SELF-ASSESSMENT BY JEAN-MICHEL BISMUT (MAI 2025)

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In my work as a mathematician, I have addressed seven areas :

(1) Stochastic optimization.
(2) Malliavin calculus and stochastic mechanics.
(3) The index theorem.
(4) The η invariants, and Ray-Singer analytic torsion, both real and complex.
(5) Hypoelliptic deformations of the Laplacian.
(6) The trace formula.
(7) Bott-Chern cohomology and the theorem of Riemann-Roch.

Before going into details, I would like to mention that probability theory and the calculus of variations play an essential role in my work. I believe that I have worked on only one and the same subject, whose unity only became apparent to me gradually. The fact that the equations I use to describe certain aspects of the trace formula are essentially the same as those I developed when working on the stochastic maximum principle is not a coincidence, but rather a necessity. Only a certain amount of work separates them.

In this summary, I review only some of my work, referring to the complete bibliography for more details.

1. Work on stochastic optimization

In my thesis, I established a maximum principle for stochastic differential equations. If one controls an ordinary differential equation of the type $\dot{x}_t = f(t, x_t, u_t)$ by making extremal a functional of the form $\int_0^T L(t, x_t, u_t)\, dt$, under adequate hypotheses, Pontryagin's maximumprinciple allows us to construct a Hamiltonian $H(t, x, p)$. The extremalization of the considered functional leads either to Euler-Lagrange equations, or to Hamilton equations.

In [Bis73, Bis78d, Bis80], I obtained a Pontryagin maximum principle for Itô stochastic differential equations of the type [1]

$$dx_t = f(t, x_t, u_t)\, dt + \sum_1^m \sigma_i(t, x_t, u_t)\, \delta w^i,$$

when making extremal a functional of the type $E \int_0^T L(t, x_t, u_t)\, dt$, where E denotes the expectation. The Hamilton equations are replaced by stochastic differential equations. The dual variable p is the solution of a stochastic differential equation with terminal condition (as in the deterministic case!), even though it does not anticipate on the Brownian motion filtration. This apparent contradiction caused perfectly correct papers to be blocked for several years.

1. Here, w is a Brownian motion in $\mathbf{R}^m$, and δw is its Itô differential.

I applied this type of results either to linear stochastic differential equations with quadratic criterion in [Bis76a, Bis78b], or to constrained stochastic systems [Bis74]. I also gave applications of the maximum principle in mathematical economics [Bis75]. The idea that one can deform a stochastic process is found throughout my later work, whether devoted to random mechanics, Malliavin calculus, the index theorem, or the theorem of Riemann-Roch .

Another series of works is devoted to variational problems on Markov processes. This led to the paper [Bis76b] devoted to the control of Markovian diffusions (where one controls the "drift" of the process). Reference [Bis78c] represents a synthesis of both viewpoints. I also extended the previous methods to more general processes, including jumps [Bis78a].

The techniques for controlling probability measures associated with Markov processes led me to consider optimal stopping problems. In works conducted partly with Skalli [Bis77a, Bis77b, Bis77c, Bis77d, BS77, Bis79a, Bis79b, Bis79c, Bis79d, Bis81a], and motivated by Rost's characterization [Ros71] of measures associated with stopping times using excessive functions of the considered Markov process, I studied questions of optimal stopping of Markov processes, optimal stopping with control, zero-sum games with optimal stopping, control of alternating processes, with a functional depending on the number and position of transitions. In these problems, I again implemented convex optimization techniques, by defining a dual problem of the initial problem, and by linking the two dual problems through classical extremality conditions.

2. Malliavin calculus and stochastic mechanics

Malliavin [Mal78] had studied the flow of a stochastic differential equation, and developed a differential calculus on the Wiener space, which, applied to stochastic differential equations, allowed to prove analytical results on the heat semigroup of a diffusion using the corresponding stochastic differential equation. Malliavin's ideas had been extended by Stroock [Str81b, Str81a].

In [Bis81c, Bis81d], I showed that Malliavin's integration by parts formula could be considered as a consequence of Girsanov's formula, related to Haussman's representation formula of Brownian motion random variables as stochastic integrals. In work conducted with D. Michel [BM81, BM82], we applied Malliavin calculus to filtering problems. In [Bis81b], I proved an Itô formula for the image of a diffusion by a stochastic flow on $\mathbf{R}^n$. I prove in particular that the flow is indeed a flow of diffeomorphisms of $\mathbf{R}^n$.

