#### DYNAMICS, GEOMETRY AND NUMBER THEORY

# A Conference in Honor of Gregory Margulis' 70th Birthday

### **Titles and Abstracts**

#### Herbert Abels (Universität Bielefeld)

"The topological generating rank of Lie groups"

We define the toplogical generating rank  $d_{top}(G)$  of a topological group G as the minimal number of elements of G which generate a dense subgroup of G. I will present joint work with Gena Noskov, partly in progress, about our attempts to compute the topological generating rank for connected Lie groups. The cases that G is semisimple, where  $d_{top}(G)$  is slightly bigger than one, and that G is nilpotent, have been known. We have a precise formula for the case that G is solvable and bounds for the general case, which we hope are sharp. The reduction steps motivated us to consider - a version of - the Frattini subgroup of G.

### Nalini Anantharaman (Université de Strasbourg)

"Quantum ergodicity on large graphs"

The topic of "Quantum ergodicity" deals with the (de)localization of eigenfunctions of Schrödinger operators, for classically chaotic systems. I will first review the rigourous results about the (de)localization of laplacian eigenfunctions, on compact riemannian manifolds. One of the main tool is the relation between the ergodic properties of the geodesic flow and the large time behaviour of solutions of the Schrödinger equation. I will then talk about my work with Etienne Le Masson, where we studied the delocalization of eigenfunctions on large regular (discrete) graphs, and work in progress with Mostafa Sabri for the Anderson model on large regular graphs.

# Yves Benoist (Université Paris-Saclay)

"Harmonic quasiisometries"

I will explain why a quasiisometric map between rank one symmetric spaces is within bounded distance from a unique harmonic map. This joint work with D. Hulin completes the proof of the Schoen-Li-Wang conjecture.

# **Victor Beresnevich (University of York)**

"Quantitative non-divergence in Diophantine approximation on manifolds"

I will start by recalling an estimate of Kleinbock and Margulis from 1998 on quantitative non-divergence of certain orbits in the space of lattices and its initial motivation. Then the main part of my talk will be to explain the role of this fundamental result in several more recent developments in the theory of Diophantine approximation on manifolds, including a problem on rational points near and badly approximable points on manifolds.

### Marc Burger (ETH, Zürich)

"Maximal representations, non Archimedean Siegel spaces, and buildings"

Maximal representations of the fundamental group G of a compact surface S into a real symplectic group Sp(V) are natural generalisations of the holonomy representations of G into SL(2,R) associated to hyperbolic structures on S. Maximal representations give quasiisometric embeddings and their images in Sp(V) can be seen as higher rank analogues of Kleinian groups. A lot of activity in the last decade has been devoted to understand which features of classical hyperbolic geometry and Teichmueller spaces generalize to this setting. In this talk I will report on joint work with Beatrice Pozzetti, were we study the structure of representations associated to points in the real spectrum compactification of the character variety of maximal representations.

### Manfred Einsiedler (ETH, Zürich)

"Measure rigidity for diagonal actions on quotients of SL(3,R)xSL(3,R)"

In joint work with E. Lindenstrauss we obtained the classification of positive entropy measures for new cases of diagonal actions.

### **Alex Eskin (University of Chicago)**

"Polygonal Billiards and Dynamics on Moduli Spaces"

Billiards in polygons can exhibit some bizarre behavior, some of which can be explained by deep connections to several seemingly unrelated branches of mathematics. These include algebraic geometry (and in particular Hodge theory), Teichmüller theory and dynamics theory on homogeneous spaces. I will attempt to explain some of these connections, and to survey some recent progress in the area.

# Simion Filip (University of Chicago)

"Teichmüller dynamics and Hodge theory"

The natural action of SL(2,R) on the moduli space of translation surfaces (i.e. holomorphic 1-forms) exhibits a rich collection of dynamical and geometric properties. Recently Eskin, Mirzakhani, and Mohammadi established measure rigidity results inspired by the homogeneous setting.

