Lattice Boltzmann scheme and finite volumes *

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We denote by $\mathcal{L}$ a lattice, Δx a typical scale associated with this lattice, Δt a time step, $\lambda \equiv \frac{\Delta x}{\Delta t}$ a typical celerity of the problem, x a vertex of this lattice, $y_j \equiv x + \Delta t \, v_j$, $(0 \leq j \leq J)$ the set of neighbouring nodes around the vertex x. We suppose that the family $(v_j)_{0 \leq j \leq J}$ of celerities is symmetric relatively to the origin :

$$\forall j \in \{0, \cdots, J\}, \quad \exists \, ! \; \sigma(j) \in \{0, \cdots, J\}, \quad v_j + v_{\sigma(j)} \, = \, 0 \, .$$

Let $f_j(x, t)$ be a distribution of particles on the lattice $\mathcal{L}$ at the vertex x and discrete time t. We assume that the density $\rho \equiv \sum_j f_j$ and the momentum $q \equiv \sum_j v_j \, f_j$ are conserved during the collision step and we denote by $f_j^*(x, t)$ the distribution after this step : $\rho = \sum_j f_j = \sum_j f_j^* = \rho^*$, $q = \sum_j v_j \, f_j = \sum_j v_j \, f_j^* = q^*$. Then the dynamics of the lattice Boltzmann scheme takes the simple form [2]

$$f_j(x, \, t + \Delta t) \, = \, f_j^*(x \, - \, v_j \, \Delta t, \, t) \, , \quad x \in \mathcal{L}, \; 0 \leq j \leq J \, .$$

We introduce a cell $K(x)$ around the vertex x such that its boundary $\partial K(x)$ is composed by J edges $a_j(x)$ separating the nodes x and y_j : $\partial K(x) \, = \, \bigcup_j a_j(x)$, $a_j(x) = \partial K(x) \cap \partial K(y_j) = a_{\sigma(j)}(y_j)$. We denote by $\mid K(x) \mid$ and $\mid a_j(x) \mid$ the measures of $K(x)$ and $a_j(x)$ respectively. Then the conservation of mass and momentum takes the discrete form

$$\frac{1}{\Delta t} \Big[\begin{pmatrix} \rho \\ q \end{pmatrix} (x, \, t + \Delta t) \, - \, \begin{pmatrix} \rho \\ q \end{pmatrix} (x, \, t) \Big] \, + \, \frac{1}{\mid K(x) \mid} \sum_j \mid a_j(x) \mid \, \begin{pmatrix} \psi_j \\ \zeta_j \end{pmatrix} (x) \, = \, 0 \, ,$$

with a mass flux $\psi_j(x)$ and a flux of impulsion $\zeta_j(x)$ between the vertices x and y_j defined according to

$$\psi_j(x) \, = \, \frac{\mid K(x) \mid}{\Delta t \, \mid a_j(x) \mid} \, (f_j^*(x) - f_{\sigma(j)}^*(y_j)) \, , \; \zeta_j(x) \, = \, \frac{\mid K(x) \mid}{\Delta t \, \mid a_j(x) \mid} \, v_j \, (f_j^*(x) + f_{\sigma(j)}^*(y_j))$$

and satisfying the conservation property :

$$\psi_j(x) + \psi_{\sigma(j)}(y_j) = 0 \, , \qquad \zeta_j(x) + \zeta_{\sigma(j)}(y_j) = 0.$$

This remark makes a clear link between the lattice Boltzmann scheme and the finite volume method. In the particular case of D_1Q_3 and D_2Q_9 models (see *e.g.* [3] for a precise description of the latter), this remark is the starting point of a complete development of a new boundary condition procedure that is more efficient than [1] and will be presented at the Conference.

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[3] P. Lallemand, L.S. Luo, *Physical Review E*, vol. **61**, n° 6, p. 6546-6562, 2000.

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