The book [Bis82] is devoted to the study of differential calculus associated with stochastic flows. A theory of random cycles associated with stochastic differential equations is developed, and differential forms are integrated over these cycles. The idea that Hamilton equations associated with a classical variational problem can be suitably perturbed by a Brownian motion is introduced, an idea that I would revisit in my work on the hypoelliptic Laplacian [Bis05].

In [Bis84b], I develop a stochastic calculus of variations for jump processes. In [Bis84c, Bis84e], I show how the theory of Brownian excursions allows the use of the time-change invariance properties of Brownian motion to obtain natural integration by parts formulas on jump processes.

In [Bis84d], I show that Malliavin calculus has a deterministic version. I prove an integration by parts formula on the Brownian motion of a Riemannian manifold. Another invariance property of Brownian motion under action of the local orthogonal group is used, which explains the appearance of the Ricci tensor in the formula. The asymptotic expansion of the heat kernel is obtained using the underlying stochastic differential equation.

In [Bis85c], the study of problems with boundary conditions leads to the study of a decomposition of Brownian paths up to a time whose law is the Lebesgue measure. A corresponding decomposition of Brownian excursions is also given, closely related to Williams' decomposition.

3. The Index Theorem

During a conference in honor of Laurent Schwartz in 1983, M.F. Atiyah [Ati85] gave a lecture on Witten's work on the index theorem. Witten had shown that formally, the McKean-Singer formula for the index of a Dirac operator acting on spinors of a Riemannian manifold X can be written in the form of a 'supersymmetric' functional integral over the loop space LX. Equivalently, one integrates over the loop space a differential form that is closed with respect to the operator $d+i_K$, where K is the velocity vector generating the natural action of S^1 on LX. An algebraic localization argument shows that this integral must localize on $X \subset LX$ viewed as the variety of zeros of K. The application of this formula leads directly and without analysis to the "proof" of the index theorem.

This lecture led me to give a probabilistic proof of the index theorem and Lefschetz fixed point formulas [Bis83, Bis84a]. The $\widehat{A}$-genus is obtained using Paul Lévy's area formula [Lév51].

Atiyah's lecture had disturbed me because it showed that there was an algebraic mechanism at work in the Brownian integral, invisible to a probabilist. In [Bis85a], I showed that Atiyah and Witten's considerations apply to all Dirac operators. In particular, one constructs a natural lift of the form on X for the Chern character to an equivariant form on LX. I develop an increasingly precise dictionary for moving from the formalism of operators to the integration of differential forms on loop space.

After attempting to extend known proofs of localization formulas to LX, I show in [Bis86b] that the proof of the index theorem via the heat equation can be considered as the application to LX of a proof that exists universally for the localization formulas of Nicole Berline and Michèle Vergne [BV83]. I show in particular that the mechanism of McKean-Singer's *fantastic cancellations* is not a miracle related to the index theorem, but exists as a universal phenomenon for all localization formulas in equivariant cohomology.

Following a suggestion by Berline and Vergne, in [Bis85b], I give a heat equation proof of delocalized formulas à la Kirillov. I read an article by Quillen [Qui85b], where he proposes a new theory of superconnections, algebraically unifying the formalism of Chern-Weil theory and the formalism of the index theorem. Quillen indicates that this formalism should lead to a proof of the families index theorem via the heat equation. The result of [Bis85b] is precisely such a proof in the equivariant context. In [Bis86a], I give a heat equation proof of the families index theorem by introducing the superconnection, called the Levi-Civita superconnection, naturally associated with a fibration.

The formalism of superconnections lends itself naturally to the calculation of transgression forms. I attend a lecture by Quillen where he proposes a connection between the families index theorem and his proof of a curvature theorem [Qui85a] for the determinant bundle of a family of connections on a bundle over a fixed Riemann surface. With Freed [BF86a, BF86b], we provide a construction by transgression of a metric and a unitary connection on the determinant bundle of a family of Dirac operators, and we give a local curvature theorem for this connection. The Levi-Civita superconnection plays an essential role in the construction. We link the holomorphicity at 0 of the η function of a Dirac operator to the mechanism of *fantastic cancellations*. We prove a holonomy theorem conjectured by Witten, indicating that the holonomy of the previously described connection over a loop in the base is the adiabatic limit of the η invariants of the cylinder constructed over this loop.

