The new methods at work: the finiteness theoem for limit-cycles.

The impetus behind the introduction of transseries and analyzable germs came from the so-called Dulac problem. Given a polynomial vector field on $\mathbb{R}^2$, proving the finiteness of isolated cycles reduces to proving the existence of only finitely many isolated fixed points for the return map $T(x)$ attached to any given polycycle $\mathcal{C}$. Once $T(x)$ has been formalized to an (intrinsically non-oscillating) transseries $\widetilde{T}(x)$, the property becomes self-evident.