D2T4 lattice Boltzmann scheme for scalar problems

François Dubois [1] [2]

[1] *Dept. of Mathematics, Conservatoire National des Arts et Métiers, Paris, France.*

[2] *Mathematics Laboratory, Paris-Sud University, Orsay, France.*

francois.dubois@math.u-psud.fr

24 february 2011 *

The importance of extending the lattice Boltzmann scheme from square regular meshes to unstructured triangulations has been recognized during the last years of 20th century ([Ch98], [KSO99], [PXDC99]). In particular the "volumetric formulation" of H. Chen [Ch98] makes a link with finite volumes, using the "Inria cells" [Vi86] around each vertex of a finite element type triangulation. This method is still under active development ([UBS03], [USB04], [PUS09]).

In this contribution, instead of adopting this volumetric formulation, we consider the lattice Boltzmann scheme as a particle method with given discrete velocities and extend to triangular meshes the approach of d'Humières [dH92].

We take a finite element type bidimensional mesh $\mathcal{T}$ composed of triangles $K \in \mathcal{T}^2$. Each triangle K has 3 edges. Each edge inside the border of K is part of the boundary of (at most) two triangles : the triangle $K_0 \equiv K$ itself and its $j^{\rm th}$ neighbor K_j. It is then natural to consider outgoing particles $(f_j)_{0 \leq j \leq 4}$ going from K towards K_j with a local velocity $v_j(K)$ chosen in such a way that the centers of both triangles K and K_j are joined in exactly one time step of duration Δt. This remark explains the name "D2T4".

We distinguish between ingoing $(f_j)_{0 \leq j \leq 4}$ and outgoing $(\widetilde{f}_j)_{0 \leq j \leq 4}$ particles linking triangle K_j and the reference triangle K. In the same way that the triangle K_j is neighbor number j relative to triangle K, the triangle K is neighbor number ℓ relative to triangle K_j. With these notations, we have $v_j(K) + v_\ell(K_j) = 0$. After a relaxation step $f \longrightarrow f^*$ described with d'Humières's method [dH92], the outgoing particles $f_j^*(K_j)$ enter inside triangle K after a time step Δt. The lattice Boltzmann scheme can be written as

$$\widetilde{f}_j(K, t + \Delta t) \,=\, f_\ell^*(K_j, t)\,.$$

In the conference, we will present first simulations obtained with the corresponding method for various linear second order scalar problems on triangular meshes of the type

* Submitted to 20th DSFD Conference, Fargo, USA, 8-12 august 2011.

proposed in Figure 1. We will analyse also the corresponding scheme with the so-called "Taylor expansion method" [Du08] and discuss conditions for higher accuracy, and give "quartic parameters" [DL09].

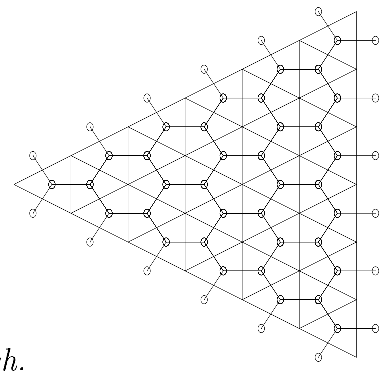

Figure 1. *Typical mesh with equilateral triangles. The four degrees of freedom of D2T4 scheme are located at the gravity center of each triangle. The links between triangles create the dual hexagonal mesh.*

The author thanks Pierre Lallemand for rich interactions all along the elaboration of this work.

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