In [Bis87b], I connect the Levi-Civita superconnection to my previous results on filtering. In [Bis87a], I give a heat equation proof of Demailly's inequalities [Dem85]. The interest of this proof is that it is parallel to the proof of the index theorem, and that Paul Lévy's formula again plays a crucial role. Naturally, this is not a coincidence.

4. η invariants and Ray-Singer analytic torsion

With Cheeger [BC89], I study the adiabatic limit of η invariants of fibered manifolds. We construct the transgression forms $\widetilde{\eta}$ in the formalism of the families local index theorem. In [BC92], we apply this result to give a new proof of Hirzebruch's conjecture on the signature of Hilbert modular varieties, which had been proven by Atiyah-Donnelly-Singer [ADS83].

In a series of articles with Gillet and Soulé [BGS88a, BGS88b, BGS88c], we prove a curvature theorem for the Quillen metric on the determinant of the direct image of a holomorphic bundle. We incorporate the formalism of the double transgression of Bott and Chern. The main result is obtained by verifying the compatibility of the constructions from [BF86a, BF86b] with complex geometry, and also by proving anomaly formulas extending Polyakov's formulas. This result is by no means a direct consequence of my work with Freed.

Gillet and Soulé's objective is to prove an arithmetic theorem in Arakelov geometry, following the path opened by Grothendieck, by using functoriality properties.

I am very interested in the problem of functoriality by immersion of Quillen metrics, especially since the functional integral reveals the importance of the role of the immersion of X into LX.

With Vasserot [BV89], responding to a question from Miyaoka, I study the asymptotics of the holomorphic analytic torsion of powers of a positive line bundle, using the methods employed in [Bis87a] for proving Demailly's inequalities.

In [Bis90b], I apply the formalism of superconnections to the question of immersions, and prove the convergence of superconnection forms as currents. In [BGS90a, BGS90b], Gillet, Soulé and I construct Bott-Chern currents associated with complex embeddings, and we show their functoriality.

The functional integral indicates to me that the holomorphic analytic torsion can be formally written as the integral of a Bott-Chern current over the space of loops. The study of the behavior of the Quillen metric under immersion can be interpreted geometrically on the loop space as a problem of intersection with excess. These are formal considerations, whose truth will only be confirmed a posteriori by rigorous proofs.

In [Bis90a], I accomplish a step that I believe is decisive in establishing the immersion formula. Gillet and Soulé [GS91] had conjectured the appearance of their genus $R(x)$, whose development reveals derivatives of the Riemann zeta function at odd negative integers. The immersion formula itself was supposed to contain this R genus. The proof scheme that I elaborate to calculate the formula reveals exotic characteristic classes, which express the excess in an intersection formula in infinite dimension. In [Bis90a], the calculation is carried out rigorously and indeed reveals the R genus.

With Bost [BB90], I study the behavior of the Quillen metric associated with a Riemann surface, when it degenerates with ordinary singularities. I revisit this question in [Bis97b] in arbitrary relative dimension.

With Cheeger [BC90a, BC90b, BC91], we prove an index theorem for a family of manifolds with boundary. The contribution of the boundary to the index formula reveals the $\widetilde{\eta}$ forms described previously. This result should be viewed as an extension of results by Atiyah-Patodi-Singer [APS75] and Cheeger [Che83].

In [BL91], extensive work with Lebeau leads us to the proof of the long-sought immersion formula for Quillen metrics. Although the general plan of the proof is simple, its implementation is technically difficult, requiring a mix of functional analysis techniques and local index theory methods. The local objects constructed in [Bis90a, BGS90a, BGS90b] actually appear in the proof, in a formula expressing global secondary objects using local secondary objects. Gillet and Soulé [GS92] will use this result to complete their proof of a Riemann-Roch-Grothendieck theorem in Arakelov geometry.

With Köhler [BK92], I then tackle the proof of anomaly formulas for holomorphic analytic torsion forms.

In [Bis92a], I show an excess formula in K-theory for Bott-Chern currents. In [Bis92b], I show another excess formula for Bott-Chern currents, which applied in infinite dimension, formally reproduces the result proven with Lebeau on the behavior of the Quillen metric under immersion. This text provides some keys to the preparatory work that led to a proof scheme for the immersion formula.

