question (Q_2) exchanges the datum and the unknown. Then the relation (6) is now a linear system with unknown $p\in K_m$ and given datum $\beta\in\widetilde{K}_n$. Of course, the system (6) is in general not correctly posed if the hypothesis (3) is satisfied. We have n equations and only m unknowns. We adopt in this contribution a least square approach and replace the system (6) by the minimization of some squared functional, say

$$J(p) \;=\; \frac{1}{2} \sum_{\gamma\in\Gamma} \Big(\sum_{\nu\in G} S_{\gamma\,\nu}\, p_\nu \,-\, \beta_\gamma \Big)^2$$

to fix the ideas. The constraint (5) has to be satisfied because the family of numbers $\big(p_\nu\big)_{\nu\in G}$ is a probability distribution. We solve a quadratic optimization problem with the functional $J(\bullet)$ and the linear inequalities constraints (5):

$$\text{find } p \,\in\, K_m \text{ such that } J(p) \,=\, \inf\, \big\{ J(q),\; q\in K_m \big\}\,. \tag{7}$$

If the matrix $S_{\gamma\nu}$ introduced at the relation (4) is of maximal rank m (and we do this hypothesis in the following), the problem (7) is the minimization of a coercive quadratic functional inside a closed non empty convex set. This problem has a unique solution; we solve it using the Uzawa algorithm (see *e.g.* the book of Gondran and Minoux [15]).

3) Two quantum models for voting process

The fact of considering quantum modelling induces a specific vision of probabilities. We refer *e.g.* to the classical treatise on quantum mechanics of Cohen-Tannoudji *et al.* [9], to the so-called contextual objectivity proposed by Grangier [16], to the approach of Mugur-Schächter [28], or to the elementary introduction proposed by Busemeyer and Trueblood [8] in the context of statistical inference.

- In a first tentative [12], we have proposed to introduce an Hilbert space V_Γ formally generated by the candidates $\gamma \in \Gamma$. In this space, a canditate γ is represented by a unitary vector $|\,\gamma\!>$ and this family of n vectors is supposed to be orthogonal. Then an elector ℓ can be decomposed in the space V_Γ of candidates according to

$$(8) \qquad |\,\ell\!> \,=\, \sum_{\gamma\in\Gamma} \theta_{\ell\gamma}\,|\,\gamma\!> \,.$$

The vector $|\,\ell\!>\in V_\Gamma$ is supposed also to be a unitary vector to fix the ideas. According to Born's rule, the probability for a given elector ℓ to give his voice to the particular candidate γ is equal to $|\,\theta_{\ell\gamma}\,|^2$. The violence of the quantum measure is clearly visible with this example: the opinions of an elector ℓ never coincide with the program of any candidate. But with a voting system where an elector has to choice only one candidate among n, his social opinion is *reduced* to the one of a particular candidate.

- Our second model [13] is adapted to the grading step of range voting [4, 29]. We introduce a specific grading space $W_{\rm G}$ of political appreciations associated with a grading family G. The space $W_{\rm G}$ is formally generated by the m orthogonal vectors $|\,\nu\!>$ relative to the opinions. Then we suppose that the candidates γ are now decomposed by each elector on the basis $|\,\nu\!>$:

$$(9) \qquad |\,\gamma\!> = \sum_{\nu\in G} \alpha_{\nu\,\gamma}\,|\,\nu\!>\,, \quad \gamma\in\Gamma\,.$$

Moreover the vector $|\,\gamma\!>$ in (9) is supposed to be by a unitary:

$$(10) \qquad \sum_{\nu\in G} |\,\alpha_{\nu\,\gamma}\,|^2 \;=\; 1\,, \quad \gamma\in\Gamma\,.$$

With this notation, the probability for a given elector to give an opinion ν to a candidate γ is simply a consequence of the Born rule. The mean statistical expectation of a given opinion ν for a candidate γ is equal to $|\,\alpha_{\gamma\,\nu}\,|^2$ on one hand and is given by the popularity polls $S_{\gamma\,\nu}$ on the other hand. Consequently,

$$(11) \qquad |\,\alpha_{\nu\,\gamma}\,|^2 \;=\; S_{\gamma\,\nu}\,, \quad \gamma\in\Gamma\,,\; \nu\in{\rm G}.$$

4) State of the opinion: a link between quantum voting models

We have at our disposal two quantum models. The first one operates in an Hilbert space V_Γ generated (formally) by the candidates $|\,\gamma>$ for $\gamma \in \Gamma$. The second uses an Hilbert space $W_{\rm G}$ formally generated by the grading G of appreciations $|\,\nu>$ for $\nu \in {\rm G}$.

