Cohesive functions: the Great Divide.

Parallel to the *acceleration transforms* $\zeta_1 \to \zeta_2$, which mirror, on the convolutive side, "strong" variable changes $z_1 \to z_2$ with $z_2/z_1 \to +\infty$, we have (going in the opposite direction but similar in their regularizing effect!) *pseudo-deceleration transforms* $\zeta_1 \to \zeta_{1\_}$ which mirror "weak" variable changes $z_1 \to z_{1\_}$ with $z_1 - z_{1\_} = o(z_1) > 0$. Both transforms:

$$\begin{aligned} &\textit{acceleration}: & \widehat{\varphi}_2(\zeta_2) &= \int_{+0}^{+\infty} C_F(\zeta_2,\zeta_1)\,\widehat{\varphi}_1(\zeta_1)\,d\zeta_1 \\ &\textit{pseudo-deceleration}: & \widehat{\varphi}_{1\_}(\zeta_{1\_}) &= \int_{+0}^{\zeta_1} C_{id+F}(\zeta_{1\_},\zeta_1)\,\widehat{\varphi}_1(\zeta_1)\,d\zeta_1 \end{aligned}$$

essentially make use of the same kernels:

$$C_F(\zeta_2,\zeta_1) := \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{z_2\zeta_2 - z_1\zeta_1}\,dz_2 \quad \textit{with} \;\; z_1 \equiv F(z_2) \prec z_2$$

$$C_{id+F}(\zeta_{1\_},\zeta_1) := C_F(\zeta_{1\_}-\zeta_1,\zeta_1)$$

Moreover, these unlikely twins in back-to-back posture have the distinction of illuminating what is arguably the central dichotomy in real Analysis, to wit the *cohesive/loose* divide.

The class *COHES* of cohesive functions is defined as the limit of all Denjoy classes DEN_α for $\alpha \uparrow \omega^\omega$ (whereas Denjoy considered only finite integer values of α). Like the analytic sort, cohesive functions are "of one piece"; they cover all quasi-analytic classes liable to arise naturally in Analysis; and they enjoy stability properties totally lacking in Carleman's or Mandelbrojt's quasi-analytic classes.

The divide between *cohesive* and *loose* (i.e. non-cohesive) is a brutal, unbridgeable chasm; an unremovable discontinuity cutting right across Analysis. Yet it finds an unexpected reflection in these two statements:

- Whatever the nature of $\widehat{\varphi}(\zeta_1)$, the accelerate $\widehat{\varphi}(\zeta_2)$ is automatically cohesive.

- Whatever the nature of $\widehat{\varphi}(\zeta_1)$, a suitable choice of pseudo-deceleration can render $\widehat{\varphi}_{1\_}(\zeta_{1\_})$ as smooth as one wishes – short of cohesive!

Both (i) and (ii) admit reciprocal statements, leading in particular to an elegant and universal procedure for cohesive continuation.