I/ Quelques schémas multipas:

I-1/ Exemples

I-1-1/ Adams Moulton

Script MUPAD

```
adams_moulton:=proc(k)
begin
xl:=[t[n]-j*h $j=-1..(k-2)];
yl:=[f[n-j] $j=-1..(k-2)];
p:=interpolate(xl,yl,s);
x[n+1] = x[n] + factor(int(p,s=t[n]..(t[n]+h)));
end_proc:
```

Exemple d'utilisation

```
"AM1:"; adams_moulton(1); "AM3:"; adams_moulton(3);
```

"AM1:"

$$x_{n+1} = x_n + h \cdot f_{n+1}$$

"AM3:"

$$x_{n+1} = \frac{h \cdot (5 \cdot f_{n+1} - f_{n-1} + 8 \cdot f_n)}{12} + x_n$$

I-1-2/ Adams Bashforth

Script MUPAD

```
adams_bashforth:=proc(k)
begin
xl:=[t[n]-j*h $j=0..(k-1)];
yl:=[f[n-j] $j=0..(k-1)];
p:=interpolate(xl,yl,s);
x[n+1] = x[n] + factor(int(p,s=t[n]..(t[n]+h)));
end_proc:
```

Exemple d'utilisation

```
"AB1:"; adams_bashforth(1); "AB3:"; adams_bashforth(3);
```

"AB1:"

$$x_{n+1} = h \cdot f_n + x_n$$

"AB3:"

$$x_{n+1} = \frac{h \cdot (5 \cdot f_{n-2} - 16 \cdot f_{n-1} + 23 \cdot f_n)}{12} + x_n$$

I-1-3/ Differentiation retrograde

Script MUPAD

```
ER:=proc(k)
begin
xl:=[t[n]-j*h $j=-1..(k-1)];
yl:=[x[n-j] $j=-1..(k-1)];
p:=interpolate(xl,yl,s);
factor(h*limit(diff(p,s),s=t[n]+h))=h*f[n+1];
end_proc:
```

```
end_proc:
```

Exemple d'utilisation

```
"ER1:"; ER(1); "ER3:"; ER(3);
```

"ER1:"

$$x_{n+1} - x_n = h \cdot f_{n+1}$$

"ER3:"

$$\frac{9 \cdot x_{n-1} + 11 \cdot x_{n+1} - 2 \cdot x_{n-2} - 18 \cdot x_n}{6} = h \cdot f_{n+1}$$

I-1-4/ Nystrom

Script MUPAD

```
nystrom:=proc(k)
begin
xl:=[t[n]-j*h $j=0..(k-1)];
yl:=[f[n-j] $j=0..(k-1)];
p:=interpolate(xl,yl,s);
x[n+1] = x[n-1] + factor(int(p,s=(t[n]-h)..(t[n]+h)));
end_proc:
```

```
"Nyst2:"; nystrom(2); "Nyst3:"; nystrom(3);"Nyst4:"; nystrom(4);
```

"Nyst2:"

$$x_{n+1} = x_{n-1} + 2 \cdot h \cdot f_n$$

"Nyst3:"

$$x_{n+1} = x_{n-1} + \frac{h \cdot (f_{n-2} - 2 \cdot f_{n-1} + 7 \cdot f_n)}{3}$$

"Nyst4:"

$$x_{n+1} = \frac{h \cdot (4 \cdot f_{n-2} - 5 \cdot f_{n-1} - f_{n-3} + 8 \cdot f_n)}{3} + x_{n-1}$$

I-2/ Erreurs de consistance

I-2-1/ Adams Bashforth

Script MUPAD

```
error_adams_bashforth:=proc(k)
begin
xl:=[t-j*h $j=0..(k-1)];
yl:=[D(x)(t-j*h) $j=0..(k-1)];
p:=interpolate(xl,yl,s);
erro:=x(t+h)-x(t) - factor(int(p,s=t..(t+h))):
return(taylor(erro,h=0,AbsoluteOrder=k+2));
end_proc:
```

Exemple d'utilisation

```
error_adams_bashforth(3)
```

$$\frac{3 \cdot h^4 \cdot x''''(t)}{8} + O(h^5)$$

$$\frac{3 \cdot h^4 \cdot x''''(t)}{8} + O(h^5)$$

I-2-2/ Adams Moulton

Script MUPAD

```
error_adams_moulton:=proc(k)
begin
xl:=[t-j*h $j=-1..(k-2)];
yl:=[D(x)(t-j*h) $j=-1..(k-2)];
p:=interpolate(xl,yl,s);
erro:=x(t+h)-x(t) - factor(int(p,s=t..(t+h))):
return(taylor(erro,h=0,AbsoluteOrder=k+2));
end_proc:
```

```
error_adams_moulton(3)
```

$$-\frac{h^4 \cdot x''''(t)}{24} + O(h^5)$$

I-2-3/ Differentiation retrograde

Script MUPAD

```
error_ER:=proc(k)
begin
xl:=[t-j*h $j=-1..(k-1)];
yl:=[x(t-j*h) $j=-1..(k-1)];
p:=interpolate(xl,yl,s);
erro:=factor(h*limit(diff(p,s),s=t+h))-h*D(x)(t+h);
return(taylor(erro,h=0,AbsoluteOrder=k+2));
end_proc:
```

Exemple d'utilisation

```
error_ER(3)
```

$$-\frac{h^4 \cdot x''''(t)}{4} + O(h^5)$$

```
error_adams_bashforth(4);
```

$$\frac{251 \cdot h^5 \cdot x'''''(t)}{720} + O(h^6)$$