After providing some background in the subject, I will explain some connections between Hodge theory and the SL(2,R) action, as well as applications to Lyapunov exponents. One consequence of these results is that orbit closures, which have an affine structure by work of Eskin, Mirzakhani, and Mohammadi, also have natural algebraic structures.

# **Hillel Furstenberg (Hebrew University of Jerusalem)**

"Are no algebraic numbers well-approximable?"

We show that the diophantine nature of algebraic numbers can be determined by ascertaining whether the orbit of rational points on a particular manifold under a rationally defined group acting on the manifold is closed.

# **Bill Goldman (University of Maryland)**

"Dynamical systems arising from moduli of geometric structures"

The classification of geometric structures on surfaces leads to interesting dynamical systems. For example, classifying Euclidean geometries on the torus leads to the usual action of the SL(2,Z) on the upper half-plane. This action is dynamically trivial, with a quotient space the familiar modular curve.

By contrast, the classification of other simple geometries on the torus leads to the standard linear action of SL(2,Z) on  $R^{^{^{2}}}$ , with chaotic dynamics and a pathological quotient space.

In these talks I will describe such dynamical systems where the moduli space correspond to two-generator groups of 2-by-2 matrices, is described by the nonlinear symmetries of cubic equations like Markoff's equation  $x^2 + y^2 + z^2 = x$  y z. Both trivial and chaotic dynamics arise simultaneously, and relate to possibly singular hyperbolic metrics on surfaces whose fundamental group is free of rank two.

# **Anatole Katok (Penn State University)**

"Entropy and Lyapunov exponents: rigidity vs. flexibility"

For dynamical systems of algebraic origin, i.e. homogeneous and affine maps and flows on homogeneous spaces the values of metic entropy with respect to Haar measure and topological entropy are always the same, and the range of their values (as well as the values of Lyapunov exponents) is limited. There is a number of situations where the values of those invariants determine algebraic systems within a large class of systems. On the other hand, beyond the algebraic case the flexibility paradigm should hold:

Under properly understood general restrictions within a fixed class of smooth dynamical systems quantitative dynamical invariants take arbitrary values.

Most known constructions are perturbative and hence at best would allow to cover a small neighborhood of the values allowed by the model, or more often, not even that, since homogeneous systems are often 'extremal'. So establishing flexibility calls for *non-perturbative or large perturbation constructions* in large

families to cover possible values of invariants.

Work on flexibility is still in its infancy and in many situations the proper "general restrictions" are not fully understood. In this talk I will discuss general conjectures and first two results that confirm those conjectures (technically still in progress since preprints are not publicly available): one (joint with J. Bochi and F. Rodriguez Hertz) deals with description of possible values of Lyapunov exponents for Anosov volume preserving diffeomorphisms of a torus and the other (joint with A. Erchenko) describes all possible pairs of values for Liouville and topological entropy for geodesic flows on compact surfaces of negative curvature.

### François Labourie (Université de Nice-Sophia Antipolis)

"Surface groups in uniform lattices of complex simple groups"

This is a joint work in progress with Jeremy Kahn and Shahar Mozes. After Kahn-Markovic work for SL(2,C) and Hamenstadt further work on rank 1 groups, we present the existence of surface groups in a large class of lattices, together with a (conjectural) quantitative statement, generalizing the classical quasisymmetry. The main tool is the study of configurations of triples in the full grassmannian.

# **Elon Lindenstrauss (Hebrew University of Jerusalem)**

"Diagonal actions in positive characteristic"

Measure classification of higher rank diagonalizable algebraic actions in positive characteristic poses additional difficulties essentially because the additive group of local fields in positive characteristic has a lot of possible subgroups. Indeed, examples due to Einsiedler show that there are strange measures of positive entropy in the positive characteristic analog of the x2 x3 system.

We show that the situation is much better for such actions on quotients of semisimple groups, providing in particular a classification of positive entropy invariant measures for quotients of SL(d). One ingredient in our proof is a measure classification result for certain groups generated by unipotents by Salehi Golsefidy and Mohammadi. Joint work with Einsiedler and Mohammadi.

