- In the whole paper, "finite group scheme" stands for "finite commutative group scheme".

- A simpler argument is available to prove Lemma 3.2. Let F be a finite Galois module over K. By Lemma 3.1 and local duality (5), there is an exact sequence

$$H^1(K,F) \to \prod_{v\in S} H^1(K_v,F) \to Ш^2_S(F')^D \to Ш^2(F')^D \to 0.$$

  Apply this exact sequence to $F = F' = \mu_n$. Since $H^1(K,\mu_n) = K^*/K^{*n}$ (and similarly with K replaced by K_v), the first map is onto by the weak approximation theorem. Hence $Ш^2_S(\mu_n) = Ш^2(\mu_n)$, which implies $Ш^2_S(\mathbf{G}_m) = Ш^2(\mathbf{G}_m)$ by Kummer theory and Hilbert 90. Now it is sufficient to prove that $Ш^2(\mathbf{G}_m) = 0$, which is the easier part of the proof of Lemma 3.2.