• The first model in space V_Γ is well adapted for determining the voting intentions throught the Born rule. In this contribution, we simplify the approach (8) and suppose that there exists some equivalent candidate $|\,\xi> \in V_\Gamma$ such that the voting intention β_γ for each particular candidate $\gamma \in \Gamma$ is equal to $|<\xi\,,\,\gamma>|^2$:

(12) $$|<\xi\,,\,\gamma>|^2 \,=\, \beta_\gamma\,, \quad \forall\,\gamma \in \Gamma\,; \quad |\,\xi> \equiv \sum_{\gamma\in\Gamma} |\,\gamma><\gamma\,,\,\xi> \,\in V_\Gamma\,.$$

• The second model in space $W_{\rm G}$ is appropriate to range voting and popularity polls. We interpret now the relation (9) in the following way: for each candidate $\gamma \in \Gamma$, there exists a political decomposition $A\,|\,\gamma> \in W_{\rm G}$ in terms of the grid G and we have

(13) $$A\,|\,\gamma> = \sum_{\nu\in G} \alpha_{\nu\gamma}\,|\,\nu>\,, \quad \gamma \in \Gamma\,.$$

By linearity, we construct in this way a linear operator $A\,:\,V_\Gamma \longrightarrow W_{\rm G}$ between two different Hilbert spaces. A state of the opinion is now modelized by a vector $|\,\zeta> \in W_{\rm G}$. Remark that the coefficients $\alpha_{\nu\gamma}$ are related to the data $S_{\gamma\nu}$ with the help of the relation (11). We suppose also $\alpha_{\nu\gamma} \geq 0$ in the following to fix the ideas.

• The questions (Q_1) and (Q_2) presented in Section 2 can now be formulated in terms of links between the equivalent candidate $|\,\xi> \in V_\Gamma$ and the state of the opinion $|\,\zeta> \in W_{\rm G}$. If the state of the opinion $|\,\zeta>$ is known, the question set by (Q_1) is now to determine the voting intentions β_γ obtained also by the relation (12). We suppose that this operation is also done for each particular candidate $\gamma \in \Gamma$ according to the Born rule *via* a scalar product between the opinion state $|\,\zeta>$ and the political decomposition $A\,|\,\gamma>$ proposed in (13):

(14) $$|<\zeta\,,\,A\,\gamma>|^2 \,=\, |<\xi\,,\,\gamma>|^2\,, \qquad \forall\,\gamma \in \Gamma\,; \quad |\,\zeta> \in W_{\rm G}\,.$$

Then there exists some phase $\varphi_\gamma \in \mathbb{R}$ for each $\gamma \in \Gamma$ and the relation (14) implies

(15) $$<\zeta\,,\,A\,\gamma> = {\rm e}^{-i\,\varphi_\gamma} <\xi\,,\,\gamma>\,, \qquad \forall\,\gamma \in \Gamma\,.$$

We introduce the "phase operator" J with a diagonal matrix composed by the different phases:

$$J \,=\, {\rm diag}\Big\{{\rm e}^{i\,\varphi_\gamma}\,,\,\gamma \in \Gamma\Big\}\,.$$

We remark that the adjoint operator J^* is the inverse J^{-1} of the operator J: $J^* = J^{-1}$. We introduce also the adjoint operator $A^* : W_{\rm G} \longrightarrow V_\gamma$. Then the relation (15) takes the form

(16) $$<A^*\,\zeta\,,\,\gamma> = <\xi\,,\,J^*\,\gamma> \equiv <J\,\xi\,,\,\gamma>\,, \qquad \forall\,\gamma \in \Gamma\,.$$

By linearity of the operators A and J, we can write the relation (16) under the compact form

(17) $$A^*\,|\,\zeta> = \,J\,|\,\xi>\;.$$

We have a response to the first question (Q_1): if the state of the opinion $\,|\,\zeta>\,$ is known, it determines an equivalent candidate $\,|\,\xi>\,$ *modulo* a phase. We observe also that the phase operator is eliminated when we consider the Born rule (14). In the following, we replace the operator $\,J\,$ by the identity and (17) by

$$A^*\,|\,\zeta> = \,|\,\xi> \ . \tag{18}$$

- The question (Q_2) can now be formulated in a simple way: if the equivalent candidate $|\,\xi>$ is known, is it possible to determine a state of the opinion $\,|\,\zeta> \in W_{\mathrm{G}}\,$ such that the relation (18) holds ? The difficulty concerns now linear algebra. Because $\,\mathrm{rank}\, A = m$, the operator $\,A^*\,$ is injective $\,W_{\mathrm{G}} \longrightarrow V_\Gamma$. But it is not a surjective operator since $\,n > m\,$ as supposed in (3). In this contribution, we propose to solve (18) in terms of least squares, *i.e.* to solve the equation obtained after multiplying the relation (18) by the operator A:

$$A\,A^*\,|\,\zeta> = \,A\,|\,\xi> \ . \tag{19}$$

Then the state of the opinion $\,|\,\zeta>\,$ can be determined without difficulty. We normalize it for our application. When the state $\,|\,\zeta>\,$ is known, the relative quantum probability $\delta_\nu\,$ of observing the particular state $\,\nu \in \mathrm{G}\,$ is equal, as consequence of Born's rule, to the square of the component $\,<\nu,\,\zeta>$:

$$\delta_\nu \; = |<\nu,\,\zeta>|^2\,, \quad \nu \in \mathrm{G}\,. \tag{20}$$

5) Spring 2012 preliminary results

We have obtained in French popular newspapers three political popularity polls in february, march and april 2012. For each case, we have chosen voting intention polls at a date as close as possible to the previous ones. The first family of data has been obtained in february 2012. Popularity data [17, 21] and result of voting intentions [17, 21] are displayed in Table 1. The names of the principal candidates to the French presidential election are proposed in alphabetic order with the following abbreviations: "Ba" for François Bayrou, "Ho" for François Hollande, "Jo" for Eva Joly, "LP" for Marine Le Pen, "Mé" for Jean-Luc Mélanchon and "Sa" for Nicolas Sarkozy. Similar data are displayed in Table 2 for march 2012 [18, 20] and in Table 3 for april 2012 [22, 23]. In this last table, we have also reported the result of the election of 22 April 2012.

	+	0	−	voting
Ba	.55	.14	.31	.125
Ho	.52	.08	.40	.30
Jo	.29	.13	.58	.03
LP	.28	.06	.66	.175
Mé	.38	.20	.42	.085
Sa	.33	.00	.67	.25

Table 1. Popularity and sounding polls, february 2012 [17, 19, 21].

	++	+	0	−	−−	voting
Ba	.08	.62	.03	.23	.04	.12
Ho	.09	.45	.00	.30	.16	.275
Jo	.02	.34	.02	.40	.22	.03
LP	.10	.24	.01	.26	.39	.17
Mé	.11	.46	.03	.31	.09	.11
Sa	.10	.31	.00	.29	.30	.28

Table 2. Popularity and sounding polls, march 2012 [18, 20].

	+	0	−	voting	result
Ba	.56	.07	.37	.095	.091
Ho	.57	.03	.40	.285	.286
Jo	.35	.10	.55	.015	.023
LP	.26	.05	.69	.15	.179
Mé	.47	.10	.43	.145	.111
Sa	.49	.05	.46	.29	.272

Table 3. Popularity, sounding polls and result, april 2012 [22, 23].

• The result of our mathematical treatment is presented in tables 4 to 7. From popularity polls and voting intentions, we evaluate a classical and a quantum state of the opinion. The first line gives the classical probability p solution of the problem (7). The second line describes the components of the quantum state of the opinion ζ compatible with the relation (19). The third line is the quantum probability, *id est* the square of the second line according to Born's rule (see (20)). We observe that the constraints (5) are active and induce values equal to zero for some classical probabilities in the first line of Table 5 and Table 6. The quantum state ζ (second line of tables 4 to 7) is unitary. We observe that the sign of some components is negative. The comparisons of classical and quantum probabilities (first and third lines of tables 4 to 7) agree globally in a first approach. Nevertheless for precise components (opinion "−" in march 2012 *id est* fourth column of Table 5, opinion "+" in april 2012 *id est* first column of tables 6 and 7) the two probabilities differ notabily.

	+	0	−
classical probability	.15	.74	.11
quantum state	.34	−.90	.26
quantum probability	.11	.82	.07

Table 4. Classical and quantum state of the opinion, february 2012.

	++	+	0	−	−−
classical probability	.57	.14	0	0	.29
quantum state	−.58	.53	−.18	−.51	.31
quantum probability	.33	.28	.03	.26	.10

Table 5. Classical and quantum state of the opinion, march 2012.

	+	0	−
classical probability	.28	.72	0
quantum state	.14	−.96	.25
quantum probability	.02	.92	.06

Table 6. Classical and quantum state of the opinion, april 2012.

	+	0	−
classical probability	.25	.73	.02
quantum state	.13	−.96	.25
quantum probability	.02	.92	.06

Table 7. Similar to Table 6, but the voting polls have been replaced by the result of 22 April (last column of Table 3).

6) Conclusion and perspectives

In this contribution, we have introduced a state of the opinion to analyse with a given degree of precision the variety of appreciations of political programs. In a classical approach the state of the opinion is a discrete law of probability. With quantum modelling, this state is a vector in an Hilbert space of political appreciations. Two questions has been formulated. On one hand, how the knowledge of the state of the opinion determines the voting intentions ? The reverse question on the other hand: how the knowledge of voting intentions can define a state of the opinion ? We have studied these two questions in both classical and quantum points of view. We have proposed responses as simple as possible in terms of mathematical modelling. We have tested the possibility to determine a state of the opinion with data issued from popularity and voting intentions polls available during the "first tour" of French presidential election of April 2012. Of course the existence of such a state of the opinion remains an hypothesis, especially in the quantum case. We suggest that a possible further step is to replace an ordered grading family of opinions by a non-ordered set of political points of view.

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