With Zhang [BZ92], we provide an extension of the Cheeger-Müller theorem : this result states that Ray-Singer torsion [RS71] in de Rham theory equals Reidemeister torsion [Rei35], a combinatorial invariant. We formulate the question as comparing two metrics on the determinant of the cohomology of a flat bundle, one constructed analytically, the other by a combinatorial method. We use for this the deformation of the de Rham complex suggested by Witten [Wit82], as well as results by Helffer-Sjöstrand [HS85] and Laudenbach (these are contained in the appendix of [BZ92]). For arbitrary flat bundles, our formula explicitly shows a local defect term in the comparison of the two metrics. In [BZ94], we extend these results in an equivariant context, and we radically simplify our previous proofs, replacing Helffer-Sjöstrand's arguments with geometric considerations inspired by Laudenbach. With Zhang [BZ93], we study the behavior of the η invariant under immersion.

With Berthomieu [BB94], we show a compatibility formula for analytic torsion forms with the composition of two submersions when the base of the second submersion is a point. The proof implements adiabatic limit techniques, the local index theorem, and methods inspired by Dai and Melrose [DM12].

In [Bis94, Bis95], I extend the results obtained for immersions to the equivariant situation. I thus obtain the equivariant extension of the R genus, as well as an immersion formula for the equivariant determinant.

In [BL95], with Lott, I prove a Riemann-Roch-Grothendieck theorem for direct images by submersion of flat bundles. The proof uses the formalism of superconnections. In [BL97], Lott and I apply this result to the case of $\mathrm{SL}(n, \mathbf{Z})$-bundles.

In [Bis97a], I prove the compatibility of holomorphic torsion forms with the composition of an immersion and a submersion.

In work with Labourie [BL99], we give a proof of Verlinde's formulas by applying the Riemann-Roch-Kawasaki formula to the moduli space of flat bundles on a Riemann surface.

With Goette [BG00], I show the compatibility of two versions of the equivariant holomorphic analytic torsion. This result confirms the principle that holomorphic torsion is the integral over loop space of a current that exists universally, even in finite dimension. The formula obtained is merely the manifestation of a functoriality principle for these currents.

With Goette [BG01], we extend the results obtained with Zhang to de Rham analytic torsion forms. In particular, we show results on rigidity of analytic torsion forms. We calculate these forms explicitly under a very strong assumption of the existence of a Morse function in the fibers.

With Goette, in [BG04], we obtain a comparison result for two versions of torsion forms in de Rham theory. The cohomological formalism on loop space is subtle. We obtain a formula expressing the difference between the objects using a new object, the V-invariant. The de Rham analytic torsion itself is formally the V-invariant of the loop space. The V-invariant naturally localizes on the critical points of a Morse-Bott function.

In [Bis04], I show that the equivariant holomorphic torsion forms of the de Rham complex are zero. This result is obtained by comparing these forms to the forms that I had constructed with Lott in de Rham theory. This torsion is indeed a not very explicit term appearing in the fixed point formula of Köhler-Rössler in Arakelov geometry. The fact that this torsion is zero is interesting and has been exploited by Burgos, Freixas and Litcanu [BGFiML14].

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5. Hodge theory and hypoelliptic Laplacian

If we accept the paradigm described previously on the V-invariant, the de Rham analytic torsion should naturally localize on the critical points of any natural functional on loop space, such as the energy functional. These considerations are the starting point of [Bis05], where I construct a deformation of the Hodge Laplacian of a Riemannian manifold X, into a family of hypoelliptic Laplacians on the total space $\mathcal{X}^*$ of the cotangent bundle T^*X, which interpolates between the usual Laplacian and the generator of the geodesic flow. The hypoelliptic Laplacian is essentially a weighted sum of the harmonic oscillator of the fiber and the generator of the geodesic flow on $\mathcal{X}^*$. The random mechanics equations developed in [Bis73, Bis81e] reappear here in a geometric context.

In work conducted with Lebeau [BL08], we prove a series of analytical results on the hypoelliptic Laplacian. An appropriate calculus is developed, which allows us to show that the hypoelliptic Laplacian is indeed a deformation of the usual Hodge

Laplacian. We verify that the analytic torsion of the hypoelliptic Laplacian equals the analytic torsion of the elliptic Laplacian. More precisely, we show that the Ray-Singer metric on the determinant of the cohomology of a flat bundle coincides with the corresponding hypoelliptic metric, which is constructed using the hypoelliptic Laplacian.

In [Bis06], we connect the construction of the hypoelliptic Laplacian in de Rham theory to the Chern-Gauss-Bonnet theorem. In article [Bis08c], we provide several heuristic motivations that all lead to the construction of the hypoelliptic Laplacian from [Bis05].

