

LABORATOIRE DE MATHÉMATIQUES  
UNIVERSITÉ PARIS-SUD PARIS-SACLAY, UMR 8628

**Stochastic Analysis**  
**and**  
**Large Scale Interacting Systems**

*le vendredi 10 mars 2017 au Bâtiment 425 en salle 117 - 119*  
*organisée par*  
*Jan Elias, Danielle Hilhorst, Perla El Kettani et Francesco Russo*

**09h55 - 10h00, Opening : Frédéric Rousset (University Paris-Sud)**

**10h00 - 10h40, Stefano Olla (University Paris-Dauphine)**

Macroscopic temperature profiles in non-equilibrium stationary states

**11h05 - 11h45, Motoko Kotani (Tohoku University)**

Applications of discrete geometric analysis to hierarchical understanding of materials

**11h50 - 12h30, Sandesh Hiremath (U Kaiserslautern)**

A stochastic model featuring acid induced gaps during cancer progression

**12h30 - 14h30, LUNCH**

**14h30 - 15h10, Martina Hofmanova (TU Berlin)**

Stochastic mean curvature flow

**15h40 - 16h20, Francesco Russo (ENSTA-ParisTech)**

BSDEs, càdlàg martingale problems and Föllmer-Schweizer decomposition

## *Titles and abstracts*

### **Sandesh HIREMATH**

*A stochastic model featuring acid induced gaps during tumor progression*

In this talk we propose a phenomenological model for the formation of an interstitial gap between the tumor and the stroma. The gap is mainly filled with acid produced by the progressing edge of the tumor front. Our setting extends existing models for acid-induced tumor invasion models to incorporate several features of local invasion like formation of gaps, spikes, buds, islands, and cavities. These behaviors are obtained mainly due to the random dynamics at the intracellular level, the go-or-grow-or-recede dynamics on the population scale, together with the nonlinear coupling between the microscopic (intracellular) and macroscopic (population) levels. The wellposedness of the model is proved using the semigroup technique and 1D and 2D numerical simulations are performed to illustrate model predictions and draw conclusions based on the observed behavior.

### **Martina HOFMANOVA**

*Stochastic mean curvature flow*

Motion by mean curvature of embedded hypersurfaces in  $\mathbb{R}^{N+1}$  is an important prototype of a geometric evolution law and has been intensively studied in the past decades. Mean curvature flow is characterized as a steepest descent evolution for the surface area energy and constitutes a fundamental relaxation dynamics for many problems where the interface size contributes to the systems energy. One of the main difficulties of the mean curvature flow is the appearance of topological changes and singularities in finite time. Further issues then arise in the mathematical treatment of the stochastic mean curvature flow, which was introduced as a refined model incorporating the influence of thermal noise.

We study a stochastically perturbed mean curvature flow for graphs in  $\mathbb{R}^3$  over the two-dimensional unit-cube subject to periodic boundary conditions. In particular, we establish the existence of a weak martingale solution. The proof is based on energy methods and therefore presents an alternative to the stochastic viscosity solution approach. To overcome difficulties induced by the degeneracy of the mean curvature operator and the multiplicative gradient noise present in the model we employ a three step approximation scheme together with refined stochastic compactness and martingale identification methods.

### **Motoko KOTANI**

*Structure understanding of materials by discrete geometric analysis*

Technology to observe and control atoms and molecules has been developed so that we considered materials as systems of hierarchical networks, where microscopic structures are discrete and macroscopic properties are continuous and recognized the importance of mathematical idea to bridge between discrete and continuum. Discrete geometric analysis offers such views. In the present talk, I would like to discuss some emerging results obtained at the AIMR based on the collaboration between materials science and mathematics.

## Stefano OLLA

*Macroscopic temperature profiles in non-equilibrium stationary states*

Systems that have more than one conserved quantity (i.e. energy plus momentum, density etc.), can exhibit quite interesting temperature profiles in non-equilibrium stationary states. I will present some numerical experiments and mathematical results.

## Francesco RUSSO

*BSDEs, càdlàg martingale problems and Föllmer-Schweizer decomposition*

*The talk will be based on joint work with Ismail Laachir (ZELIADE and ENSTA ParisTech) SIAM Journal on Financial Mathematics., vol. 7, pp. 308-356, 2016.*

The aim of this talk consists in introducing a new formalism for the deterministic analysis associated with backward stochastic differential equations driven by general càdlàg martingales. When the martingale is a standard Brownian motion, the natural deterministic analysis is provided by the solution of a semilinear PDE of parabolic type. A significant financial application concerns the hedging problem under basis risk of a contingent claim  $g(X_T, S_T)$ , where  $S$  (resp.  $X$ ) is an underlying price of a traded (resp. non-traded but observable) asset, via the celebrated Föllmer-Schweizer decomposition. We revisit the case when the couple of price processes  $(X, S)$  is a diffusion and we provide explicit expressions when  $(X, S)$  is an exponential of additive processes.