# Alex Lubotzky (Hebrew University and ETH-ITS)

"Ramanujan complexes and topological expanders"

Expander graphs in general, and Ramanujan graphs, in particular, have played a major role in computer science in the last 4 decades and more recently also in pure math. The first explicit construction of bounded degree expanding graphs was given by Margulis in the early 70's. In mid 80' Margulis and Lubotzky-Phillips-Sarnak provided Ramanujan graphs which are optimal such expanders.

In recent years a high dimensional theory of expanders is emerging. A notion of topological expanders was defined by Gromov in 2010 who proved that the complete d-dimensional simplical complexes are such. He raised the basic question of existence of such bounded degree complexes of dimension d>1.

This question was answered recently affirmatively (by T. Kaufman, D. Kazdhan and A. Lubotzky for d=2 and by S. Evra and T. Kaufman for general d) by showing that the d-skeleton of (d+1)-dimensional Ramanujan complexes provide such topological expanders. We will describe these developments and the general area of high dimensional expanders.

# **Gregory Margulis (Yale University)**

"Mathematical and autobiographical recollections"

# **Amir Mohammadi (University of Texas, Austin)**

"Effective equidistribution of certain adelic periods"

We will discuss a quantitative equidistribution statement for adelic homogeneous subsets whose stabilizer is maximal and semisimple. An application to certain equidistribution theorems will also be given. This is a joint work with Einsiedler, Margulis and Venkatesh.

### Pierre Pansu (Université Paris-Saclay)

"Large scale conformal maps"

Benjamini and Schramm's work on incidence graphs of sphere packings suggests a notion of conformal map between metric spaces which is natural under coarse embeddings. We show that such maps cannot exist between nilpotent or hyperbolic groups unless certain numerical inequalities hold.

## Alireza Salehi Golsefidy (University of California, San Diego)

"Super-approximation and its applications"

Let G be a finitely generated subgroup of GL(n,Q). Under certain algebraic conditions, strong approximation describes the closure of G with respect to its congruence topology. Super-approximation essentially tells us how dense G is in its closure! Here is my plan for this talk:

- 1. I will start with the precise formulation of this property.
- 2. Some of the main results on this subject will be mentioned.
- 3. Some of the (unexpected) applications of super-approximation will be mentioned, e.g. Banach-Ruziewicz problem, orbit equivalence rigidity, variation of Galois representations.
- 4. Some of the auxiliary results that were needed in the proof of super-approximation will be mentioned: sumproduct phenomena, existence of small solutions.

# Anna Wienhard (Ruprecht-Karls Univ., Heidelberg)

"On rigidity and flexibility of (some) discrete subgroups of Lie groups"

I will discuss several developments regarding representations of surface groups, and more generally of hyperbolic groups into Lie groups and put them into context.

I will describe relations to proper actions on homogeneous spaces and to deformation spaces of geometric structures.

The focus lies on the interaction between rigidity and flexibility phenomena.

# **Amie Wilkinson (University of Chicago)**

"Partial hyperbolicity, exponents, and superrigidity"

A superrigidity phenomenon, in its loosest formulation, occurs when one can reconstruct the action of a Lie group from that of a discrete subgroup. In Margulis's superrigidity theorem, the Lie group is large (higher rank semisimple) and the subgroup is a lattice. In this talk, I will indicate how one of the core ideas behind Margulis's superrigidity also lies behind a superrigidity phenomenon in partially hyperbolic dynamics: under relatively mild hypotheses, one can reconstruct a flow from the action of a diffeomorphism. Here is Lie group is quite small (the real numbers), but hidden in the wings is the action of a much larger group.

## **Tamar Ziegler (Hebrew University of Jerusalem)**

"Concatenating cubic structures and patterns in primes"

We describe concatenation results for uniformity norms in ergodic and combinatorial categories. Roughly speaking we show that if a system exhibits nil-behavior with respect to two different communing actions, then it exhibits nil-behavior (of higher step) with respect to their joint action. We describe an application to counting polynomial configurations in primes.