Analyse asymptotique non linéaire au 4e ordre des schémas de Boltzmann sur réseau

François Dubois*†

Groupe de travail "Schémas de Boltzmann sur réseau"
Institut Henri Poincaré, Paris
10 janvier 2018

* Conservatoire National des Arts et Métiers, LMSSC, Paris
† Department of Mathematics, University Paris-Sud, Orsay, France

Partial differential equations

$$\partial_t W = \Gamma_1 + \Delta t \, \Gamma_2 + \Delta t^2 \, \Gamma_3 + \Delta t^3 \, \Gamma_4 + \mathrm{O}(\Delta t^4)$$

Γ_j: vector obtained after j space derivations
of the conserved moments W
and the equilibrium vector $\Phi(W)$.

Non-Conserved moments:

$$Y = \Phi(W) + S^{-1}\big(\Delta t \, \Phi_1 + \Delta t^2 \, \Phi_2 + \Delta t^3 \, \Phi_3\big) + \mathrm{O}(\Delta t^4)$$

Φ_j analogous to Γ_j but not with the same dimension!

Main result: explicitation of the vectors Γ_j and Φ_j

$$\partial_t W = \Gamma_1 + \Delta t\, \Gamma_2 + \Delta t^2\, \Gamma_3 + \Delta t^3\, \Gamma_4 + \mathrm{O}(\Delta t^4)$$
$$Y = \Phi(W) + S^{-1}\big(\Delta t\, \Phi_1 + \Delta t^2\, \Phi_2 + \Delta t^3\, \Phi_3\big) + \mathrm{O}(\Delta t^4)$$

$$\Gamma_1 = A\, W + B\, \Phi(W)$$
$$\Phi_1 = C\, W + D\, \Phi(W) - \mathrm{d}\Phi(W).\Gamma_1$$
$$\Gamma_2 = B\, \Sigma\, \Phi_1$$

with Σ the Hénon matrix: $\Sigma \equiv S^{-1} - \frac{1}{2}\,\mathrm{I}$

Notations: $\gamma_j \equiv \mathrm{d}\Phi(W).\Gamma_j$

$$\partial^2\Psi.\Gamma_1 \equiv \mathrm{d}^2\Psi(W).(\Gamma_1, \Gamma_1) + \mathrm{d}\Psi(W).\,\mathrm{d}\Gamma_1(W).\,\Gamma_1 = \mathrm{d}\big(\mathrm{d}\Psi(W).\,\Gamma_1\big).\,\Gamma_1$$

$$\Phi_2 = D\, \Sigma\, \Phi_1 - \gamma_2 - \Sigma\, \mathrm{d}\Phi_1.\Gamma_1$$
$$\Gamma_3 = B\, \Sigma\, \Phi_2 + \frac{1}{6}\, B\, \mathrm{d}\Phi_1.\Gamma_1 - \frac{1}{12}\, B_2\, \Phi_1$$
$$\Phi_3 = D\, \Sigma\, \Phi_2 - \gamma_3 - \Sigma\, \mathrm{d}\Phi_1.\Gamma_2 - \Sigma\, \mathrm{d}\Phi_2.\Gamma_1 + \frac{1}{6}\, D\, \mathrm{d}\Phi_1.\Gamma_1 - \frac{1}{12}\, D_2\, \Phi_1 - \frac{1}{12}\, \partial^2\Phi_1.\Gamma_1$$
$$\Gamma_4 = B\, \Sigma\, \Phi_3 - \frac{1}{4}\, B_2\, \Phi_2 + \frac{1}{6}\, B\, D_2\, \Sigma\, \Phi_1 + \frac{1}{6}\, A\, B\, \Phi_2 - \frac{1}{6}\, B\,\big(\mathrm{d}\gamma_1.\Gamma_2 + \mathrm{d}\gamma_2.\Gamma_1\big) - \frac{1}{6}\, B\, \Sigma\, \partial^2\Phi_1.\Gamma_1$$

Survey

- Compact iteration of lattice Boltzmann schemes
- Block decomposition of the moment-velocity operator matrix
- Asymptotic expansion of the equivalent partial differential
  equations and of the non conserved moments
- Explicitation of the coefficients up to order 4
  with less than 7 terms
- Intensive use of differential calculus
- Validation of the nonlinear expansion for fluid flow
  and thermal problems up the the order 3
- Simplified formulae in the linear case
- Validation of the linear expansion
  for the fluid flow D2Q9 scheme

Merci de votre attention !