In [Bis08a], we show that any Dirac operator on a compact manifold X has a hypoelliptic deformation acting on the total space $\mathcal{X}$ of the tangent bundle TX. This deformation is different from the one treated in [Bis05], although at the level of operator symbols, it is of the same nature. For complex Kähler manifolds, it allows us to obtain a deformation of the Hodge Laplacian associated with the Dolbeault complex. In this case, the hypoelliptic deformation involves the Koszul complex of the fiber TX. We relate the hypoelliptic Quillen metric to the elliptic Quillen metric by a formula involving the R genus of Gillet-Soulé.

In [Bis08d], we provide a synthetic presentation of the construction of the hypoelliptic Laplacian in de Rham theory, and also for the Dirac operator.

6. Hypoelliptic Laplacian and trace formula

In [Bis08b], we begin applying the hypoelliptic Laplacian theory to symmetric spaces. This article is devoted to a new derivation of the Poisson formula for the heat kernel on a compact Lie group using a hypoelliptic Laplacian. This work builds on previous work by Frenkel [Fre84] and Atiyah [Ati85].

Kostant's Dirac operator plays a key role in constructing the hypoelliptic Laplacian adapted to the situation under consideration. This operator is distinct from the operators described previously. Its construction has two motivations.

(1) We want to show the connection between the heat kernel on the group and the equivariant cohomology of its loop space, with the hypoelliptic deformation explicitly realizing the proof of the corresponding Berline-Vergne localization formula.

(2) We aim to test the methods that we intend to apply later to symmetric spaces of non-compact type.

In [Bis09, Bis11b], we implement the program described above. Let G be a reductive group with Lie algebra $\mathfrak{g}$, let $K \subset G$ be a maximal compact subgroup, and let $X = G/K$ be the corresponding symmetric space. Let $\gamma \in G$ be a semi-simple element, and let $\rho : K \to \mathrm{Aut}\,(E)$ be a unitary representation of K, such that E descends to a Hermitian bundle F on X. In [Bis09, Bis11b], we give an explicit formula for the orbital integral associated with $\exp\left(t\Delta^X/2\right)$, where $-\Delta^X$ is the action of the Casimir on $C^\infty(X, F)$. More generally, if $\mu : \mathbf{R} \to \mathbf{R}$ is an even function with rapid decay whose Fourier transform has Gaussian decay, we obtain a corresponding formula for the orbital integral associated with $\mu\left(\sqrt{-\Delta^X + c}\right)$. We again use Kostant's Dirac operator. Here, the hypoelliptic Laplacian acts on $G \times_K \mathfrak{g} \simeq X \times \mathfrak{g}$. The proofs essentially use probabilistic techniques from Malliavin calculus, and also a systematic application of Toponogov's theorem, in order to quantitatively control the convergence of the hypoelliptic diffusion toward the geodesic flow, when the hypoelliptic deformation parameter b tends to $+\infty$. The formulas in [Bis11b] resemble, at least formally, the Atiyah-Singer index formulas. The role of the $\widehat{A}$-genus is played here by a function $J_\gamma\left(Y_0^k\right)$ defined on the compact part $\mathfrak{k}(\gamma)$ of the Lie algebra $\mathfrak{z}(\gamma)$ of the centralizer $Z(\gamma)$.

In [Bis11a, Bis12], we present the set of ideas that connect index, integration over loop space, localization formulas, and hypoelliptic Laplacian. These two articles provide a non-technical approach to the ideas described previously.

Work by Bergeron and Venkatesh [BV13] on one hand, and by Müller [Mül12] on the other, has shown the interest in studying the asymptotics of analytic torsion in de Rham theory, either when ascending the tower of coverings of the considered manifold, or when suitably increasing the flat bundle toward "infinity." With Ma and Zhang [BMZ11, BMZ17], we provide a calculation of the asymptotics of analytic torsion of a compact manifold, when the flat bundles are holomorphic direct images F_p of powers L^p of a positive line bundle L along the complex fibers of a flat fibration above the considered manifold. Under a strong non-flatness assumption of a metric, we calculate the asymptotic torsion. When the manifold is a locally symmetric space, we recover these results using the trace formulas from [Bis11b].

In [Bis19], we apply the methods of the hypoelliptic Laplacian to reproduce results by Moscovici and Stanton [MS89] on calculating eta invariants of Dirac operators on locally symmetric spaces. Such an extension was not straightforward. In [Bis11b], we had only treated orbital integrals associated with the Casimir operator. Here, we need to involve the classical Dirac operator, whose square coincides with the Casimir up to a constant operator. To resolve this difficulty, Quillen's superconnection formalism applies again. We had indeed shown that the eta invariant naturally writes as a transgression in the superconnection formalism. In the context of the trace formula, this formalism can again be used, but with a different meaning. The hypoelliptic deformation now has two parameters, the original parameter $b > 0$, and the parameter ϑ which expresses a rotation in the Clifford algebra.

In a paper with Shen [BS22, BS19], we extend the formulas from [Bis11b] to all natural kernels on a symmetric space associated with the center of the enveloping algebra. Our results take as a starting point the formulas from [Bis11b] where only the Casimir appears. We use deformation methods inspired by Harish-Chandra to evaluate the semi-simple orbital integrals when γ is regular, then limit arguments when γ is semi-simple without necessarily being regular. The hidden symmetries of the function $J_\gamma\left(Y_0^\gamma\right)$ play a role in the proofs. Indeed, the function J_γ can be expressed simply using the imaginary roots.

7. Bott-Chern cohomology and the Riemann-Roch theorem

Let $\pi : M \to S$ be a proper holomorphic projection, let F be a holomorphic bundle on M, and let $R^\cdot\pi_*F$ be its derived direct image, which we assume is locally free. Let $H_{\mathrm{BC}}^{(=)}(S, \mathbf{C})$ be the Bott-Chern cohomology of S, which is the quotient of the space of closed forms that are sums of forms of type (p,p) by the image of $\overline{\partial}\partial$. In [Bis11c, Bis13], we prove the expected Grothendieck-Riemann-Roch theorem in $H_{\mathrm{BC}}^{(=)}(S, \mathbf{C})$, without any other hypothesis. In the case where the varieties are projective, or even Kähler, this result was already known. In the general case, traditional analytical proofs fail for fundamental reasons. More specifically, generally, the "extraordinary vanishings" that we had used with Gillet and Soulé in [BGS88a, BGS88b] no longer occur. Moreover, the hypoelliptic deformation of the

Dolbeault-Hodge complex that we had introduced in [Bis08a] does not allow us to obtain the result. In [Bis11c, Bis13], we construct an exotic hypoelliptic deformation of Hodge theory for the Dolbeault complex. In the corresponding hypoelliptic Laplacian, the traditional quadratic potential associated with the harmonic oscillator of the fiber is replaced by a potential of order 4. For this new Laplacian, we can show that the "extraordinary vanishings" still occur, which allows us to obtain the result.

In work with Shen and Wei [BSW21], we remove the restrictions imposed by the results of [Bis13]. The article [BSW21] contains two significant advances.

(1) The first is the construction of a Chern character defined on $K_{\cdot}(X)$, the Grothendieck group of coherent sheaves on X, which takes values in $H_{\mathrm{BC}}^{(=)}(X, \mathbf{R})$. We use a category equivalence proven by Block [Blo10] between the derived category $D^{\mathrm{b}}_{\mathrm{coh}}(X)$, and the category of antiholomorphic superconnections. We can then develop a Chern-Weil theory for Block's superconnections, and obtain a Chern character for coherent sheaves.

(2) The proof of the Grothendieck-Riemann-Roch theorem for antiholomorphic superconnections. For immersions, we use the deformation to the normal cone ; for submersions, we use the hypoelliptic Laplacian as in [Bis13].

8. Fried sections

In work with Shen [BS24], we give a very general formulation of Fried's conjecture [Fri87]. Let Z be an Anosov vector field on a compact connected manifold Y, and let F be a complex flat vector bundle on Y equipped with a flat metric. We can define an associated dynamical zeta function, following Ruelle's methods. Fried's conjecture predicts that this function has a meromorphic extension to the entire complex plane, and moreover that if the function is well-defined at 0, the modulus of its value at 0 equals the corresponding Reidemeister or Ray-Singer torsion.

In the article, we construct a canonical non-zero section $\tau(i_Z)$ of $\det H(Y, F)$. When there is no resonance (which means that L_Z is invertible), this section can be identified with the value at 0 of Fried's zeta function. The article uses the techniques of Faure-Sjöstrand [FS11] and Dyatlov-Zworski [DZ19] related to the spectral theory of Anosov vector fields.

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